The Art of Guesstimation - Part 2 Using Fermi Problems in a Classroom Setting

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In the first part of this article published in the March 2024 issue, we learned about *Fermi Problems* and looked at some examples. Fermi problems challenge us to make educated guesses to solve complex questions with limited information. These guesses are based on intuition and common sense, which makes the problems engaging and relatable. Often connected to real-life scenarios, they allow us to apply basic mathematical concepts in practical ways. This makes them a valuable teaching tool in the classroom. In this second part, we will explore more examples of Fermi problems suitable for classroom use and discuss effective ways to present them to engage students.

Planning the classroom activity

As we saw in the first part, solving Fermi problems involves various strategies, assumptions, and estimations. In the classroom, it's effective to treat these problems as discussion-based activities. This approach requires careful planning and selection of problems that are appropriate for the learner's age, local context, and mathematical preparedness. In this section, we will explore a sample activity along with a few suggestions.

Every good activity starts with an engaging introduction. To kick off the discussion, the teacher can pose an intriguing, amusing, or surprising question that requires students to estimate an answer. For instance, consider these prompts:

- 'For how many seconds are your eyes closed in a year?'
- 'How long would it be if you laid all the strands of hair on your head end to end?'

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The key to a good introduction is simplicity and lack of ambiguity, ensuring every student can immediately engage by making and justifying their own guesses. With the provided examples, students might wonder whether blinking counts, how long they sleep each day, the average duration of sleep; the average length of a hair strand, or how many strands of hair they have on their heads. Such questions might further stimulate instant experiments!

Next, the teacher can guide students in making informed guesses and estimations. This might involve introducing the concept of Fermi problems with some standard examples - such as those discussed in the first part of the article -and working through their solutions. Discussing Enrico Fermi's story briefly could also be enlightening. The class could then be divided into small groups of two or three students each, with each group tackling a specific problem.

The teacher continues the activity by reading out the problem and facilitating a discussion on what the question means and how students might interpret it. It's useful to discuss that a range of answers might be acceptable. The students could make their immediate guesses, which they need to record. Then they could be asked to come up with a strategy to find a better estimation. This could involve a series of smaller estimation problems, each requiring reasoning and calculations based on everyday experiences and common sense. Observing how different groups approach the problem and the assumptions they make at each step can be a valuable learning experience. It is helpful if the problems are chosen so that a few steps can be experimentally verified (for example, measuring).

The teacher can help the groups define variables and come up with formulas to solve their steps better. She could then ask groups for their estimates to show the range of answers possible, and also periodically share the various approaches with the class. Students need to understand that we are seeking an order of magnitude estimate, not an exact number, which in the context of Fermi problems, is not possible to calculate.

Students should identify numbers that are likely to be too small or too large to be the answer. Next, they should make guesses that they think are close to the correct answer. During the final discussion, students should share their assumptions and estimates. They can compare their guesses to the answers they found and the data they gathered/justification of their answers to be made. It is useful to discuss whether the answer might change if the experiment were repeated, and what factors might contribute to different answers. Finally, the class could discuss open questions that generate more questions to investigate.

A sample activity

Let's explore an example of how an ideal classroom activity might unfold. Having already introduced the Fermi problems and divided the students into groups, the teacher now engages them in the problem-solving activity. The scenario is set in a 7th grade classroom.

Teacher: Alright, now that we know what Fermi problems are and how to approach them, here is a question for *you* to solve: How many people would be needed to surround the state of Madhya Pradesh if they all held hands?

[Students murmur, some look intrigued, others puzzled.]

Teacher: Before we start guessing, let's try to take up an easy problem. Suppose all of us stand side-by-side holding each other's hand and form a circle, how large will the circle be?

One way of answering this question is to find the length of your arms while they are stretched to the sides, making a 'T' shape. Then we can multiply the length by the number of children in the classroom. This would give us the *perimeter* of the circle. Do you think there will be any problems with this approach?

Student 1: Yes, some students have longer hands. So, the answer will depend on whose arm length is measured.

