10 What Ails Mathematics Teaching?

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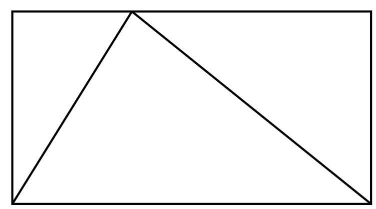
Ask children in school which subject they dislike the most; chances are that 9 out of 10 will say Mathematics. In fact, if you probe a little deeper, you will discover that 7 out of 10 children say they are 'terrified' or 'mortally scared' of the subject. Why children, ask any adult and you will find a similar pattern. I spoke to a random sample of my friends inside and outside the Azim Premji Foundation and asked them 'what words come to your mind when I say Mathematics?' Barring a microscopic minority, the descriptors were very negative – fear of failure, very difficult, getting beaten in school, incomprehensible, no link to life, formulae, memorization, I am stupid since I don't know Math, its only for the intellectual, dry, boring and so on. One response summed it all up – 'Oh my God! Many of the tragic cases of children taking the extreme step after their exams can be traced to Mathematics. The problem is often not limited to that subject alone but can actually result in the child ending up with an aversion to the entire process of 'schooling'.

The reasons for this are not too far to seek; the way the subject is handled in schools by our teachers is a big contributor. In a study carried out among primary school teachers, it was found that most of them are from an 'Arts' background and Math has been one of their weakest subjects too. When they themselves are fearful of the subject, it is but natural that they transmit the same to the children. Unfortunately, the focus of Math teaching is on definitions, memorization, recall, calculation. There is also a lot of effort in providing the 'right context' to the problems and making it 'useful' in real life.

Some where, at the back of the minds of many people, Math is closely associated with and seen to be similar to 'Science'. While Mathematics is certainly used in science, the two are starkly different. Science is grounded strongly in experiments while Mathematics is imaginary and abstract. You require no special equipment or laboratory to do it. If it is handled well, it can be extremely enjoyable. Paul Lockhart in his article 'A Mathematicians Lament' says "Mathematics is an art and it is just that our culture does not recognize it as such. The fact is that there is nothing as dreamy and poetic, subversive and psychedelic as Mathematics. It is as mind blowing as Cosmology or Physics (mathematicians conceived of black holes long before astronomers actually found any). We, as a culture do not know what Mathematics is. The impression we are given is of something very cold and highly technical, that no one could possibly understand – a self fulfilling prophecy if there ever was one."

According to Lockhart, there is no ulterior practical purpose in Mathematics. It is just playing, wondering and amusing yourself with your imagination. He gives a beautiful example with a figure. He says "imagine a triangle inside a box:

Figure 1

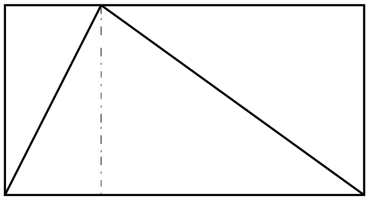


Source: A Mathematicians Lament, by Paul Lockhart

He then goes on to ask "does the triangle take up two – thirds of the box? Imagination is the only way to get at the truth. In this case, one way is to cut the box into two pieces like in figure 2.

> *The idea should be to make the subject a journey of enquiry, discovery and excitement. It is all about patterns and finding ways to look at and explore them, make mistakes, asking oneself further questions, looking for more answers and letting your mind traverse unexplored territory.*

Figure 2



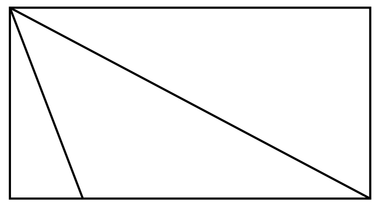
Now, one can see that each piece is a rectangle cut into two by the sides of the triangle. So, there is as much space inside the triangle as outside. That means the triangle takes up exactly half the box. Now, how did the idea of drawing the line come? It is inspiration, experience, trial and error, dumb luck. That is the art of it. The relationship between the two shapes was a mystery till the line made it obvious. I couldn't see, then all of a sudden I could. Somehow, I was able to create a profound simple beauty out of nothing. Isn't that what art is all about?"

