Azim Premji University

At Right Angles

A RESOURCE FOR SCHOOL MATHEMATICS

Azim Premji University

PATTERNS & PRE-ALGEBRA

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PATTERNS AND PRE-ALGEBRA

Mathematics is the study of patterns - in numbers and in geometry, more commonly - but also in the most unexpected places.

Patterns are beautiful and catch our attention. All of us notice them either in our surroundings, in clothes, in constructions, and so on.

How do we describe these repetitive patterns? Some are visual patterns which share some common characteristics. Some of these are number patterns with sequences based on addition or subtraction. Some patterns are based on multiplication and division. If we know the first few numbers, we are able to predict the next number or other numbers in the sequence.

Children have also seen them and often depict them in their artwork. They have an intuitive understanding of patterns and are able to predict what comes next.

The teaching of algebra needs to build on this intuitive understanding that students have and help them to articulate their understanding in clear, concise language. The ability to generalize patterns will build the needed scaffolding for learning algebra.

Exposure to patterns in pre-algebra and articulation of the patterns using language leads to the ability to form mathematical statements. At a later point, when variables are introduced, students learn to express those same statements using variables and operations.

Here is an example related to the pattern shown in Figure 1. 'The number of lines is 4 times the number of rocket shapes plus 2.' This same statement will be expressed later as 'l equals 4 times n and 2 more.' (where l stands for the number of lines, n stands for the number of rocket shapes) or '$l = 4n + 2$'.

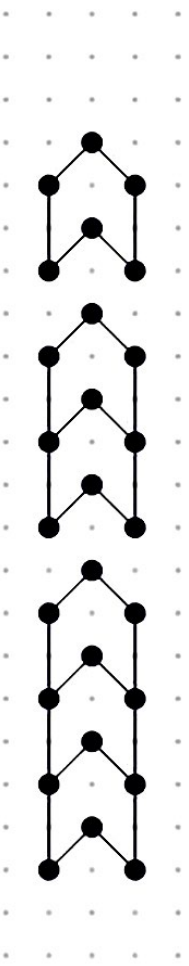


Figure 1

Keywords: Pattern, sequence, rule, connection, communication, language.

Pattern 1: Rockets, dots and lines

Figure 1 shows a number of rocket shapes made into a pattern with dots and line segments. Each line segment connects one dot to the nearest dot.

Number of Rocket Shapes	Number of lines	Number of dots
1	6	6
2	10	9
3	14	12
4	18	15

Let the students observe and note down the number of lines used in the first rocket shape. How many lines and dots have been drawn to make the second shape? Third shape? Students can record the information in a table format.

Students will notice that the pattern is increasing by 4 line segments each time. How many such lines will be needed to make 20 rocket shapes?

They will also notice the pattern in the number of dots.

Do they see any relationship between the number of dots and the number of lines?

Pose the question: How do the numbers in the second column (number of lines) relate to the numbers in the first column (number of rocket shapes)?

Number of Rocket Shapes	Number of lines	
1	6	$1 \times 4 + 2$
2	10	$2 \times 4 + 2$
3	14	$3 \times 4 + 2$
4	18	$4 \times 4 + 2$

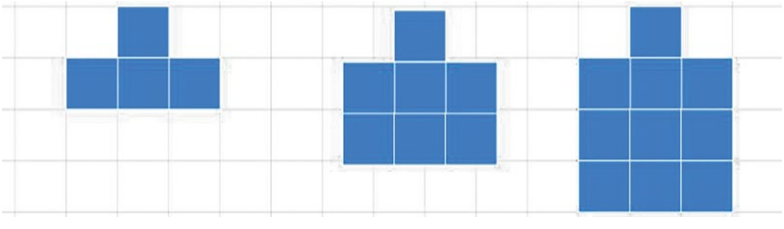
Students will see that the number of lines are not multiples of the number of shapes. However, they will be able to state that each subsequent rocket shape uses four more lines. If the students are not able to state the relationship, the teacher can ask some leading questions. Can they use the fact that the first one had two more than the rest?

