

Azim Premji University

At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS



MATH TALK IN THE CLASSROOM

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MATH TALK IN THE CLASSROOM

Keywords: *pedagogical strategies, discussion, reasoning, communication, inclusion.*

Does Math Talk have a place in the teaching of mathematics?

To answer the above question, let us ask ourselves a few more questions. Does a classroom where the teacher asks questions that can be answered with a simple 'yes' or 'no' provide scope for students to share their true understanding of knowledge? Does it throw light on their misconceptions? Is there an opportunity for students to learn from one another? Does it expose students as to how their peers look at the question? The answer is obviously 'no'.

Here are a few important ways in which a student can benefit from a classroom engaged in math talk.

Students will:

- Learn to reason mathematically and articulate clearly, justifying their answers
- Develop self-reliance in testing the correctness of their reasoning
- Engage in stimulating peer interactions and collaborative work
- Develop a deeper appreciation of concepts and processes
- Retain their learning due to this more in-depth engagement

Is math talk possible at the primary level? Are these objectives too ambitious for students who are being introduced to mathematics? If the approach to teaching mathematics in the primary classroom is top-down, teacher-centric, and authoritarian, then it is highly unlikely that children can switch to a different approach in middle school. This may mean that they never develop the confidence to reason, make mistakes, check their own work and learn to try different methods.

Right from Class 1 onwards, teachers can encourage students to demonstrate using objects and informal conversation why 2 and 3 make 5, why 7 is more than 4, etc.

They can also pose questions to get students to express their thinking.

In how many ways can this blank be filled? Why do you think so?

8 > _

Can you show using counters that 2×4 is the same as 4×2 ?

Why should multiplying a number by 4 give the same answer as multiplying the same number by 2 and then multiplying the answer you got again by 2?

Many teachers may feel diffident about trying such a format in their classes due to a fear of loss of control over the class, or lack of conviction in such a process as a way of learning. Also, there is a lack of clarity on how to proceed. However, it is possible to conduct classes in which meaningful math talk happens. An illustration of such a discussion from the NCERT book, Class 5, is shown in Figure 1. The discussions listed in this article portray such scenarios.

Many Ways to Multiply

Do you think they are all correct? Why do you think so?

What is 18 × 5



First, I doubled 18 to get 36. Then I doubled 36 to get 72 and then I added 18 to 72 to get 90.

Half of 18 is 9. 9×5 is 45 and 9×5 is 45. I added 45 and 45 together to get 90.



$18 \times 5 = 9 \times 10$. So, 90.

I separated 18 into 8 and 10. 8×5 is 40. 10×5 is 50. then I added 40 and 50 together to get 90.

I did 20×5 , which is 100. Then I took away 2×5 , which is 10. So, $100 - 10 = 90$.

Figure 1: Reproduced with permission from NCERT Class 5 textbook.

What can a teacher do to create such spaces and support productive math talk?

In this article, we will explore a few discussions centred on questions that lend themselves to math talk among peers. These discussions can be organised in different ways.

The teacher can get pairs of students to work together on a problem. They may solve it independently but check with each other if they got the same answer. Or the teacher can get students to work in groups of four sitting around a table, using hands-on material when required. The teacher's prompts will help the students to develop the ability to work together by dividing up the work among themselves.



Figure 2

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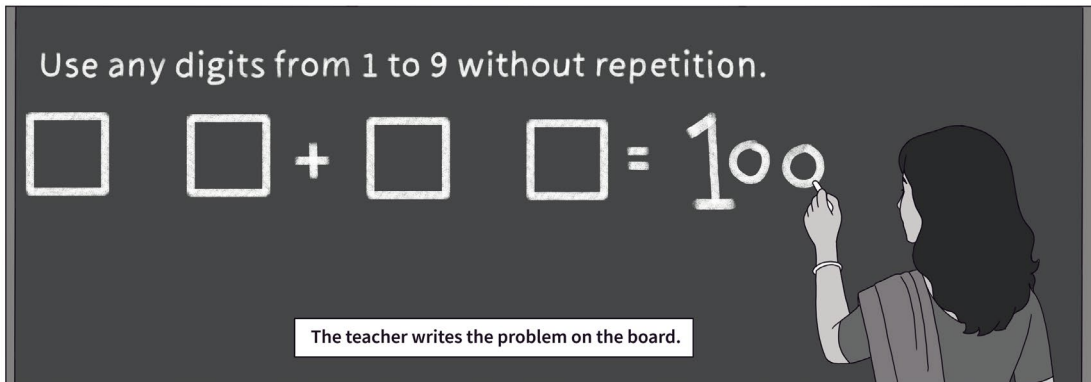
It is good to establish some norms for such a discussion, such as asking them to speak in turns, to listen fully before talking, and so on. This will ensure equal opportunity for all the students to share their thinking and engage in careful listening.

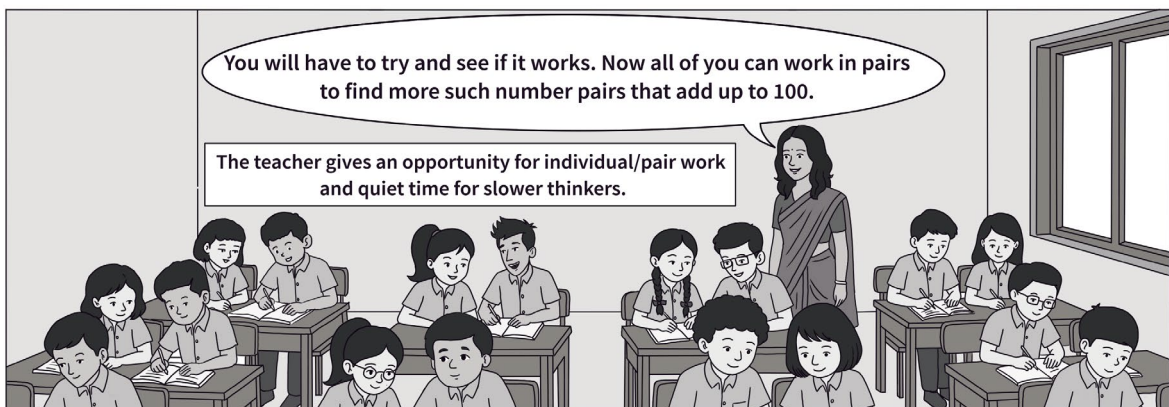
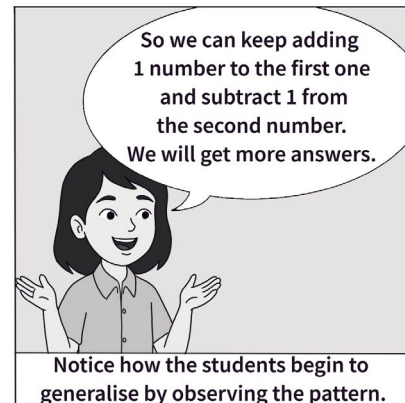
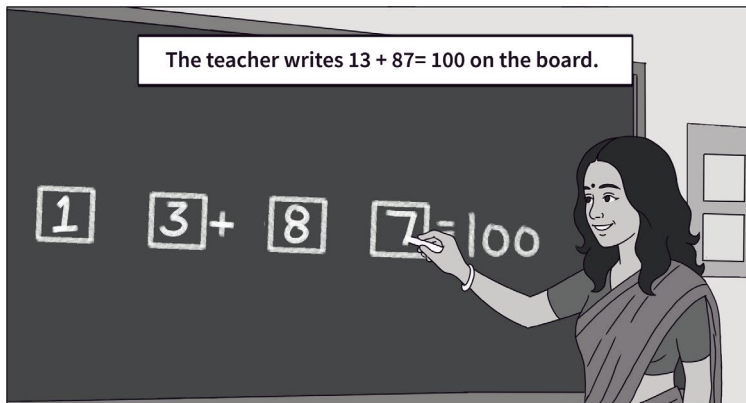
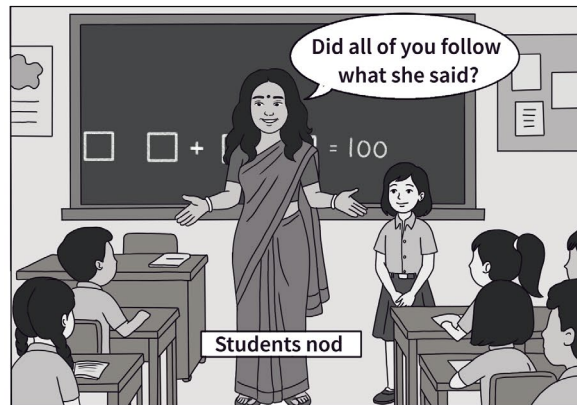
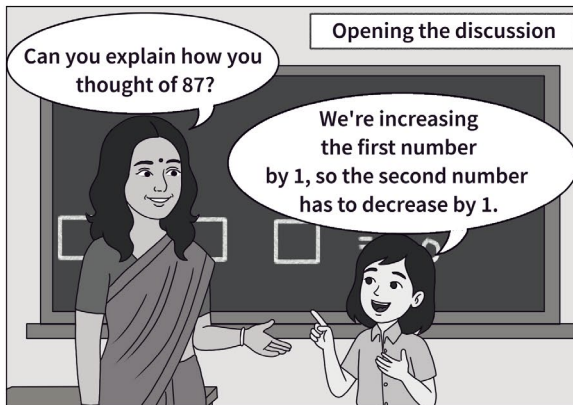
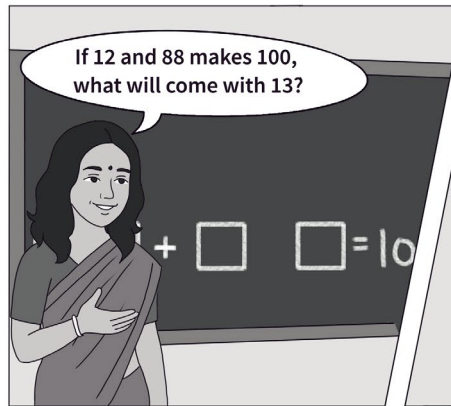
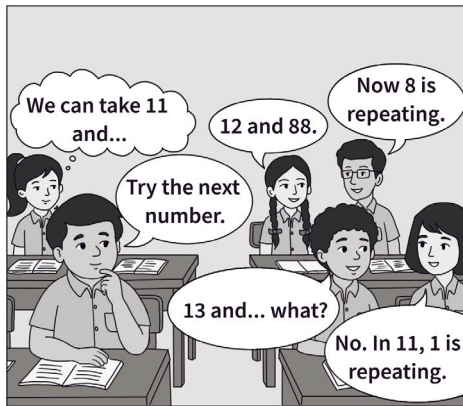
The teacher should pose questions such as 'Can you repeat what he said?' or 'What did she mean by that?'. This will develop among the students the skills of rephrasing another student's reasoning and justifying their answers.

