Teacher Notes

Using Ten-Frames to Find the Sums of Even and Odd Numbers

Math Space

Ten-frames are 2 × 5 frames that can be used to show the first ten natural numbers 1, 2, 3, … 10. You can find more details including how to make these at https://bit.ly/4kmFdoh

The poster *Sums of Even and Odd Numbers* is intended to be displayed in class for students to move from the concrete model of the ten-frame to representations of selected combinations of numbers from it. Students should be given adequate time to observe patterns, think about the questions posed and discuss their thoughts with their peers. Even a simple listing of the addition problems shown helps them to realise how many such combinations are possible and if all such combinations have been shown. From simple observation to generalisation is a powerful step in mathematics and this poster scaffolds such learning. The teacher notes given below are intended to help the teacher facilitate such a journey and it should be noted that simply focusing on the answers will not help the student develop their mathematical thinking skills. The questions are designed to provide the starting points, and the peer discussion will enable student-led discovery.

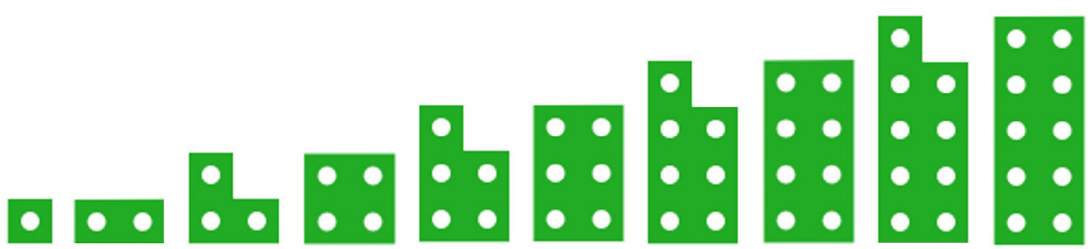

Figure 1

A careful observation using the trigger of the first question *What do you notice about the tops of alternate numbers?* in the poster shows that every alternate number beginning at the first, has a single 'odd' dot at the top, while the second, fourth, etc., numbers have a flat or 'even' top. This visualisation can help in connecting the 'parity of a number,' i.e., whether the number is 'odd' or 'even', and the mathematical concepts associated with these terms. Observe that this visualisation provides meaning to the standard algebraic approach of representing an even number as $2n$ and an odd number as $2m - 1$ or $2m + 1$ at a later stage.

Our answers for the second question *In each of the images in the three groups A, B and C, two numbers (shown in contrasting colours) are added. What do you notice about these numbers and about their sum?* are given below:

In Group A, we see the sums of all possible combinations of odd (orange) and even (blue) single-digit numbers.

In Group B, we see the sums of all possible combinations of two even single-digit numbers.

Similarly in Group C, we see the sums of two odd single-digit numbers.

Listing all the combinations of digits and observing their sums will help students work systematically.

Keywords: ten-frame, odd-even, parity, observation, conjecture

Teacher Notes

Questions that may be posed:

In Group A	In Group B	In Group C
The top number is ___________ (because ___________) and the bottom one is ______________ (because _________________) So, the sum is ______________ (because _________________)	Both numbers are ___________ (because _________________) So, the sum is ______________ (because _________________)	Both numbers are ___________ (because _________________) But, the sum is _____________ (because _________________)

- What is the smallest total in each group? And the largest?
- What is common to the sums of the numbers in each group? Can we generalise?
  (i) Odd number + even number = _____ number
  (ii) Even number + even number = _____ number
  (iii) Odd number + odd number = _____ number

These visualisations help one understand how adding an even number doesn't change the parity of the sum from that of the number added to, i.e., odd + **even** = odd (as illustrated by A) and even + **even** = even (as depicted by B).

But adding an odd number changes the parity. When two odd numbers are added, they compensate for their oddness and make the sum even; the two odd ones sum to an even two as shown in C. In other words, **odd** + odd = even while B illustrates **odd** + even = odd.

Extension Question: Is zero odd or even?

There are multiple reasons why zero can't be odd. For example, if it is odd, then it needs to have an 'odd' dot at the top. But zero can't have any dot. So, it can't be odd. Therefore, it has to be even.

Also, adding an even number does not change the parity. So, odd + even = odd and even + even = even. Zero preserves this since a number does not change when zero is added to it. Therefore, zero must be even.

The next set of questions:

- *Can you make 13, 18, or any whole number greater than zero if you have enough ten-frames? How?*
- *What about 125? 602? 1234?*
- *Why does the units digit of a number determine whether it is odd or even?*

are again a step in the direction of generalisation.

Any whole number ≥ 10 is a bundle of tens and a 1-digit number (0, 1, 2, … 9). For example, 43 = 4 × 10 + 3, 602 = 60 × 10 + 2, 1234 = 123 × 10 + 4. So, each of these numbers can be represented with as many full-frames and then the 1-digit number, e.g., 43 can be represented as in Figure 2.

Figure 2

From this understanding, the generalisation that the units digit of a number determines its parity follows immediately.

If you find this interesting, check the review of ten-frames in the Nov 2022 issue of this magazine at https://bit.ly/4r79Qkb. It includes a worksheet.