Help the students to express the relationship in words.

The number of lines is 4 times the number of the rocket shape and 2 more.

Pattern 2: Increasing blocks

Let the students note down the number of squares in the table to discover the relationship between the number of each block and the number of squares it is composed of.



Block 1 Block 2 Block 3

Figure 2

Block	1	2	3
Number of squares			

If the number of squares in this sequence is 67 what will be the number of the block?

Pattern 3: Faces

Here is a series of faces made with dots and lines.

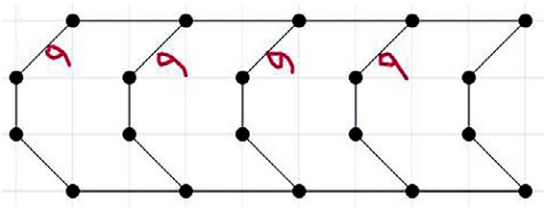


Figure 3

Students can make a table to record the information of the lines and dots used and find the relationship between the number of faces and the number of lines used. What is the connection between the number of dots and the number of lines used? How many lines will the complete figure have if there are 7 faces in it?

Pattern 4: Expanding shapes

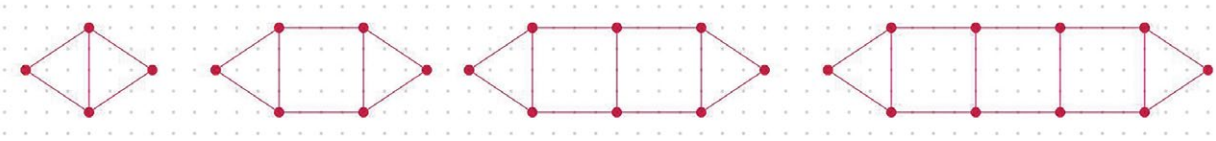


Figure 4

Create a table for these figures to discover the relationship between the number of the shape and the number of lines.

Pattern 5: Growing squares

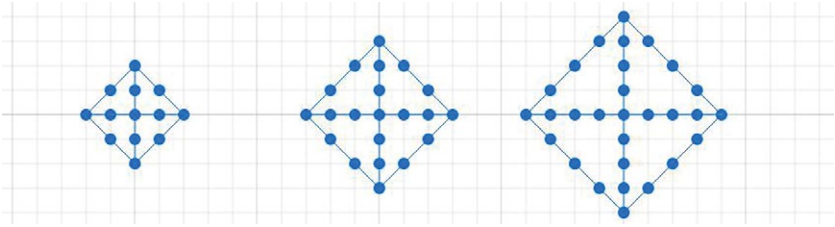


Figure 1 Figure 2 Figure 3

Figure 5

What is happening in these figures? Let the students express their understanding of these figures. Answers will vary depending on their observations.

Some students might count the dots, and some might count the lines.

Can they identify the dots that are repeating in all the three figures?

Figure Number	1	2	3
Number of dots			

How many are they? Will that number appear in the relationship of the number of dots to the number of the square?

What is happening to the other dots?

Students can count the dots and observe how the dots are increasing as the size of the square increases.

Pose the question: 'If the number of dots is 77, what will the number of the square be?'

Can the students create a figure that can be repeated and extended and verbalise the pattern?

Pattern 6: Tables and stools

Here is a classroom arrangement of tables and stools.

How does the number of stools increase if the pattern continues?