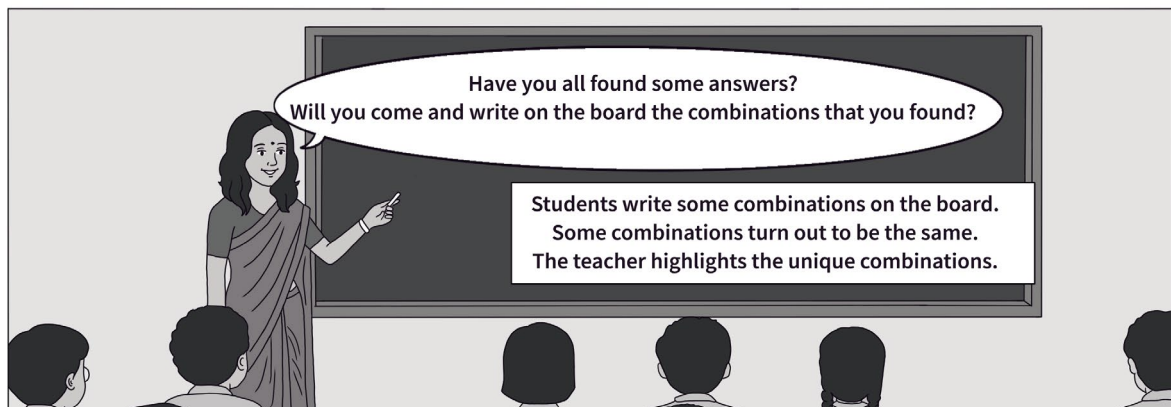
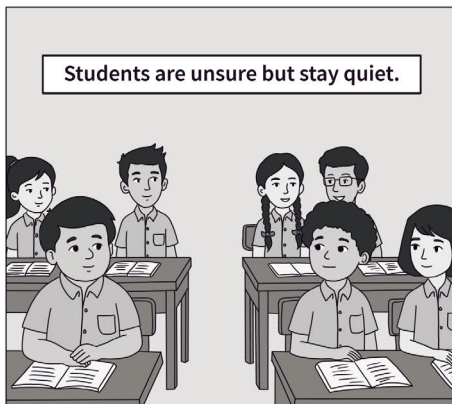
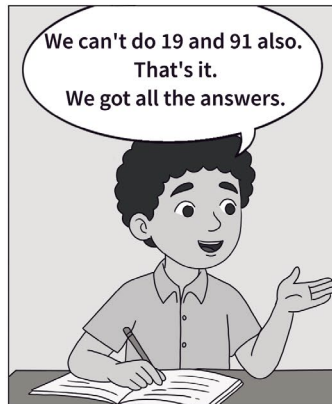
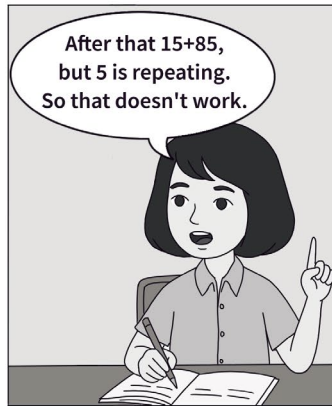
The results of these discussions may be presented to the rest of the class in different ways. The children's written work and drawings can be put up on the bulletin board. If they have come up with a different way to solve a problem, that can be highlighted. Solutions to open-ended problems can be put up. This will help in improving children's appreciation of multiple approaches and make way for peer learning.

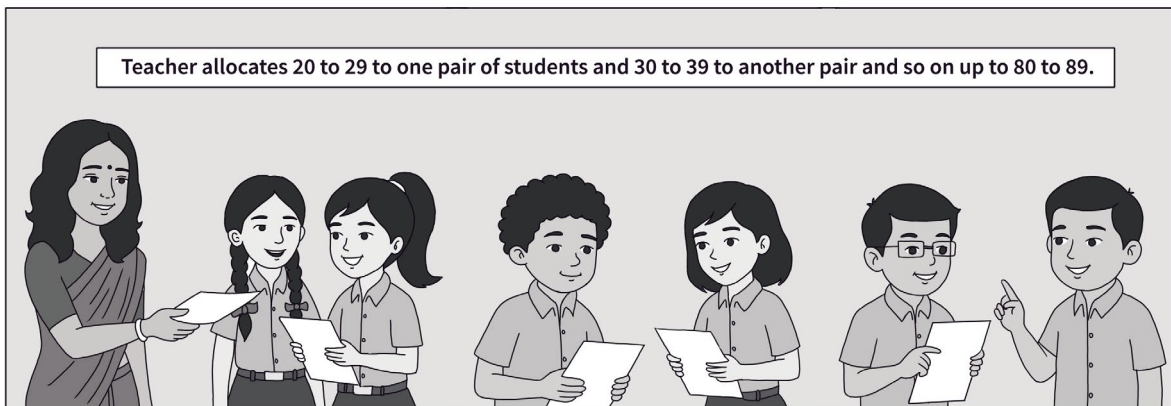
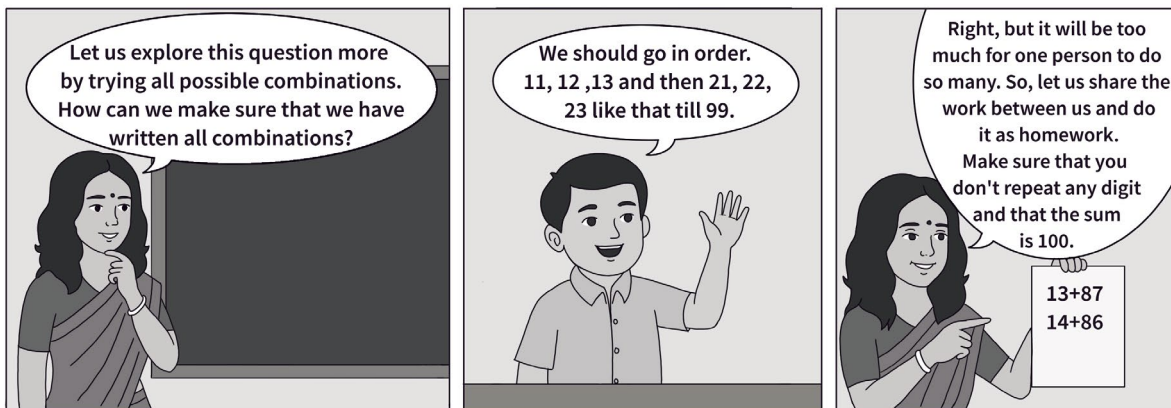
DISCUSSION 1

Grade Level 4, students work in groups of 4.



