



Traditional patterns of Math education in schools have largely remained unchanged for almost a century now, even when other things like our culture, beliefs and way of life have tremendously changed. Till date, Math learning in schools is synonymous with rigorous mundane practice. Teaching in schools has lacked focus especially in primary classes. Math has largely been a confusing subject with teachers themselves not knowing what Mathematics actually is; they remain blissfully unaware of the pedagogical issues relating to the subject. Teachers are ill prepared because of this lack of understanding and as a result the confusion continues and prevails throughout the students' school days. It is a paradox that even high scorers in school Math are unprepared for college level Math. Mushrooming tuition centers stand a testimony to this fact. This lack of understanding has profound effect even in the tasks that a person does as a common man, i.e. tasks like finding the percentage or calculating taxes. This leads to the discussion is exam Math indicative of mathematical understanding?

NCTM (National Council of Teachers of Mathematics) has identified two categories as standards for learning Math. The standards are (a) thinking Math standard (b) content Math standard. The 'thinking Math' standard focuses on the nature of mathematical thinking (i.e.) problem solving, communication, reasoning and connections. The 'content Math' standard consists of specific Math topics such as number sense, estimations, geometry, measurement, statistics, probability, fractions, patterns and relationships etc. While content of Mathematics is very important, it is equally important that all of the 'thinking Math' areas be woven into the content. The whole experience of learning Math at school should turn out to be one fabric for the learner in which students can paint their experience to discover new ideas and develop mathematical thinking. Mathematical skills are often cumulative in nature, one skill building upon the previously learned one. For eg algebraic concepts cannot be imbibed without proper understanding of basic arithmetic. This being true lack of understanding of basic concepts has a far reaching and cumulative effect on students. It is essential that primary students have clear understanding of the basic concepts and not just memorize formulas and facts at the dictate of a teacher.

Experiential learning provides concrete experience and ensures maximum student involvement in the process. This type of learning focuses on understanding and enhancement of critical thinking, as students learn to plan, act, discuss, communicate and conclude by themselves. This is especially true of Mathematics, as the subject is abstract.

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One practical way of providing this experiential learning to students is the use of 'manipulatives'. Manipulatives are objects that can be touched and moved by students. These can be used to either introduce or reinforce particular Math content. While use of manipulatives will ensure active involvement of students, it is the teachers' role to design activities which will help in delivering the content and also achieve requisite levels of 'thinking Math'. Research suggests that manipulatives are especially helpful in learning Math, as they help the learner move from concrete to abstract levels. Class activities should be carefully designed and structured such that they form a bridge between the concrete and the abstract. It is very important to note that just the use of manipulatives will not produce the desired outcome unless experiences using the

manipulatives have been carefully designed. This brings us to the practical question of why, when, what, how and with whom manipulative material should be used. While it is beyond the scope of this article to discuss all the questions asked above, it is certain that a teacher can ask the question to oneself and answers are not difficult to find or arrive at.

We shall discuss experiential learning in the perspective of optimal use in class rooms. The experiences using manipulatives should be such that students draw their own conclusions and achieve mathematical insight. The approach should be explorative and teachers should remain guides or facilitators, permitting students at times to even attempt tasks that seem to be ridiculous or non productive. To take the discussion further, let us take a look at what typically happens in a class room.

Situation 1: Introduction of number sequence using links – links being used as external representation

In a typical class, the teacher assumes the role of a ring master and gives step wise instruction to the class with little room for any exploration. Instructions like “take one red link, put it on the table. Take two red links, join them and lay them next to one” and “do as I say”. The activity gets completed with the teacher having satisfied herself of delivering the content, but, for the student, the activity is abrupt and its for sure the student has not imbibed any of the 'thinking Math' content.

A sensitive teacher would put the manipulatives to better use by withholding instructions like 'use red links' or 'put two links next to one' and so on. This teacher would give minimum instruction and allow students to internalize the concept by allowing colour choices or mixture of colours, while remaining focused that students need to make strings of various lengths ,arrange them and discover for themselves the number sequence and arrive at 'one more gives the next number' .