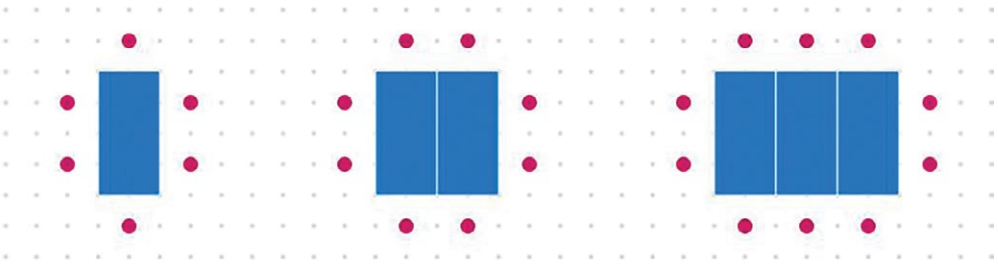


Figure 6

Number of tables	1	2	3	4
Number of stools	6			
Relationship	....			

Pattern 7: Designs with squares

How does the number of dots increase if the pattern continues?

How many dots will be there in the tenth figure in this pattern?

Figure 7

Pattern 8: Clothes and pegs

Clothesline problem: How many pegs are there in relation to the number of clothes? Will this relationship continue if the number of clothes increases?

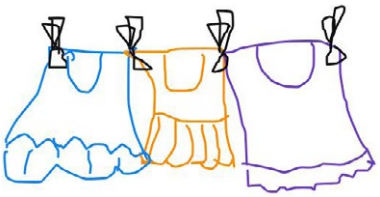


Figure 8

Pattern 9: Triangles and Lines

Look at the triangles within the orange triangles to answer the following questions.

How many white lines are used to make the inner triangles in Figure 9(1) ?

How many white lines are used to make two rows of triangles in Figure 9(2)?

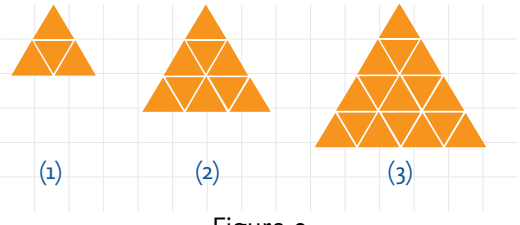


Figure 9

If we extend the pattern, how many white lines are used for three rows of triangles? How many white lines will be used to make 4 rows of triangles?

It is difficult to see a direct connection linking the row number and the lines used. However, a pattern can be found by looking at the additional lines used.

What is the relationship between the row number and the number of lines used? In the first row it is 1×3.

In the second row it is $1 \times 3 + 2 \times 3$

In the third row it is $1 \times 3 + 2 \times 3 + 3 \times 3$.

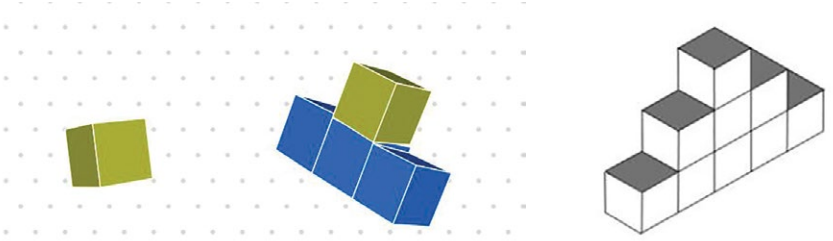
Hence, in the fourth row it will be .....

Rows	Number of lines	
1	3	3
2	9	3 + 6
3	18	3 + 6 + 9
4		

Pattern 10: Up and down staircases

In this pattern the first figure has 1 block. The second figure has 4 blocks. To continue the pattern, how many blocks are needed to make the third figure?

Can the students build a table and study the incremental increase at each stage to spot the pattern?



(1) (2) (3)

Figure 10

Pattern 11: Patterns and rules of counting numbers

Let us look at sets of three consecutive numbers:

6, 7, 8 11, 12, 13

Figure 11

What patterns can we find in these sets?

6 + 8 is 14 and 14 is double of 7.

Does this hold for the other set?

11 + 13 is 24 and 24 is double of 12.

Will this happen for all sets of three consecutive numbers? Let the students check and see. Why does it happen? How is 6 related to 7? By how much is it less? How is 8 related to 7? By how much is it more?