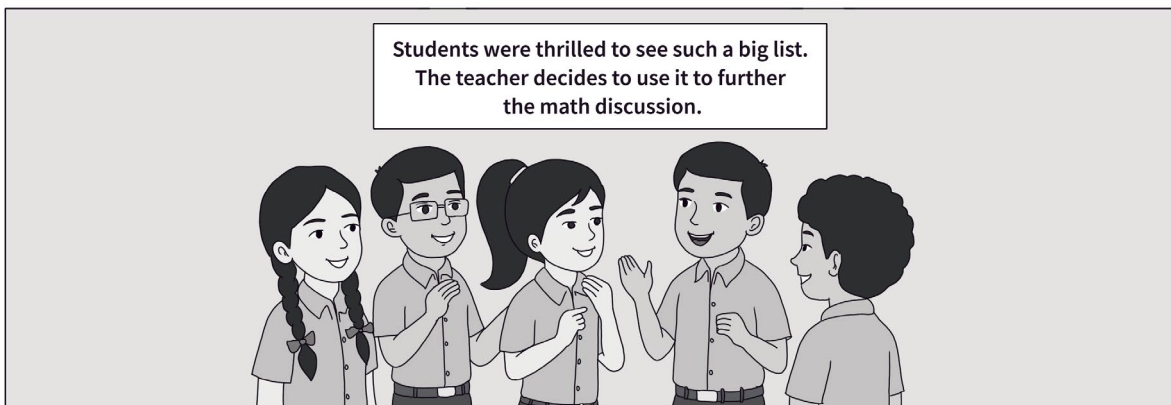


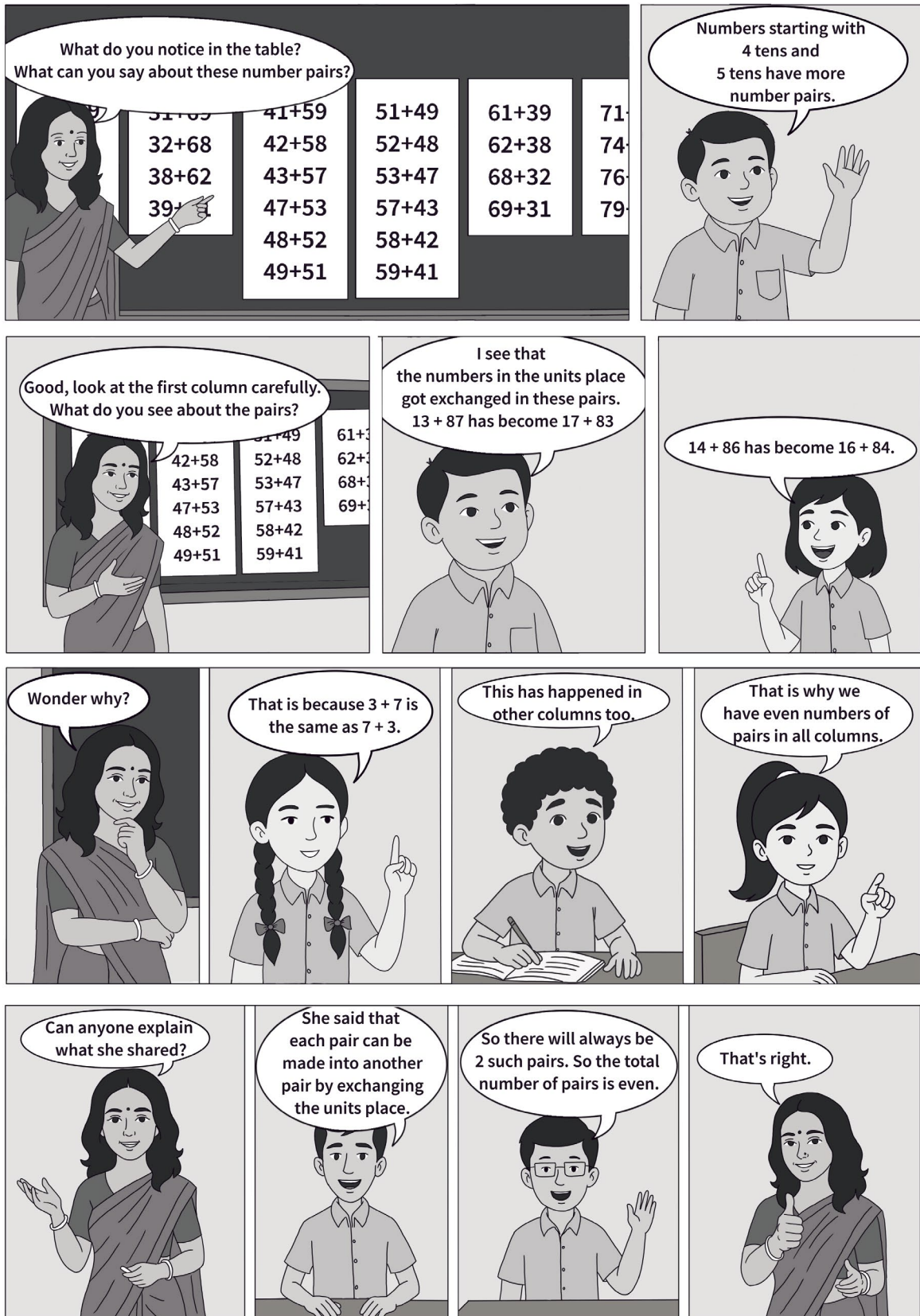


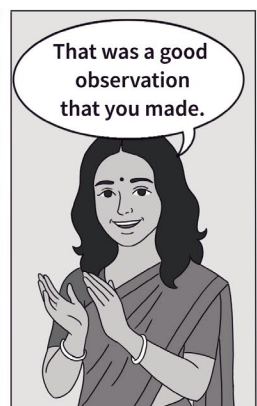
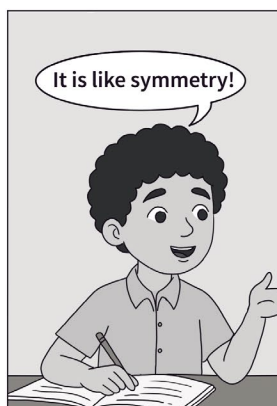
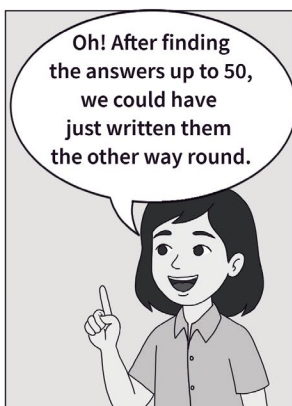
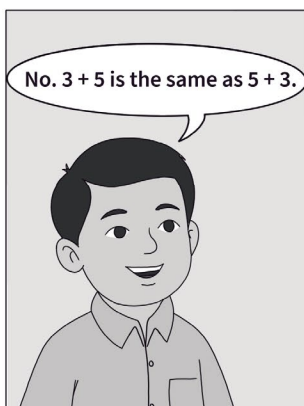
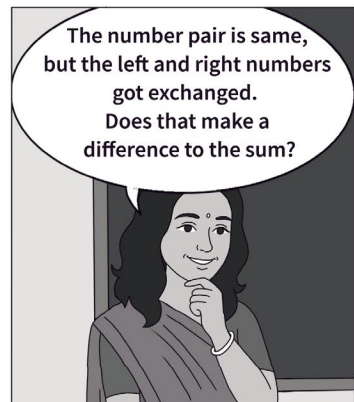
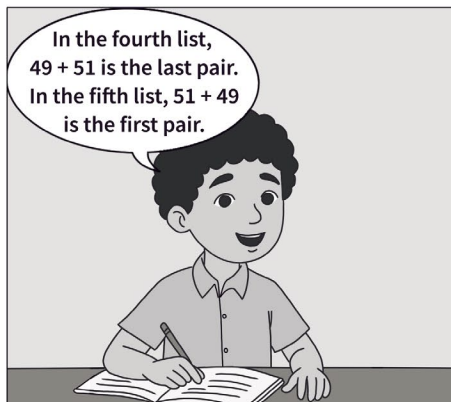
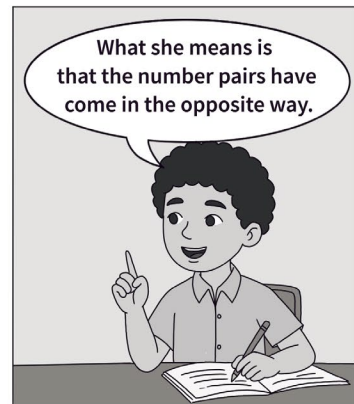
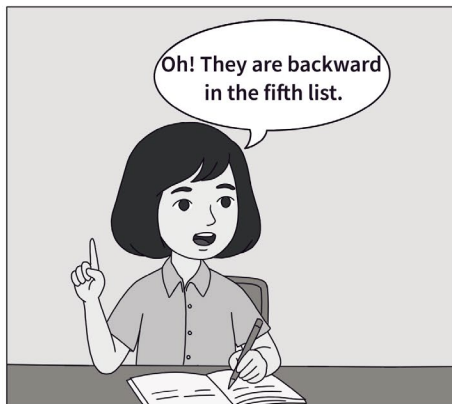
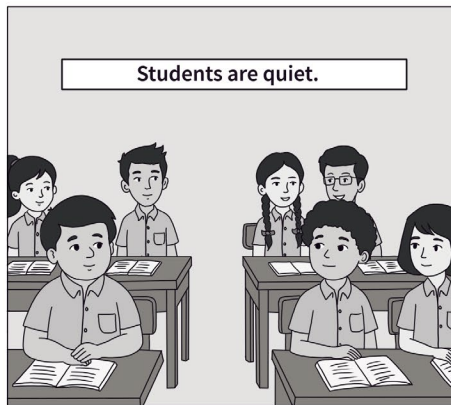


Here is the compiled list they put together on the bulletin board the next day.