Isn't doing this so much more fun than asking children to memorise - Area of a triangle is equal to half its base times its height - and applying it over and over again. Lockhart is not objecting to formulas or to memorizing interesting facts in some context. He says "what is critical is the process of generating options and how that might inspire other beautiful ideas and lead to creative breakthroughs in other problems". He goes on to add "Mathematics is the art of explanation. If you deny students the opportunity to engage in this activity - to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs - you deny them Mathematics itself. Students are not aliens. They respond to beauty and pattern, and are naturally curious like anyone else. Just talk to them! And more importantly, listen to them!"

So do our Mathematics teachers give children the time to make discoveries, conjectures, or choose engaging problems for them? Are they (teachers) creating an atmosphere of healthy discussion, enquiry and provide space for curiosity to be satisfied? Lockhart says "I doubt that most teachers even want to have such a relationship with the children. It is far easier to use material in some book and follow the 'lecture, test and repeat' method – the path of least effort. If adding fractions is to the teacher an arbitrary set of rules, and not the outcome of a creative process, then of course it will feel that way to the poor students. Teaching is about having an honest intellectual relationship with your students. It requires no method, no tools, and no training".

Now, take the above example of the triangle in the box. What if the triangle was slanted? How does one draw the line? What can be done?

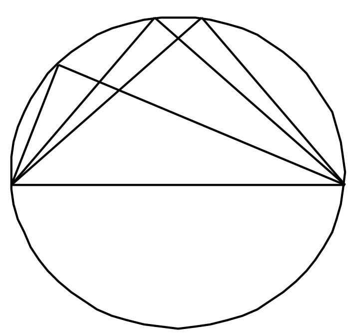
Figure 3



Go ahead and try various possibilities and discover. That is what Mathematics is all about.

Geometry in our schools is made very boring and is invariably reduced to a sequence of – Theorem - - - Proof - - - Rider - - - - solution and then one more Rider. Lockhart has some scathing comments to make about high school Geometry. "The student – victim is first stunned and paralyzed by an onslaught of pointless definitions, propositions and notations and is then slowly and painstakingly weaned away from any natural curiosity or intuition about shapes and their patterns by a systematic indoctrination into the stilted language and artificial format of so-called formal geometric proof, The geometry class is, by far the most mentally and emotionally destructive component of the entire K-12 Mathematics curriculum". He gives a beautiful example of a triangle inside a semicircle and the fact that no matter where you place the tip of the triangle on the circle, it always forms a right angle.

Figure 4



At first sight this seems unlikely. The question then to ask is how can this be true? Here is an opportunity to let students explore this and attempt to find out why this could be so? Or maybe even try and see if this is not true? But all we do is give them a standard 'proof' which they have to remember.

In Geometry, how about asking the students to find out what is the minimum number of colours needed to shade a map so that no two adjacent states have the same colour? Can you think of any special shapes which may require more than these? What special shapes can be managed to be coloured by less number of shades?

As another example, many jig saw puzzles with geometric shapes can be created and that can be a great source of joy and learning for the children just by playing around, turning the pieces around, discovering different shapes in the process and so on.

Children (and for that matter adults) are thrilled to make discoveries. The discovery is all the more exciting if one stumbles upon it accidentally. We get our children to memorise tables. How much more fun it would be for the child to explore patterns in numbers? How exciting would it be to let the child discover this herself?

Take the case of the number 9.

9 x 1 = 9	0 + 9 = 9
9 x 2 = 18	1 + 8 = 9
9 x 3 = 27	2 + 7 = 9
9 X 4 = 36	3 + 6 = 9
.....	
.....	And so on.

Now, ask the children to discover interesting facts about other numbers like 3, 5, 11, 15 ....

You would have seen lot of people these days on buses and trains busy writing, correcting, rewriting numbers on the newspaper, all the time chewing the pencil. They are at Sudoku, the latest craze. There is no practical relevance or use of it other than fun. The simpler forms of this – magic squares - can be given to children to play with. The simplest is a 3 X 3 grid which needs to be filled up with numbers from 1 – 9 so that the total of each row, each column and each diagonal totals to 15. It is possible to construct several such magic squares of larger size.