Can they now explain why the sum of 6 and 8 has to be double of their middle number 7?

What if we multiply the numbers 6 and 8? $6 \times 8 = 48$. If we multiply 7 by itself, it is 49. 6×8 is one less than 7×7.

Does this work for 11, 12, 13?

$11 \times 13 = 143$. $12 \times 12 = 144$

11×13 is one less than 12×12.

Ask students to make a dot array for 6×8 and 7×7 to understand the connection.

In the array of dots showing 6×8, if we remove one column of 6 dots, and turn it into one more row , we will have 7 columns of 7 dots each except for the last column.

Ask the students to show why 11×13 is 1 less than 12×12 using dot arrays.

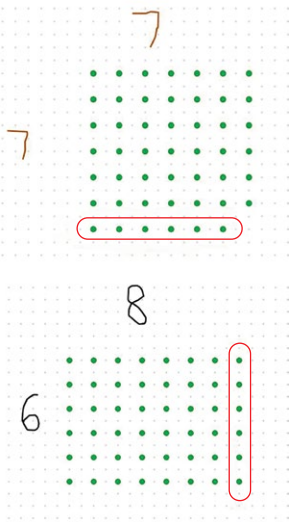


Figure 12

Pattern 12: 4 consecutive numbers

Students can now study sets of 4 consecutive numbers.

9, 10, 11, 12 4, 5, 6, 7

Figure 13

How do the numbers 9, 10, 11, 12 relate to each other? Is there a similar relationship between 4,5,6,7?

Students can try summing different pairs in these sets to look for relationships.

They may try adding 4 to 5 and 6 to 7.

Sum of 4, 5 is 9 and sum of 6, 7 is 13. 9 is 4 less than 13.

What if they try the same operation with another set of 4 consecutive numbers, say 9, 10, 11, 12.

Sum of 9, 10 is 19 and 11, 12 is 23. Again 19 is 4 less than 23.

Can the students give a justification for this result?

What if they add 4, 7 and 5,6. Sum of 4,7 is 11 and 5,6 is also 11. Will the same happen for 9,10,11 and 12?

How can they express their findings using statements?

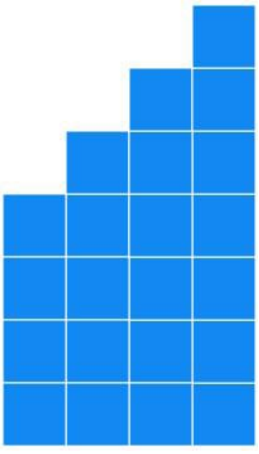


Figure 14

Pattern 13: Sum of 4 consecutive numbers

Let them add the first 4 consecutive numbers 1 to 4. They get a sum of 10. What will they get if they add the numbers from 2 to 5? 3 to 6? 4 to 7?

What pattern do they notice in the sums? Can they explain the reason for the pattern? Can they express the pattern as a statement?

Let the students add the numbers and record the sum.

Sum	1 to 4	5 to 8	9 to 12	13 to 16
	10			

What pattern do they notice here? Can they explain why it works?

Pattern 14: Consecutive multiples

Here is a set of 4 consecutive multiples of a number that have been increased by 1:

13, 16, 19, 22

This sequence is a multiple of __ , increased by 1.

Find the number which has been multiplied and the number which has been subtracted to give the following numbers.

22, 26, 30, 34

The relationship can be expressed as a sentence.

This sequence is a multiple of __ , decreased by __.

It can also be considered as a sequence that is a multiple of ....., increased by 2. Can the students explain why this happens?

Teachers can apply a rule to a series of multiples of a number and generate sequences that students can decipher. They can express the rule in the form of a statement.

How did the students work out the table and increase/decrease each time? Did they use any method?

Will the method always work?

Pattern 15: Bags of coins

Here are 3 bags with 5-rupee coins, 2-rupee coins and 1-rupee coins. Each student is allowed to pick exactly five coins from any of the bags. What coins could each one of them select to make a sum of 9 rupees? Students can express the statement as four 1- rupee coins and one 5-rupee coin makes a sum of 9 rupees.

