ClassRoom

Divisibility Rules for 7

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Divisibility rules are one of the important topics of study in school mathematics, especially in upper primary classes. They enable us to quickly identify if one number is divisible by another. We know various methods for checking the divisibility of a number by 2, 3, 4, 5, 6, 8, 9, 10 etc. It is also clear that checking of divisibility of the given number by some numbers is quite easy, while for some numbers is a bit complicated.

Divisibility by 7 is a challenging one, with many attempts made to simplify the rule. Chika's divisibility rule for 7 is a recent one among them. Here, we shall discuss three different divisibility methods for 7, using existing methods which add new dimensions to the concept.

Method 1: Doubling the unit digit

Take the given number	Remove the unit digit and write the truncated number	Double the unit digit which was removed	Subtract the doubled digit from the truncated number	If the difference is either 0 or a multiple of 7, then the original number is divisible by 7. (Repeat if necessary)
532	53	2 × 2 = 4	53 – 4 = 49	49 is divisible by 7 so 532 is also divisible by 7
427	42	2 × 7 = 14	42 – 14 = 28	28 is divisible by 7 so 427 is also divisible by 7
29792 2975 287	2979 297 28	2 × 2 = 4 2 × 5 = 10 2 × 7 = 14	2979 – 4 = 2975 297 – 10 = 287 28 – 14 = 14	Repeat for 2975 Repeat for 287 14 is divisible by 7 so 29792 is also divisible by 7
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With the above examples, we understand that this method is useful for checking divisibility by 7 without performing long division for a 3-digit number, but is quite lengthy for 4 or more-digit numbers.

Keywords: Factors, divisibility, checks, rules, justification

Justification of the rule

Suppose $N = 1000\, a_3 + 100\, a_2 + 10\, a_1 + a_0$

(Where a_0, a_1, a_2, a_3 are the digits of the 4-digit number N)

According to the rule, we write the truncated version (say N_T) without the unit digit of N and then take away (subtract) from N_T twice the unit digit to get a new number (say M).

$N_T = 100\, a_3 + 10\, a_2 + a_1$ (Note the change in the place values after the number is truncated)

$M = N_T - 2a_0 = 100\, a_3 + 10\, a_2 + a_1 - 2a_0$

Our rule says that if M is a multiple of 7, then N is also a multiple of 7.

Assume that M is a multiple of 7, i.e. $M = 7k$ for some whole number k.

Then, $M = 7k = 100\, a_3 + 10\, a_2 + a_1 - 2a_0$ or $100\, a_3 + 10\, a_2 + a_1 = 7k + 2a_0$

Substituting this in N, we get

$N = 1000\, a_3 + 100\, a_2 + 10\, a_1 + a_0$

$N = (1000\, a_3 + 100\, a_2 + 10\, a_1) + a_0 = 10(100\, a_3 + 10\, a_2 + a_1) + a_0$

$= 10(7k + 2a_0) + a_0 = 70k + 21a_0 = 7(10k + 3a_0)$

So, if M is a multiple of 7, then so is N.

This can easily be generalized to any number of digits.

Method 2: Multiplying the unit digit by 5

Take the given number	Remove the unit digit and write the truncated number	Multiply the unit digit by 5	Add the result to the truncated number	If the sum is either 0 or a multiple of 7, then the original number is divisible by 7 (Repeat if necessary)
378	37	8 × 5 = 40	37 + 40 = 77	77 is divisible by 7 so 378 is also divisible by 7
2464	246	5 × 4 = 20	246 + 20 = 266	Repeat for 266
266	26	5 × 6 = 30	26 + 30 = 56	56 is a multiple of 7, So 266 and 2464 are divisible by 7
29792 2989 343	2979 298 34	2 × 5 = 10 9 × 5 = 45 3 × 5 = 15	2979 + 10 = 2989 298 + 45 = 343 34 + 15 = 49	Repeat for 2989 Repeat for 343 49 is a multiple of 7 so 343, 2989 and 29792 are divisible by 7
Try 2308012 now				

One may provide a justification, which is very similar to the previous one as follows.