Teacher: Good observation! I have brought a measuring tape. Let us quickly measure the lengths of five children and check how much the lengths vary.

[The teacher measures and records the lengths on the board.]

Teacher: As you can see, we are measuring the lengths in *metres*. Depending on whose measurement is taken, the final answer changes. So, 0.9 metres multiplied by 25 (persons) =22.5 metres, and 1.1 metres multiplied by 25 = 27.5 metres. Because we are finally calculating a very large number, let us not worry so much about the decimals or one's place, but only the tens place. So roughly speaking, the length of your arms would be around 1 metre, and that multiplied by the total number of students would come between 20-30 metres, right?

Now, let us try to answer a slightly related, but different question: Suppose we are inside our rectangular classroom, standing close to the walls with our hands held together. How many children would be required to cover the room?

Student 2: To find that we should measure the perimeter of the classroom.

Teacher: Exactly! [Assuming the classroom is in a rectangle shape, the teacher measures the length and breadth of the room and asks the students to compute the perimeter.] Now that we know the perimeter of the room in metres, and we assume that each student's arms-length is 1 metre, so we can mention the unit of the length in terms of number os persons. This means we can say 25 persons instead of 25 meters.

Can you guess how many students it would take to surround the school building?

Figure 1: [Students make guesses without measuring the lengths.]

Teacher: Now, let us make our question a little interesting. How many people would it take to surround the school ground? What about the school compound? What about the village?

[Students make guesses and share their reasoning.]

Teacher: Do you think there will be any issue when we surround the village? Is the ground surface even?

Student 3: No, there is a lake nearby and some places are in higher elevation compared to others.

Teacher: Yes, this is a problem we would have to address by again assuming that the ground level is the same around the village. Also, we typically get the measurements in kilometres when we measure large distances, so we have to convert them to metres.

Now, let's come back to our original problem, here is the map of Madhya Pradesh. As you can see the boundary of the state is not even like a circle, so we need to make a few assumptions and find the perimeter of the state. Since most of the people are adults, whose arms-length is bigger than that of children, let us also assume that the arms-length vary from 1 metre to 1.5 metres. With these additional assumptions, let us now try to find the number of people it would take to surround the state.

[The activity continues.]

A few tips in order:

- This activity is also a great opportunity for students to practice their written and verbal communication skills. Students often enjoy sharing their findings with other students at the end of the activity.
- Solving Fermi problems often involves converting units and hence introducing *dimensional analysis* as a bookkeeping tool can be useful.
- Providing students with simple calculators can also expedite calculations.

In summary, it's helpful to guide students through a structured process instead of just making random guesses. Here's a revision of the step-by-step process:

1. Start with the question and make sure everyone understands it.
2. Make a rough guess without doing any calculations.
3. Make an educated guess with reasoning and calculations based on everyday experiences and estimates.
4. Define variables and create formulas to solve the problem.
5. Conduct experiments, measure things, and find information to improve estimates and figure out the smallest, largest, and most likely values for the answer.
6. Summarise the conclusions, note possible errors, and interesting facts learned, and suggest directions for future investigations.

Below, we provide a set of sample Fermi problems sourced from the internet, categorized by age range. Teachers can adapt these problems as needed to fit their classroom settings and local contexts.

Fermi problems for the age range of 4-8

Young students should have concrete Fermi problems that they can understand and complete. They should choose two occasions to pause and check their guesses - one early on and one around halfway through. After completing the task, students should use pictures, numbers or equations, and words or sentences to show what they did and what they learned.

Here are a few suggestions for Fermi questions for this age level.

1. How many blocks do we need to stack to reach your height?
2. How many stickers would cover this notebook?

3. How many punctuation marks (or letter a's, etc) are in this book?
4. How many rangolis would cover this hall floor?
5. How many bananas do we eat in a week?
6. How many jumps would you need to travel across the room?
7. How many beads would it take to make a bracelet that fits perfectly on my wrist?
8. How many tablespoons of water would fill this container?
9. How many cars pass by the school bus stop in a minute?
10. How many times do you blink in a minute?