What combinations will give a sum of 18 rupees?

Figure 15

Pattern 16: Square counting

How many squares are in each figure?

How will you describe the rule for the number of squares in this series for example?

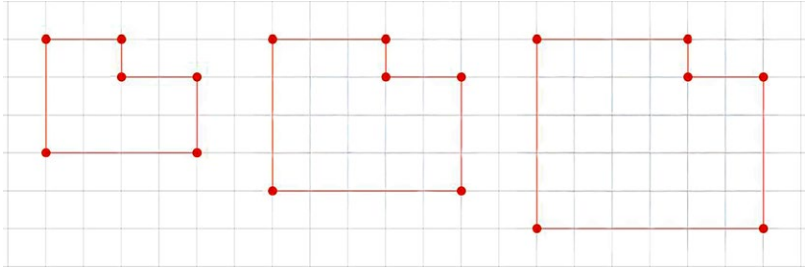


Figure 16

Pattern 17: Square designs

What fraction of the squares are shaded in blue in each figure? What fraction of the squares are shaded in grey in each figure? Are the fractions increasing or decreasing?

How does the number of blue squares increase in this series? How does the number of grey squares increase in this series?

How many blue squares will be in the 10th figure? How will you describe the sequence of the number of blue squares?

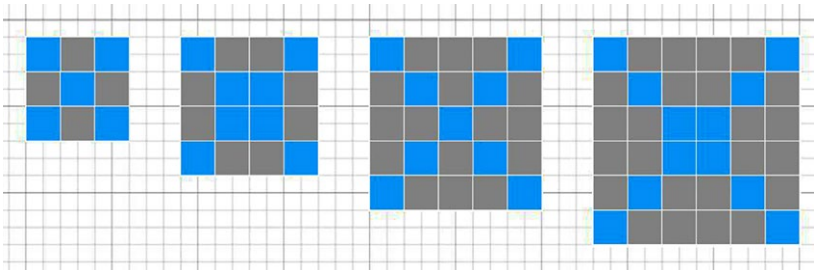


Figure 17

How are the blocks in Figure 18 increasing?

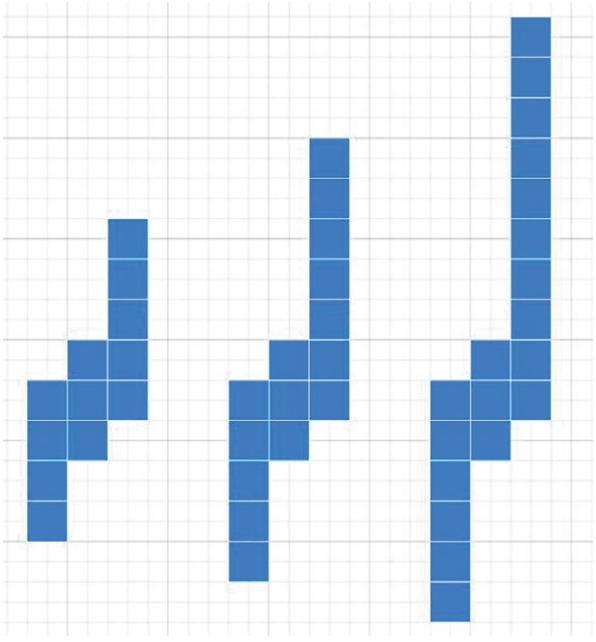


Figure 18

Challenge question: Flower pattern

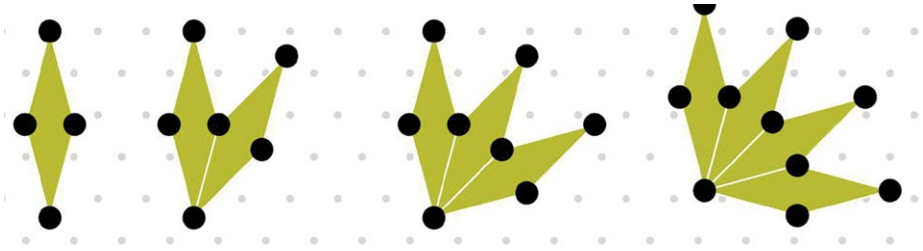


Figure 19

Here is a flower pattern. How are the figures increasing?