13+87	21+79	31+69	41+59	51+49	61+39	71+29	83+17
14+86	24+76	32+68	42+58	52+48	62+38	74+26	84+16
16+84	26+74	38+62	43+57	53+47	68+32	76+24	86+14
17+83	29+71	39+61	47+53	57+43	69+31	79+21	87+13
			48+52	58+42			
			49+51	59+41			







Further questions can be posed

Do you notice if any digit never appears as the unit digit?

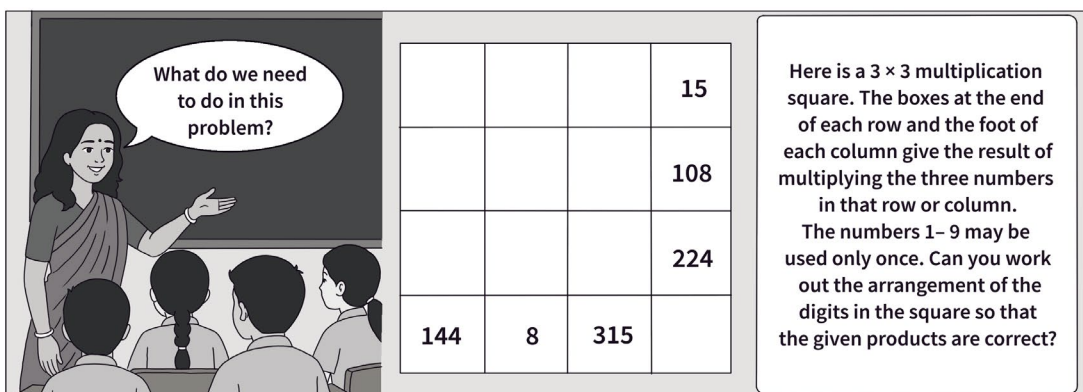
Can you find other ways to find all possible pairs?

Strategies used by the teacher

- ❖ The teacher used the problem to get students to engage in math talk.
- ❖ Students explained the problem in their own words.
- ❖ The discussion highlighted the constraints of the problem.
- ❖ Together, they reasoned out the answers they got and were provided the opportunity to clarify their understanding of such reasoning.
- ❖ Pair work ensured greater participation, particularly among the quieter students.
- ❖ The teacher guided the discussion towards a systematic way of finding all possible solutions.
- ❖ Homework was an extension of the class work but became an input for individual work and further exploration.
- ❖ The students were goaded to search for patterns and find explanations for these. By asking one student to rephrase another's explanation, the class was encouraged to listen to one another. Contributions were valued.
- ❖ Usage of appropriate math language was facilitated.
- ❖ Arithmetic laws were reinforced and revisited.
- ❖ Just one such session could achieve multiple objectives efficiently.
- ❖ Did you notice any other strategies that the teacher used?
- ❖ In the following discussions, try and pick up some other strategies. What is the teacher doing differently? What is the same? Can you recognise her objectives?

DISCUSSION 2

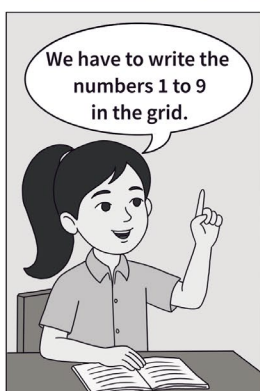
Grade Level 5, students work in pairs.



What do we need to do in this problem?

			15
			108
			224
144	8	315	

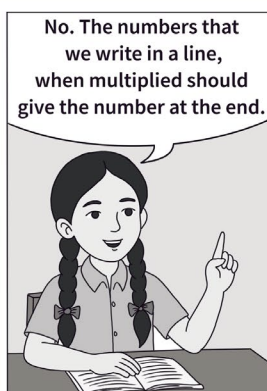
Here is a 3×3 multiplication square. The boxes at the end of each row and the foot of each column give the result of multiplying the three numbers in that row or column. The numbers 1–9 may be used only once. Can you work out the arrangement of the digits in the square so that the given products are correct?



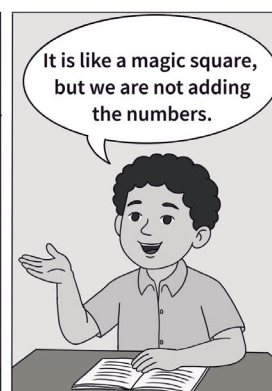
We have to write the numbers 1 to 9 in the grid.



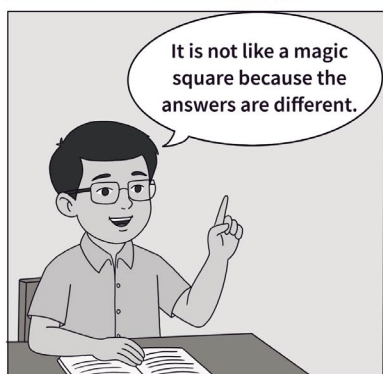
Can we write in any order?



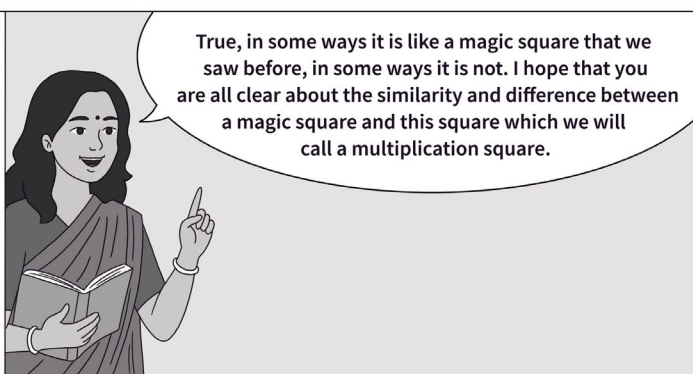
No. The numbers that we write in a line, when multiplied should give the number at the end.



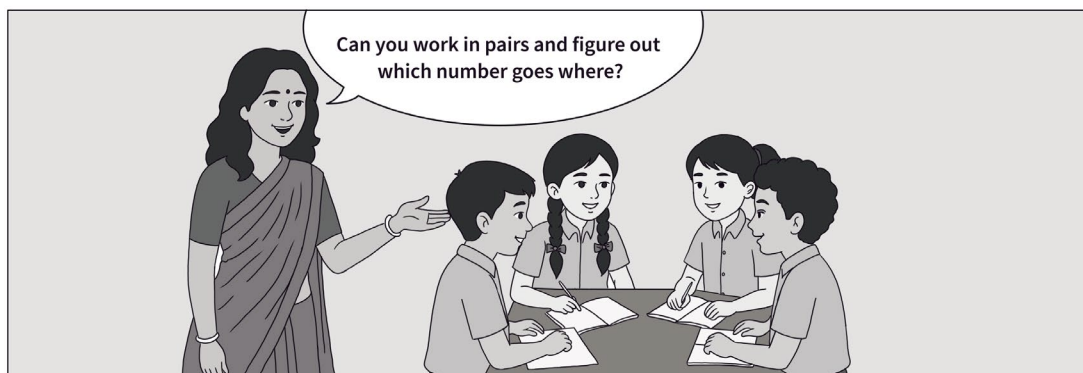
It is like a magic square, but we are not adding the numbers.