Fermi problems for the age range 9-11

Students in this age range usually understand Fermi questions better if they can touch and interact with the things involved. They should at least start doing what the question asks physically, though it may not be practical to complete it.

Here are a few ideas for Fermi questions for this age level.

1. How many one rupee coins are needed to equal your height, the height of the school, the tallest building in the world, Mount Everest, and outer space?
2. How many grapes are needed to equal your weight, the weight of a car, the weight of the school building, of the Earth?
3. How many times would a one rupee coin roll to travel down the hall?
4. How many laddus are required to fill this room?
5. How many people would be needed to surround the state of Madhya Pradesh if they held hands?
6. If you prepared a tank with all of the air you need to breathe in one day, how large would it be?
7. How many blades of grass are in our school ground?
8. How many seconds does a student sleep per week?
9. What would it take to make a mural that runs the length of our school wall? (How much time, cost, people, material).
10. How much water is wasted by a leaking water tap in one day?

Helping students solve a Fermi problem

A good Fermi problem makes students ask more meaningful mathematical questions and solve them. For instance, if we ask about the cost of housing people in a rescue camp for a week, students would need to think about questions like how to source food, clothes, and other essential items. They'd also consider the cost and time for getting, storing, and transporting these things, which involves concepts learned from various units including geometry, ratio proportions, etc.

The internet is a great resource for finding many Fermi problems that can be modified for local contexts. It is also useful to remind the students of instances from local contexts where Fermi-type estimates are used. For instance, a farmer would like to estimate the yield of mangoes on his farm, a fisherman would like to estimate the yield of fish he would catch in a particular season, or a vendor would like to estimate the number of buyers during a festival season etc. It is also highly recommended that the students come up with their own Fermi problems, find strategies to solve them and share them in the classroom.

Conclusion

One of the aims listed under mathematics education in the NCFSE-2023 reads that

> "Developing an intuition for what should or should not be true in Mathematics is often just as important as the more formal 'paper-pencil' doing of Mathematics. Focusing on the common themes and patterns of reasoning across mathematical areas, guessing correct answers (in terms of, e.g., 'order of magnitude') before working out precise answers, and engaging in informal argumentation before carrying out rigorous proofs are all effective ways of developing such mathematical intuition in students."

Fermi problems provide a great opportunity for the students to not only develop intuition in terms of estimation skills but also provide them an opportunity to hone their mathematical communication skills.

Also, NCFSE-2023 lists, among others, one of the current challenges with respect to mathematics learning that

- "Mathematics learning has traditionally been more 'robotic' and 'procedural' rather than creative and aesthetic. This is a misrepresentation of the nature of Mathematics and must be addressed in the school curriculum.
- Very often, the content presented in textbooks to illustrate mathematical concepts is far removed from the contextual realities of the learners. Young students find some mathematical concepts easier to absorb when they are directly connected to their experiences. Textbooks, classroom activities, and examples should aim to be motivated by and related to students' lives whenever possible.
- There has also been a mistaken and exclusive emphasis on symbolic language and formalism in Mathematics teaching and learning, rather than on the informal argumentation and development of mathematical intuition that is so important for mathematical discovery."

Again, Fermi problems provide opportunities for the mathematics teachers to pose meaningful mathematical questions creatively that have direct relevance to the students' everyday lives. They provide another way to look at mathematics and explore. We urge you to try a few Fermi problems in your classroom and write back to us sharing your experiences.

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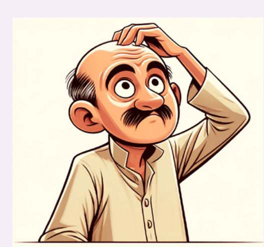
Here is a fun Fermi problem: Consider every person living on the earth. Suppose each person p has H_p number of hairs on their head. What is the product of all such H_p?



Answer: Zero. All that it takes is one bald person to make the calculation easy!