How many dots to create a closed figure? How many petals will there be in the shape altogether?

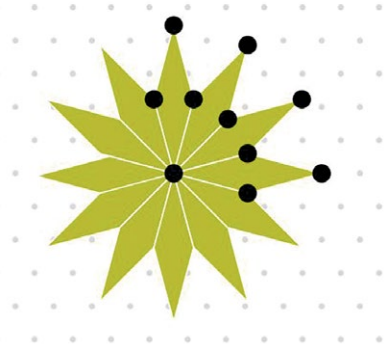


Figure 20

Figure 20 gives the answer

Pattern 18: Number line movements

On this number line an engine goes forward by 3 steps and moves backwards by 1 step in one round, then moves forward by 5 steps and goes backward by 2 steps in the second round, then moves forwards by 7 steps and goes backwards by 3 steps in the third round. The pattern goes on repeatedly forwards and backwards. Where will the engine be after 8 rounds? How will you describe this sequence?

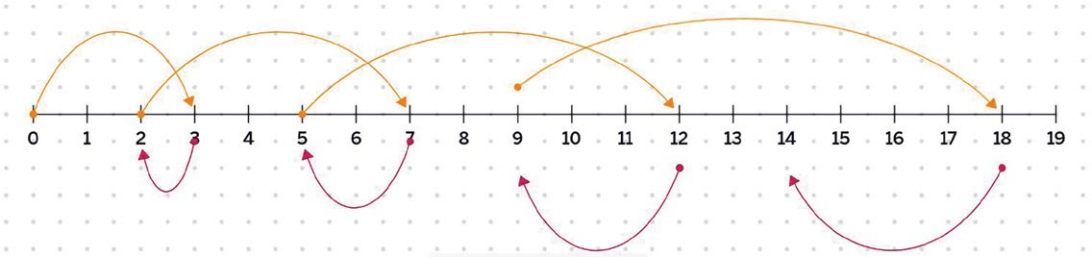


Figure 21

Pattern 19: Fractions in squares

In these series of squares the top row of the first square is shaded fully. The second row has one less shaded square than the top row, and so on. What fraction of the squares are shaded? What pattern do you notice in the fractions? How are the denominators increasing? How are the numerators increasing? How will you describe this sequence?

Is the fraction of shaded squares increasing or decreasing?

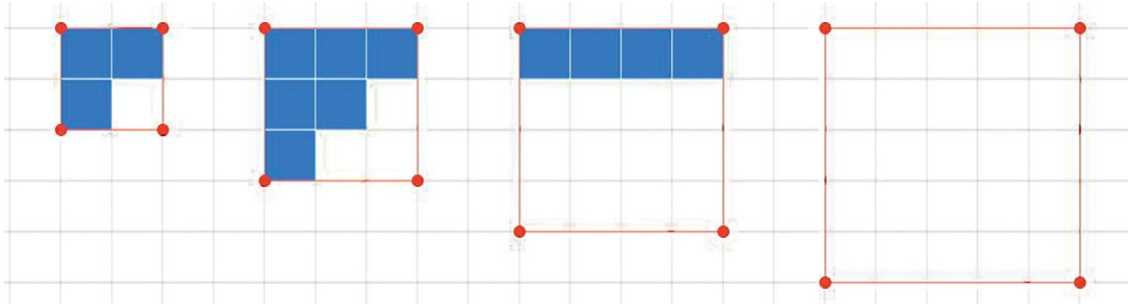


Figure 22

Pattern 20: Tables

What is happening in these tables? Try with a combination of operations to figure out the rule. How do you describe the rule?