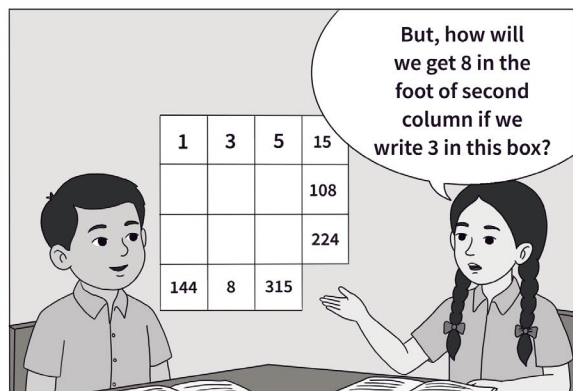
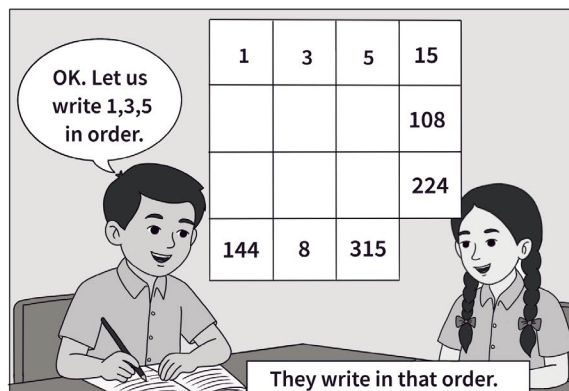
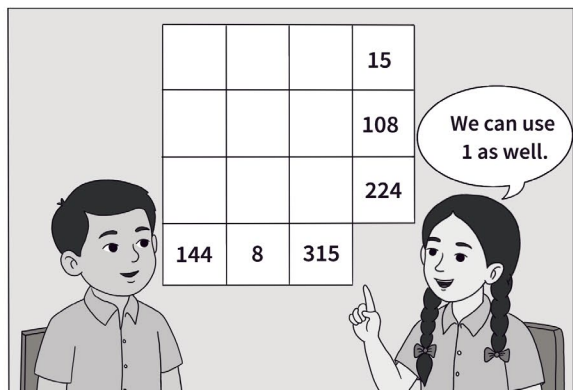
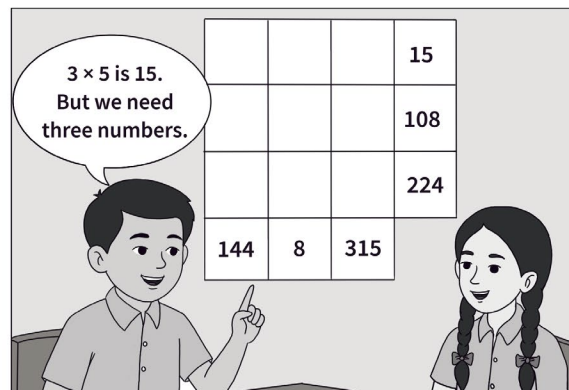
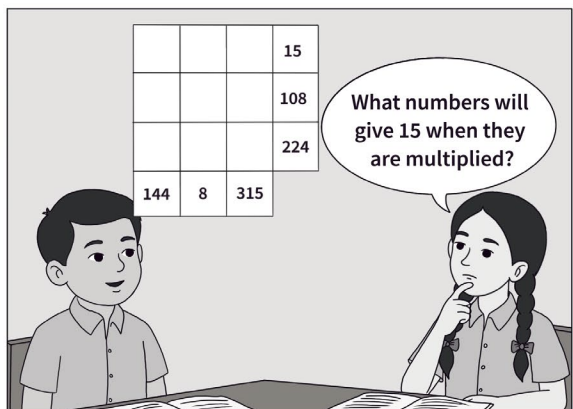
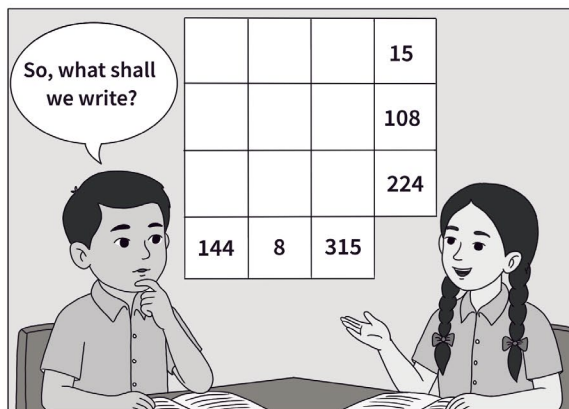
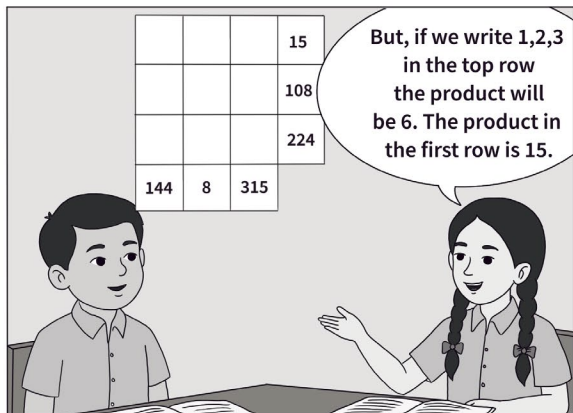
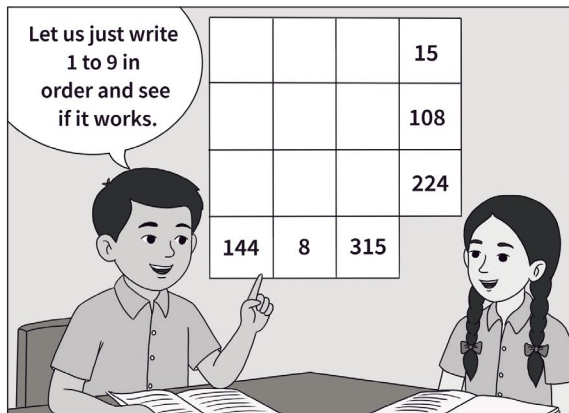
It is not like a magic square because the answers are different.

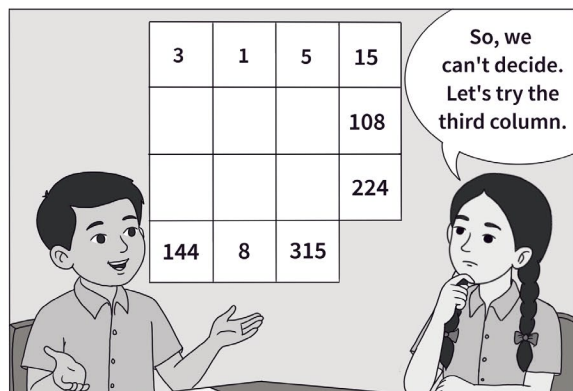
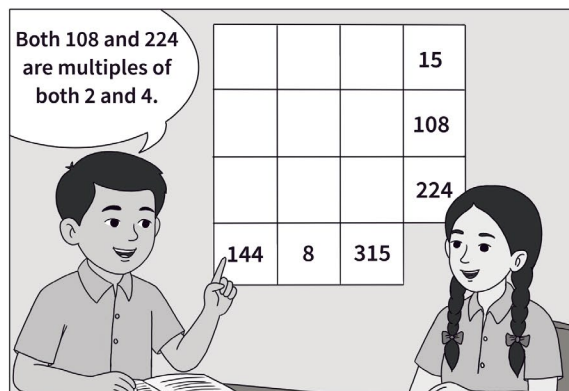
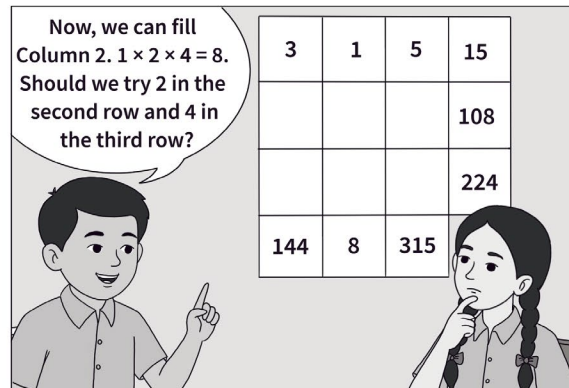
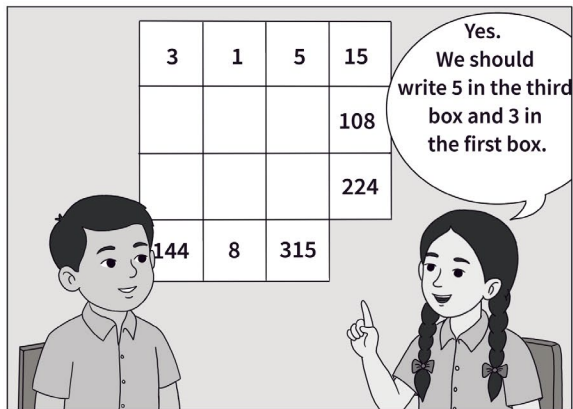
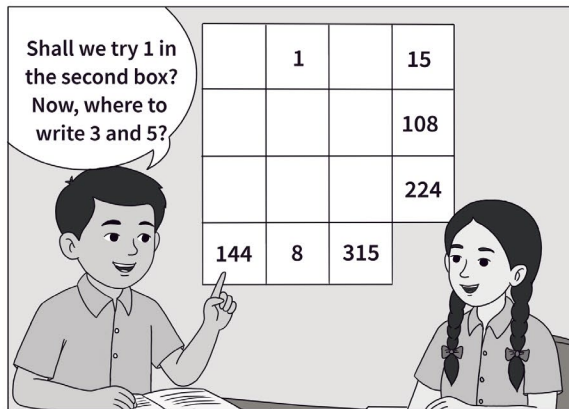
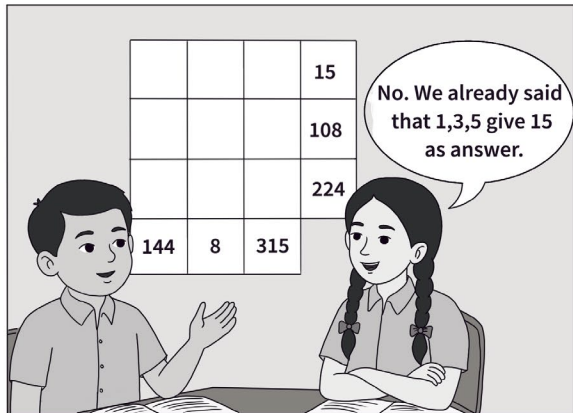
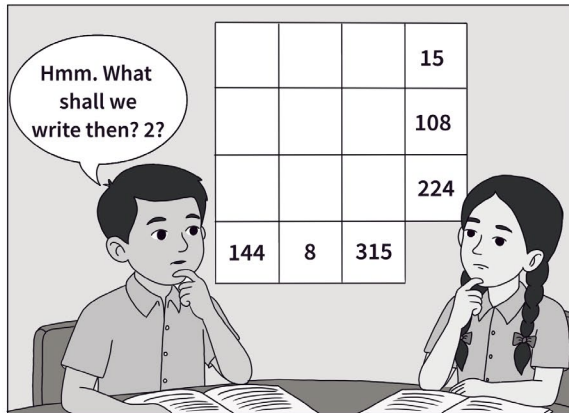