Situation 2: Simple addition – counters being used as representation of computational algorithm

When a teacher attempts teaching addition (eg. $3 + 2$ makes 5), students pick 3 red counters and 2 white counters and count all of them to arrive at the answer 5. In this activity, manipulatives were used as representations.

Such experiences are a very good starting point; but, soon the teacher should move on to the next level where manipulatives are a reference point to student's 'thinking ways' to solve mathematical problems. Students should explore and find for themselves the commutative or the additive identity (addition of zero) property of addition. Teachers as facilitators should only help in students knowing the symbols and communication associated with it.

Situation 3: Word problem – place value manipulatives used as external representations of thinking process

Given word problem: A school has 156 boys and 212 girls. How many students in all?

To solve this problem, students use 'place value manipulatives (The place value manipulative has small squares representing 1s, strips as 10s and big squares of 10s strips bundled together as 100s). Here, the manipulatives are used as an external representation of a thinking process or sequence (i.e.) students imagine the manipulative to be external representation of her thinking process and not use it just for doing the operation. This means that students know what they have to do and use the materials only as a support to arrive at the answer. These experiences are fine but lack mathematical insight. Mathematical insights have to be one's own realization.

Situation 4: Tables – sticks placed in crisscross pattern used as external representation of a computational algorithm

Students use 'tables manipulative' (sticks placed in crisscross patterns to learn tables). This experience will remain as an activity where the manipulatives are used as an external representation of a computational algorithm unless students are made to realize the mathematical idea that multiplication is a short form of repetitive addition and also see how numbers grow while multiplying (i.e.) be able to visualize that 6×4 is $6+6+6+6$ and also how big 6 would grow into. In such experiences, the manipulatives are used to support structural elements of the concepts.

Again, this is an incomplete mathematical perspective. Experiences which encourage discussions and explanations leading to deductions are the best way to support Math

learning. (Note: I am tempted to share my experience with a class of young gypsy students, all school drop outs in a school run by a NGO in Kancheepuram. I was stunned to find a student in my class explaining to his mate what they were doing in the class as 'tables'. He explained saying multiplication is just the same as selling chains. He went on to explain – “how do we find the cost of the articles? Let us say if we sell –one chain which costs Rs. 6, then, 4 chains would cost Rs. 24. This comment is significant considering the fact that the students were still grappling with numerals and had no exposure to Math symbols like \times or \div)

Situation 5: Understanding area – square tiles used as external representation of thinking process and later as a reference point for discussion and exploration

The manipulatives are used by the teacher to design and experience where by arranging tiles into square and rectangles, the students deduct $L \times B = \text{Area}$. This is the bear minimal learning that this experience can offer. This experience attains significance only when students are able to realize that not only square of areas 4, 9 or 16 square units can be formed, but, squares of any area is possible (provided the manipulatives are not a limitation). In this experience, manipulatives are not only used to support conceptual understanding, but, also share the understanding. The discussion that the class has should lead the class to deduce that it is possible to have rectangles of different perimeter with area being constant. Arranging and re-arranging and after several attempts,

the students may also arrive at the hidden agenda that for a given area, square shapes are most compact with least perimeter. This would be optimum use of the manipulatives.

From all the above examples cited, it is clear that experiential learning is not just synonymous to an activity using manipulatives. Teachers have to take the students through a series of activities where initially manipulatives are used as representations and then as external representations of a thinking process and later as a reference point for discussion and exploration. It is to be noted that initially, the experiential learning focuses on individual students and gradually shifts to the class as a whole. The way in which a manipulative is used rests wholly on the teacher initially and it is highly teacher dependent. This gradually should become the complete responsibility of students. This is when 'thinking Math' gets incorporated into 'content Math'.

Experiential learning especially in primary classes with manipulatives is good. But, that is only the beginning, just one time use of the manipulatives or using it to show case it once with lot of instructions and unfocussed writing practice will not help in achieving 'thinking Math' standards.

Careful choice of manipulatives combined with intelligent planning by the teacher with optimal instructions, and discussions that follow after using the manipulatives are sure to work wonders with young learners leading to intellectual and independent learning with desired outcomes.

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