Magic Squares

Magic Squares have fascinated Mathematicians from the ancient times. The concept seems to have originated in (where else) China around 2800 BC. In India, references to this idea are seen in 11th or 12th century and some examples have been found in the ancient town of Khajuraho.

A magic square is defined as an arrangement of sequential numbers in a grid, starting in such a manner that each number appears exactly once and the sum of the numbers in each row, each column and each main diagonal is the same. Numbers in many magic squares start with 1. These are called simple magic squares.

The smallest (and most trivial) magic square is a 1 X 1 grid.

The next larger magic square is a 3 X 3 grid with numbers from 1 to 9 in it. The total of each row, column and main diagonal in this case is 15 as shown below. **(It is not possible to have a 2 X 2 magic square. Can you say why?).**

4	9	2
3	5	7
8	1	6

From one magic square, it is possible to create multiple squares by transposing the rows and columns.

As we go to larger squares, the interesting possibilities and variations increase. A 4 X 4 square has the total of 34 and can be designed to have a special property where the four corner cells also add up to the same total (shown in the article). There are other interesting variations in larger squares like 'Multiplication Magic Square' where the multiplication of cells in each row, each column and each main diagonal is identical. Yet another special square is one where the addition is the same **and** the multiplication is the same for each row, column and main diagonal.

One other variation is an Anti Magic Square. This is a grid where the cells are filled up with numbers sequentially from 1 onwards but the totals of each row, each column and each main diagonal are **all different and are in a serial sequence**. See the 4 X 4 square below.

15	2	12	4
1	14	10	5
8	9	3	16
11	13	6	7

Can you check out the totals to be in a serial sequence for this?

As per the above definition, it is not possible to have an Anti Magic square smaller than 4 X 4. Some consider the following to be an Anti Magic square. However, it does not satisfy the condition that the totals are in a serial sequence.

7	6	5
8	9	4
1	2	3

You can figure out that this square has been created by starting with the lower left corner moving in an anti-clockwise spiral.

Another variation of the magic squares is the 'Latin Square'. Here, the numbers in the cells are repeated but not in the same row or the same column. The simplest, of course is the 2 X 2 square.

1	2
2	1

.

Can you try and create the next simplest level of a 3 X 3 Latin Square? It is not too difficult. You would have realized that the 'Sudoku' is a 9 X 9 Latin Square.

There are several other amazing magic figures – cubes, circles, stars besides a large number of innovative variations.

One of my favorite set of magic squares is the Alpha Magic square. Look at the squares below:

5	22	18
28	15	2
12	8	25

4	9	8
11	7	3
6	5	10

The contents of the square on the right are derived from the contents of the square on the left. Can you figure out the connection? (Hint: write the contents of the square on the left in English)

The beauty of the magic square lies in discovering patterns in the square and looking for alternate solutions from a given solution. Look at the interesting example below which has 16 cells with numbers from 1 to 16.

1	8	12	13
14	11	7	2
15	10	6	3
4	5	9	16

Here, you can see that each row, each column and each main diagonal add up to 34. Do you notice some other patterns?

If you observe closely, you will find that the four upper right hand corner cell numbers (13, 12, 7 and 2) also add up to 34; and for that matter so do the numbers in the four cells at each corner. There are several other cells in the grid which also add up to 34. Can you find them? What other solutions for this exist? Will every grid with even number of cells exhibit the same corner property?

Isn't this a more interesting and fun way to teach Math? There are several other ways to make Mathematics learning an enjoyable process. It is not my attempt to say that every single thing in Mathematics can and should be taught through this route. The idea should be to make the subject a journey of enquiry, discovery and excitement. It is all about patterns and finding ways to look at and explore them, make mistakes, asking oneself further questions, looking for more answers and letting your mind traverse unexplored territory. The process should become more important than the result. The aim should be to take the 'fear' out of Mathematics. This could just lead to the children starting to enjoy not just that subject but the entire process of schooling since the 'scary' part has vanished. The process of course needs to begin with our teachers who themselves need to re-learn the art of 'facilitating' the exploration of beauty in Mathematics.

This article is inspired by and largely based on the article 'A Mathematician's Lament' by Paul Lockhart. The original article can be accessed at http://www.maa.org/devlin/LockhartsLament.pdf. This is a 'must read' for all who dislike Math and can not be missed by all who love the subject.

References:

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