Input	Output
7	41
9	51
3	21
6	36

Input	Output
8	63
13	168
11	120
15	224

Pattern 21: Grids

Here is a grid made up of a pattern. The numbers go horizontally from left to right in steps of 6. The numbers go vertically down in steps of 5.

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24
5	11	17	23	29
10	16	22		
15	21			

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24
5	11	17	23	29
10	16	22		
15	21			

What is the rule for the numbers that are shaded with the same colour in this grid?

0	6	12	18	24	29
5	11	17	23		
10	16	22			
15	21				

Fill the grid with numbers and colour the numbers according to a rule. Can your friend figure out the rule?

0	1	2	3	4	5	6	7	8	9
10	11	12	13						

Pattern 22: Intersecting squares

How is the perimeter of the whole figure increasing?

How is the area of the whole figure increasing?

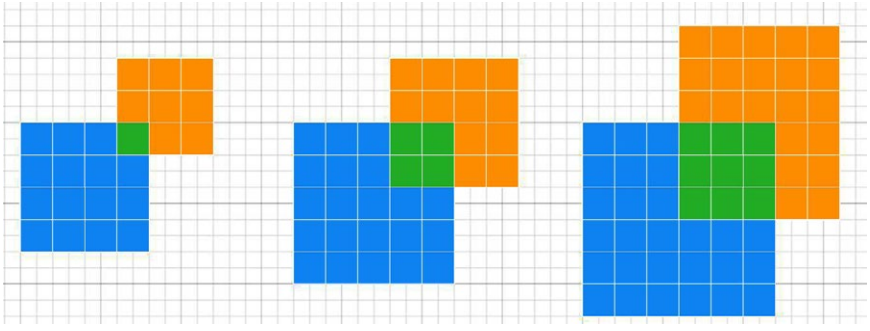


Figure 23

Pattern 23: Graphs and dots

What rule will connect dots of the same colour?

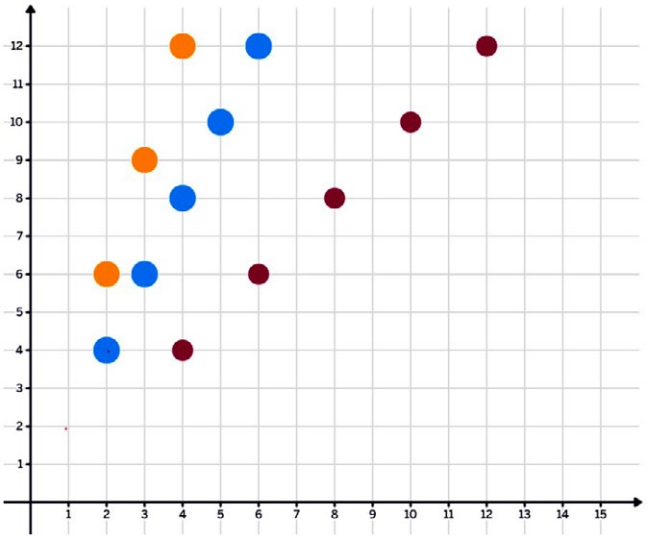


Figure 24

Pattern 24: Magic squares

Here is a magic square. What is the magic sum of this square?

What patterns do you see in the number arrangement in the square?

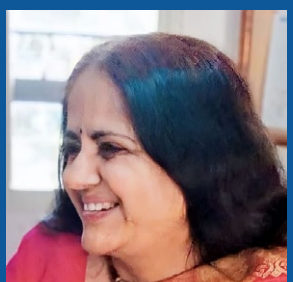
13	**8**	**15**
14	**12**	**10**
9	**16**	**11**

Figure 25

How do you describe the numbers at the opposite corners in relation to the number at the centre?

How do you describe the numbers: (a) above and below (b) to the left and right

in relation to the number at the centre of the square?



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