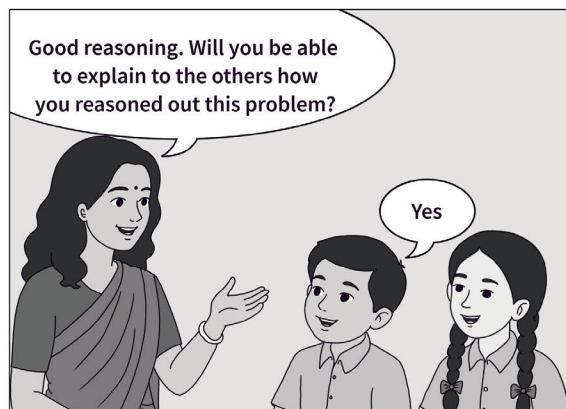
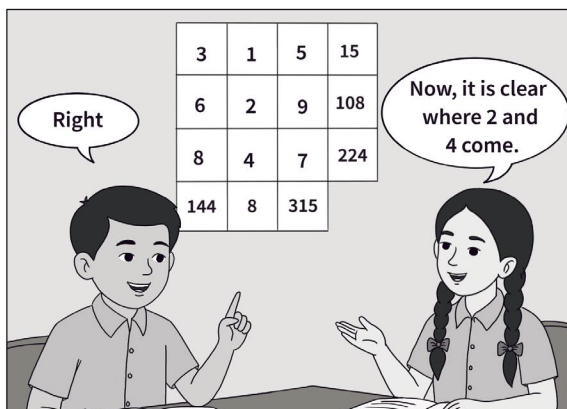
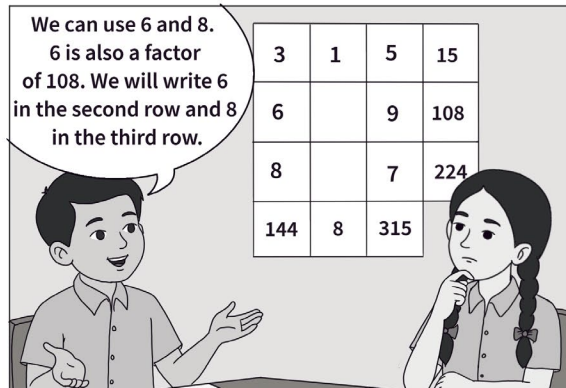
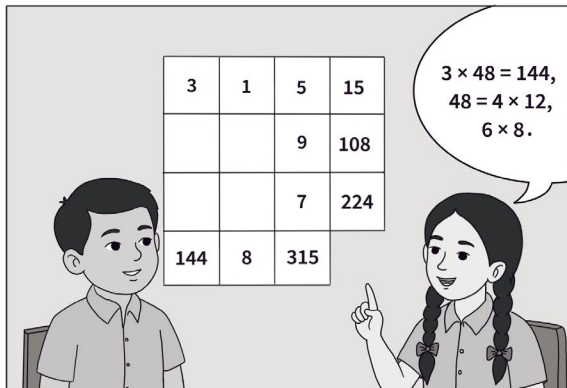
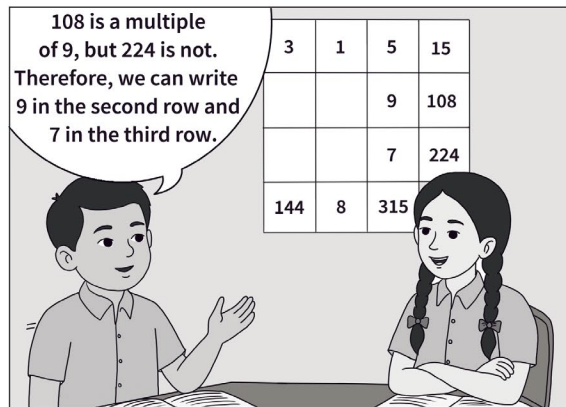
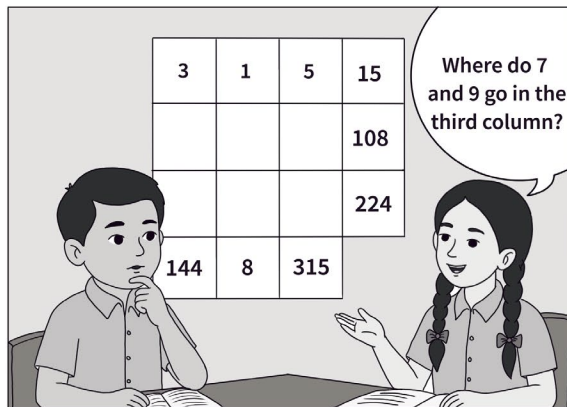
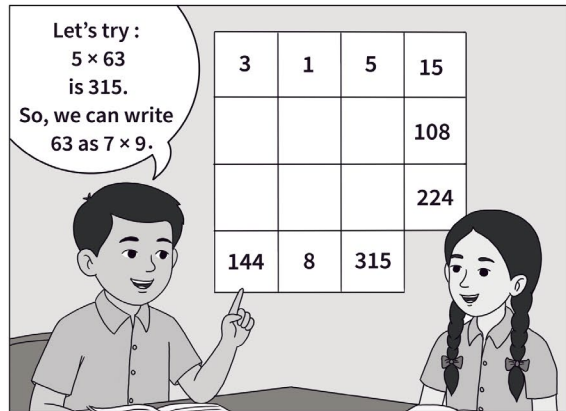
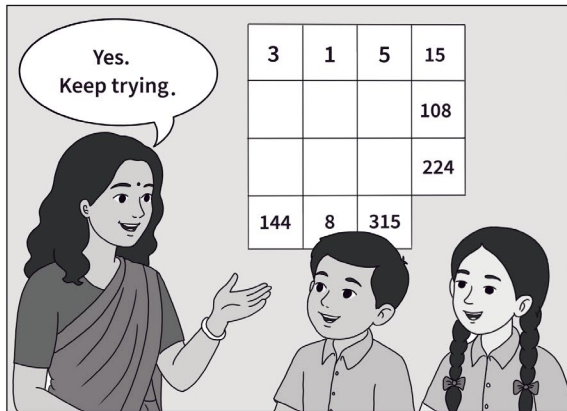
True, in some ways it is like a magic square that we saw before, in some ways it is not. I hope that you are all clear about the similarity and difference between a magic square and this square which we will call a multiplication square.



Can you work in pairs and figure out which number goes where?









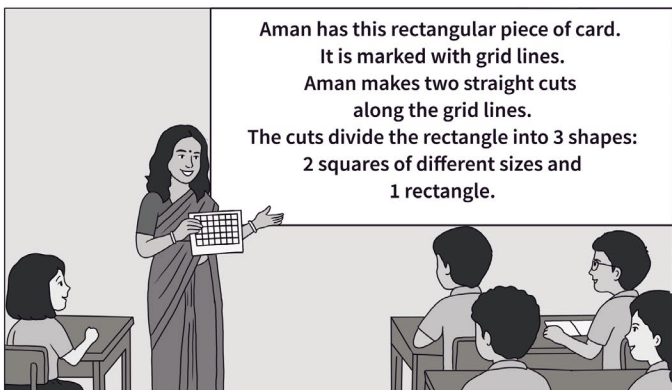
This discussion showcases most of the opportunities mentioned in Discussion 1. In addition, the students saw a connection between this problem and another.

Sometimes, relating to familiar problems can aid in understanding or solving a problem. The teacher left the first part of the discussion to the students and came into it in the middle, building a scaffold by posing questions that helped the students to proceed further.

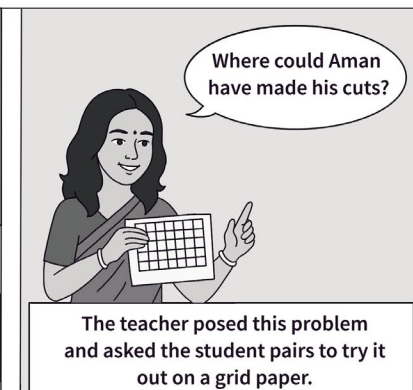
Though there was only one solution to the problem, the emphasis was clearly on the process and how the answer could be arrived at via different routes.

DISCUSSION 3

Grade Level 6, students work in groups of two or three.



