The Art of Guesstimation - Part 1

MOHAN R

This article explores the Fermi method, a quick and effective way to estimate large numbers with a minimal number of guesses. In this first part, we'll demonstrate the method with a few examples. Stick around for a list of intriguing Fermi problems for you to solve at the end!

The Fermi Method

Enrico Fermi (1901–1954) was an Italian physicist who made significant contributions to the fields of nuclear physics and quantum mechanics. His groundbreaking research on nuclear reactions caused by slow neutrons earned him the Nobel Prize in Physics in 1938. Right after winning the Nobel Prize, he moved to the United States to flee Mussolini's fascist regime. Four years later, he had produced the first sustained nuclear reaction in Chicago. This discovery paved the way for the development of atomic bombs and nuclear fission reactors, revolutionizing our understanding of nuclear energy and its potential applications.

Fermi's intellectual prowess extended beyond his scientific pursuits. He possessed a remarkable ability to make accurate estimates and solve complex problems with limited information. His unconventional approach to problem-solving, often relying on intuition and common sense, became known as the "Fermi method" or "Fermi estimation."

Fermi often entertained his friends and students by creating and solving unusual problems, such as "How many piano tuners are there in Chicago?". A 'Fermi Problem' asks for a quick estimate of something that seems hard or impossible to measure accurately.

Keywords: Fermi Method, Estimation, Creative Problem-solving, Reasoning

Fermi's approach to these problems was to use common sense and rough estimates to get a general idea of the answer.

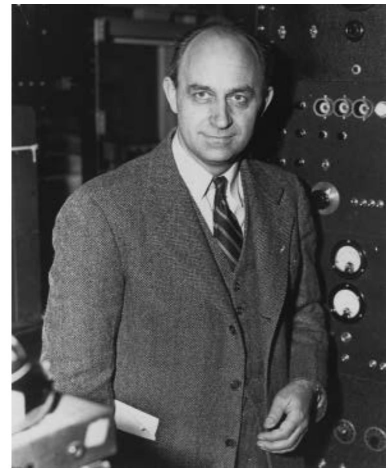

Figure 1. Portrait of Enrico Fermi. Source: Enrico Fermi. (2024, January 26). In *Wikipedia*. https://en.wikipedia.org/wiki/Enrico_Fermi

What are Fermi problems and how to solve them?

Let us consider, for instance, the problem of determining the number of piano tuners in the city of Chicago. How is this different from another estimation problem of determining how many seconds there are in a year? To solve the latter problem, we need to know the number of days in a year, the number of hours in a day, the number of minutes in an hour, and the number of seconds in a minute. All of them have a definite answer. Then we simply have to convert the unit of time into seconds per year.

$$\frac{365 \text{ \sout{days}}}{1 \text{ year}} \times \frac{24 \text{ \sout{hours}}}{1 \text{ \sout{day}}} \times \frac{60 \text{ \sout{minutes}}}{1 \text{ \sout{hour}}} \times \frac{60 \text{ seconds}}{1 \text{ \sout{minute}}} \approx 3 \times 10^7 \frac{\text{seconds}}{\text{year}}$$

In some sense, the latter problem can be solved through logical deductions with the data given in the statement of the problem itself. Fermi problems, on the other hand, differ markedly from the usual mathematical problem in that the answer to a Fermi problem cannot be verified by logical deductions alone and is always approximate. To know the exact number of piano tuners in Chicago, you may need to conduct a head count of all piano tuners in the city! All piano tuners may either not be listed in the telephone directory or cannot be found using a Google search.

Here's one way you could figure out how many piano tuners there are in Chicago:

1. Start by guessing how many people live in Chicago.
2. Guess how many homes there are in Chicago.
3. Estimate the fraction of homes that have a piano.
4. Guess how often each home gets their piano tuned.
5. Estimate how long it takes to tune one piano.
6. Guess how many hours a piano tuner works in a week.