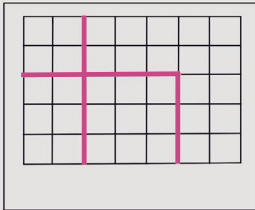
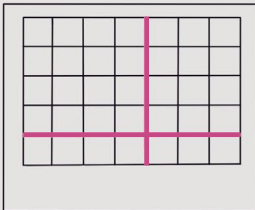
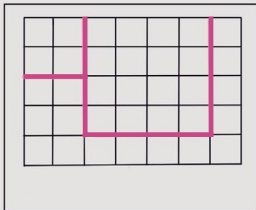
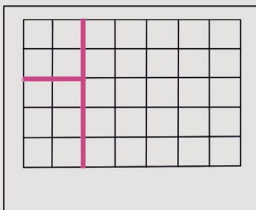
Aman has this rectangular piece of card.
It is marked with grid lines.
Aman makes two straight cuts
along the grid lines.
The cuts divide the rectangle into 3 shapes:
2 squares of different sizes and
1 rectangle.



Where could Aman have made his cuts?

The teacher posed this problem and asked the student pairs to try it out on a grid paper.

Students spent about 20 minutes drawing 2 lines on the grid in various ways and checking if it gave rise to the three required shapes.

Here are examples of their attempts. Finally, all arrived at the same solution.

How did you try the problem?

We first drew a 2×2 square, then a 3×3 square. But that did not leave one rectangle, and it needed more cuts.

Then we drew a 2×2 square and a 4×4 square. That also did not work.

Then we tried 2×2 square and a 5×5 square. Then we got it.

Group A

We first drew the largest possible square, that was 5×5 . Then we were left with a rectangle. We drew a 2×2 square in it. Finally, we got it.

All of us arrived at the same solution. Let us look at the solution carefully. What are the areas of the shapes?

The area of the smaller square is 4 square units. It is of size 2×2

The area of the bigger square is 25 square units. It is of size 5×5 . The rectangle is 6 square units and of size, 2×3 .

Group B



Notice that the teacher did not stop when the class arrived at one solution. What advantage do you see in the extended discussion? What are the various ways through which the teacher encouraged math talk in this discussion?

DISCUSSION 4

Grade Level 5/6, Students work in groups of 4.

Problem posed:

On the first day of Christmas a donor gave an orphanage one toy.
On the second day he gave another pair of toys plus a copy of what he gave on Day 1.
On Day 3, he gave three new toys, plus another copy of everything he'd already given.
If he keeps this up, how many toys will he give after twelve days?

What is the problem telling us?
What did the donor do?

The teacher decided to introduce the problem to the whole group initially, develop it up to a certain point and then let the groups work on it.

On Day 1, he gave 1 toy.
On Day 2, he gave $2 + 1$, i.e., 3.
On Day 3, he gave....

Hey! We can't do this mentally.

Let's write it out.

Will it help if we write in an organised way?

Yes. I'm going to make a table.

1	1
2	$2 + 1$
3	$3 + 2 + 1$
4	$4 + 3 + 2 + 1$
5	$5 + 4 + 3 + 2 + 1$
6	$6 + 5 + 4 + 3 + 2 + 1$
7	$7 + 6 + 5 + 4 + 3 + 2 + 1$
8	$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$
9	$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$
10	$10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$
11	$11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$
12	$12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

Writes on the blackboard.

Alright. Now, let us figure out how to get the total number of toys received over 12 days.

Students form groups of 4 to solve the problem.

Group A

Shall we total each row and then sum the totals of the rows?

Let's do that.

So, first row sum is 1, second row sum is 3 (begins to write in the third column of the table)

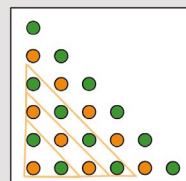
Students arrive at the answer 364.

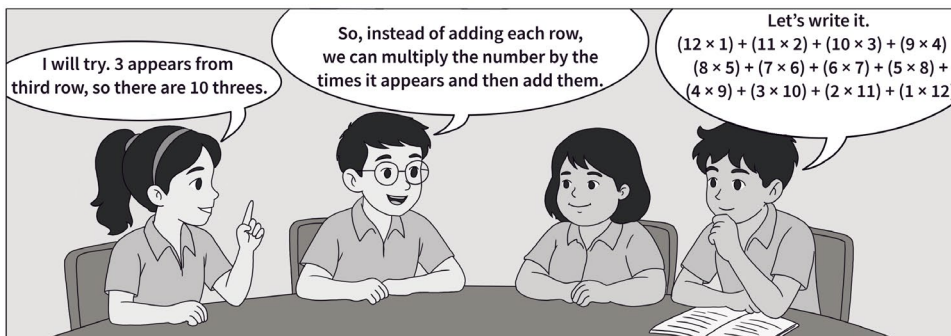
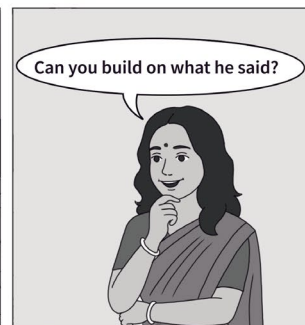
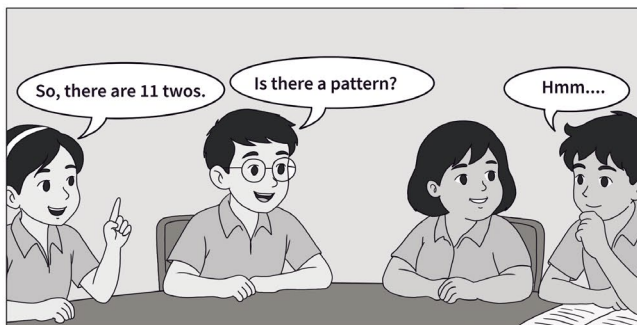
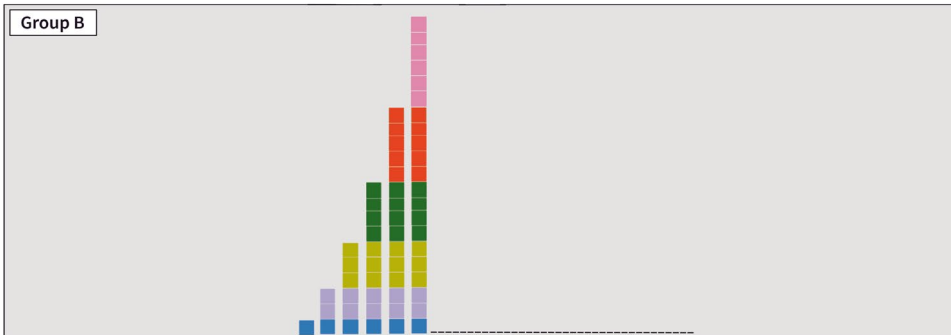
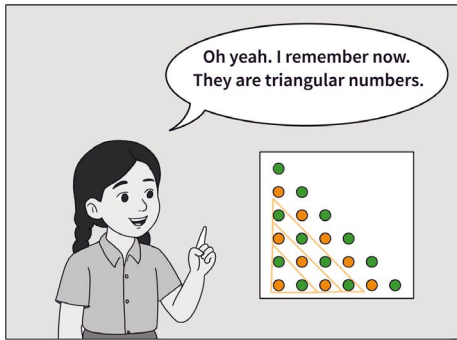
1	1	1
2	2 + 1	3
3	3 + 2 + 1	6
4	4 + 3 + 2 + 1	10
5	5 + 4 + 3 + 2 + 1	15
6	6 + 5 + 4 + 3 + 2 + 1	21
7	7 + 6 + 5 + 4 + 3 + 2 + 1	28
8	8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	36
9	9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	45
10	10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	55
11	11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	66
12	12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	78

Do you notice any patterns in the totals?
1, 3, 6, 10, 15

The second is two more than the earlier sum, the third is three more than the previous sum, the fourth is four more.

Right. What do these numbers remind you of?







Group C

1	1
2	2 + 1
3	3 + 2 + 1
4	4 + 3 + 2 + 1
5	5 + 4 + 3 + 2 + 1
6	6 + 5 + 4 + 3 + 2 + 1
7	7 + 6 + 5 + 4 + 3 + 2 + 1
8	8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
9	9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
10	10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
11	11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1
12	12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

Hey! Why don't we use the Gauss method we learnt earlier for adding numbers of each row?

What do you mean?

Remember the story when Gauss had to add all numbers from 1 to 100, he saw them as pairs, 1 and 99 make 100, 2 and 98....

That is an interesting idea. Try and see.

Oh yes. Let us look at the last row first. What can we pair here?

12 and 1 is 13.
11 and 2 is 13.

That will make 6 pairs adding up to 13. Let's put it down. 6×13 .

Let's do the same with Row 11. Here we can pair 11 and 1 and 10 and 2 as each pair adds up to 12.

6 will be left without a pair.

So, we will add 6 to 5 twelves. $(5 \times 12) + 6$.

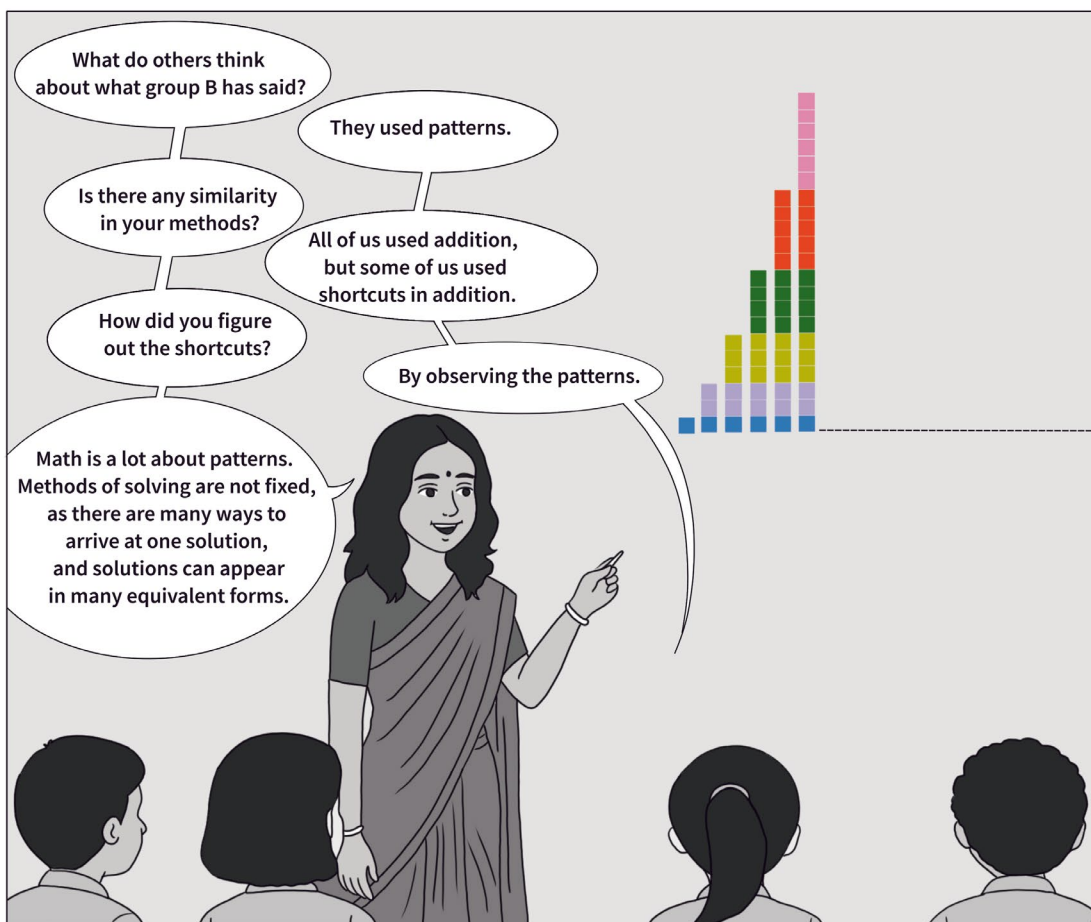
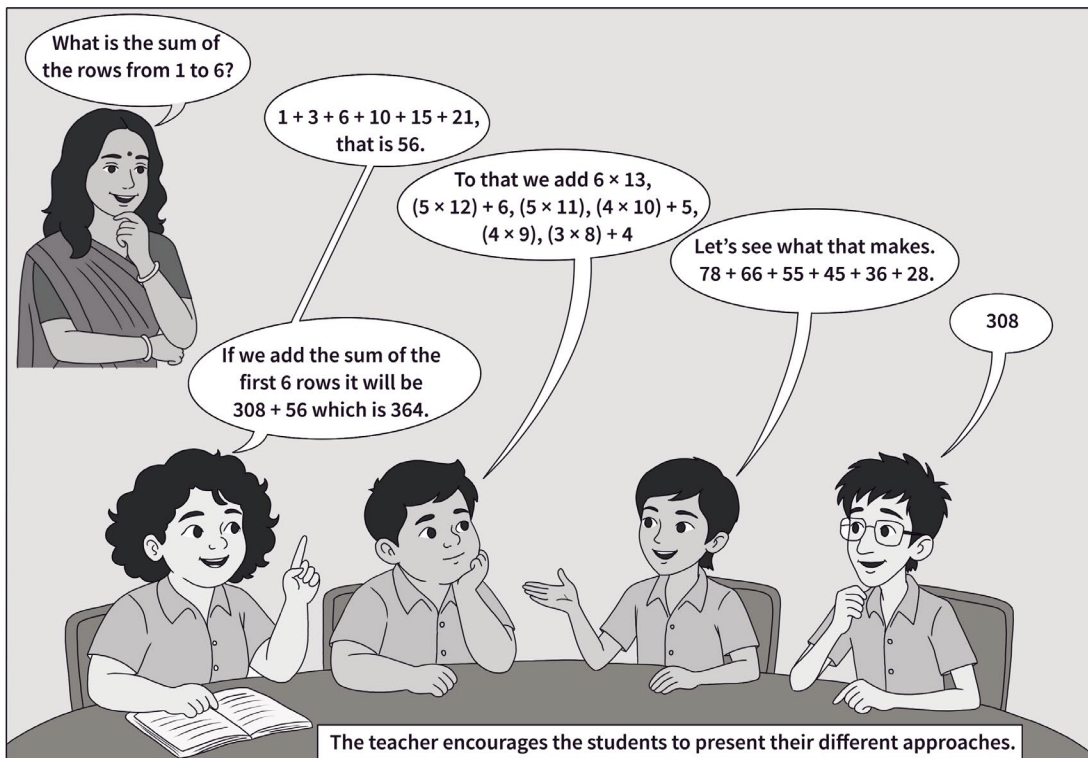
Let's look at Row 10. All 5 pairs add up to 11. So, 5×11 .

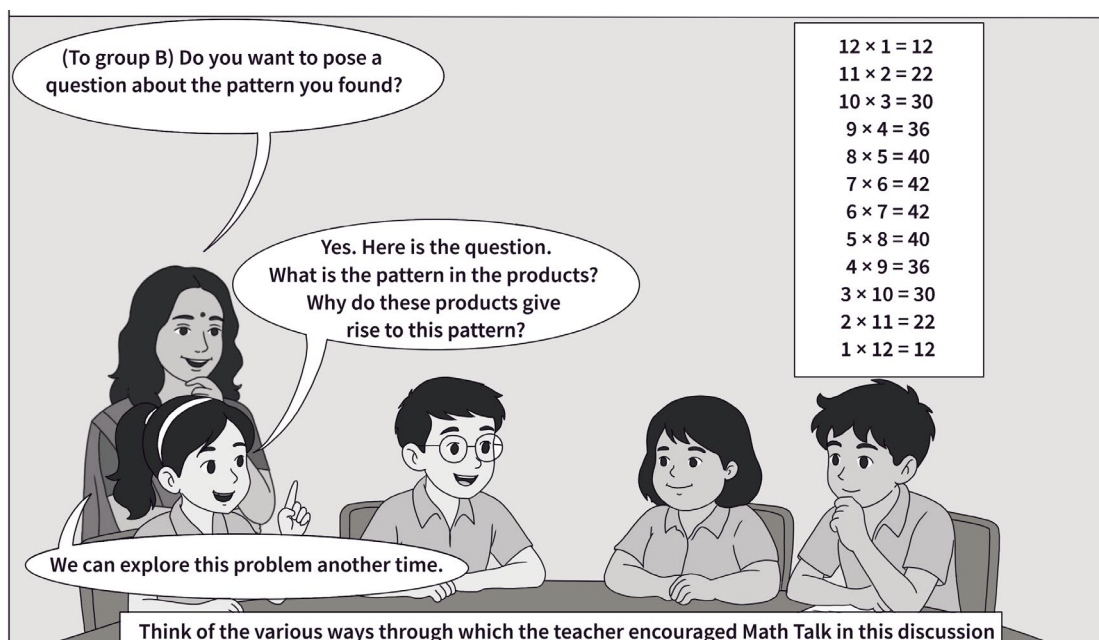
Row 9 will be 4 tens and Row 5 will be $(4 \times 10) + 5$.

Row 8 is 4 nines. 4×9 .

Row 7 will be 3 eights and 4, $(3 \times 8) + 4$.

I guess we can do the other rows by just adding.





Please do share with atrighthangles.editor@apu.edu.in any math talk that can be generated by these, or other examples.

We have showcased interesting questions and even more interesting discussions which allow students to discover things. A natural question would be about how practical it is for the average teacher to incorporate this, first by understanding the nuances of each question and second, by managing her time with portions to complete, deadlines to meet, and so on. In general, whenever we meet teachers, we recommend that they try such investigations and discussions at least once a month. Math talk promotes mathematical thinking and that is an important aim for the teaching of mathematics.



PADMAPRIYA SHIRALI

PADMAPRIYA SHIRALI is part of the Community Math Centre based in Valley School (Bangalore) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. In the 1990s, she worked closely with the late Shri P K Srinivasan. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as ‘School in a Box.’ She is currently part of the NCERT textbook development group. Padmapriya may be contacted at padmapriya.shirali@gmail.com