When we're estimating, we break down the problem into smaller steps, and in each step, we have to make some guesses. Since we're aiming for a rough estimate, not an exact one, we just need to make sure our guess is within the right order of magnitude. We might overestimate or underestimate at each stage. For example, not only houses have pianos – public places and businesses might too. Some pianos might be tuned more or less often than we think. The law of probabilities says that if we make errors in estimates in different directions, they can balance each other out, and our final estimate will be closer to the right answer.

A similar solution for estimating the number of piano tuners in Chicago can be watched in the following TED-Ed video: A clever way to estimate enormous numbers - Michael Mitchell

Here are some sample Fermi problems and strategies to solve them.

Problem 1: How many bald people are there in the world?

Figure 2. It is impossible to count the total number of bald people in the world

A solution: There are about 8 billion people in the world. About half of them are women, who don't typically get bald. So, that leaves us with about 4 billion men. Most people who get bald are over 30 years old. So, we can divide the 4 billion men into two groups: 2 billion who are 30 or younger and 2 billion who are older than 30. The older group is the one with the bald people. We can estimate that about 10% of the men in the older group are bald. That means that, by this reasoning, there are about 200 million bald people in the world.

Problem 2: How many bicycle repair shops are there in Bengaluru?

A solution: This problem is similar to the piano tuner problem. Bengaluru is a metro city in India and a typical metro city would have a population of 10 million people. If there are roughly 4 people in a household, there will be 2.5 million households in the city. If we assume that roughly one in two households own a bicycle, then there are approximately 1.25 million bicycles in the city. Assuming that a bicycle requires a repair once every year, there are 1.25 million repairs each year, which is roughly 100,000 repairs per month. Assuming that a repair shop can handle 50 repairs per month, to meet the demand, we need to have 2000 repair shops in the city. Note that we cannot be sure if the number of bicycle repair shops is 2000 or 6000, but we know that the order should be in thousands. So the number cannot be 200 or 20,000.

Here are some practical Fermi problems from different areas:

- In environmental policy: "If we stop using plastic grocery bags, how much less trash will we produce?"
- In educational policy: "If a state limits the maximum class size to 35 students, how much more money will it cost each year to run the schools?"
- In public health: "There's a serious flu going around, and everyone in our country needs a vaccination from a health care worker. How fast can we get everyone vaccinated?"
- Personal finance: "A homemaker wants to start a morning tiffin shop to help with household expenses. Does she need to take a loan, and can she manage the business on her own?"

These examples illustrate the diverse range of applications for Fermi problems, demonstrating their usefulness in various fields where precise measurements are not always feasible. Here are more sample Fermi problems.

1. How many people in the world are talking on their mobile phones at this instant?
2. If everyone in your district donated one day's wage to a good cause, how much money could be raised?
3. How many kilometres of roads/rivers are there in your state?
4. How much petrol does a typical motorcycle use during its lifetime?
5. How far does a butterfly fly each day?
6. What is the current population of mosquitoes in your city?
7. What is the average lifetime of a pencil?
8. How much does it cost to leave a tube light on for an entire week/month/year?
9. How many hours of TV will you watch in your lifetime?
10. How long will it take to count to one million? to ten million?
11. How much milk is produced in India each year?
12. Assuming that a suitable drawing surface could be placed along the entire route, how many pencils would it take to draw along the equator of the Earth?
13. If you posted an advertisement in a newspaper, how many people would be likely to see it?
14. How much food does your school consume in a month?
15. How many trees would need to be planted to lower the average global temperature by one degree? (assuming global warming is reversible)

There is no doubt that Fermi problems are fun to pose and solve, but can they be used in a classroom situation? Let us explore that question in Part 2 of the article.

MOHAN R teaches mathematics at Azim Premji University. An algebraist by training, he is also interested in mathematics education and mathematics communication. He is the regional coordinator for the Mathematical Olympiad for Karnataka. He may be contacted at mohan.r@apu.edu.in