Suppose $N = 1000\, a_3 + 100\, a_2 + 10\, a_1 + a_0$

(Where a_0, a_1, a_2, a_3 are the digits of the 4-digit number N)

According to the rule, we write the truncated version (say N_T) of N and add five times the unit digit to get a new number (say M).

$N_T = 100\,a_3 + 10\,a_2 + a_1$

$M = N_T + 5a_0 = 100\,a_3 + 10\,a_2 + a_1 + 5a_0$

Our rule says that if M is a multiple of 7, then N is also a multiple of 7.

Assume that M is a multiple of 7, i.e. $M = 7k$ for some whole number k.

Then, $M = 7k = 100\,a_3 + 10\,a_2 + a_1 + 5a_0$ or $100\,a_3 + 10\,a_2 + a_1 = 7k - 5a_0$

Substituting this in N, we get

$N = 1000\,a_3 + 100\,a_2 + 10\,a_1 + a_0$

$N = (1000\,a_3 + 100\,a_2 + 10\,a_1) + a_0 = 10(100\,a_3 + 10\,a_2 + a_1) + a_0$

$\quad = 10(7k - 5a_0) + a_0 = 70\,k - 49\,a_0 = 7(10k - 7\,a_0)$

So, if M is a multiple of 7, then so is N.

This can easily be generalized to any number of digits.

Method 3: Grouping of digits (Rule – 1-3-2)

Take the Number	Make groups of three digits starting from the unit digit	Multiply the right-most digit by 1, the next by 3 and the left-most by 2 in each group	Add all odd-numbered groups	Add all even-numbered groups	Difference $\lvert c–d \rvert$
	a	*b*	*c*	*d*	*e*
N_1 = 672	672	6 × 2 + 7 × 3 + 2 × 1 = 35	35	0	35
Result	$\lvert c–d \rvert$= 35 is divisible by 7. So, the number N_1 is also divisible by 7.				
N_2 = 4704	004 704	4 × 1 = 4 7 × 2 + 0 × 3 + 4 × 1 = 18	18	4	$\lvert 18 – 4 \rvert$ = 14
Result	$\lvert c–d \rvert$= 14 is divisible by 7 So, the number N_2 is also divisible by 7.				
N_3 = 32921	032 921	3 × 3 + 2 × 1 = 11 9 × 2 + 2 × 3 + 1 × 1 = 25	25	11	$\lvert 25 – 11 \rvert$ = 14
Result	$\lvert c–d \rvert$= 14 is divisible by 7. So, the number N_3 is also divisible by 7.				
N_4 = 197526	197 526	1 × 2 + 9 × 3 + 7 × 1 = 36 5 × 2 + 2 × 3 + 6 × 1 = 22	22	36	$\lvert 22 – 6 \rvert$ = 14
Result	$\lvert c–d \rvert$= 14 is divisible by 7. So, the number N_4 is also divisible by 7.				
N_5 = 164953525268	164 953 525 268	1 × 2 + 6 × 3 + 4 × 1 = 24 9 × 2 + 5 × 3 + 3 × 1 = 36 5 × 2 + 2 × 3 + 5 × 1 = 21 2 × 2 + 6 × 3 + 8 × 1 = 30	30+36 = 66	21+24 = 45	$\lvert 66 – 45 \rvert$ = 21
Result	21 is divisible by 7. So, the number N_5 is divisible by 7.				

This is yet another way of checking for divisibility by 7. Let's illustrate it step by step.

1. Starting from the unit place of the number, make groups of three digits. The last group will contain the remaining digits.
2. In each group, multiply the right-most digit by 1, the next by 3 and the left-most by 2.
3. Add all the products obtained in each group.
4. Find the sums of the odd and even-numbered groups.
5. If the difference of these two sums is divisible by 7 or is 0, then the original number will be divisible by 7.

Justification of the rule

Suppose $N = 100000\,a_5 + 10000\,a_4 + 1000\,a_3 + 100\,a_2 + 10\,a_1 + a_0$

(Where $a_0, a_1, a_2, a_3, a_4, a_5,$ are the digits of the 6-digit number N)

$S_1 = a_0 \times 1 + a_1 \times 3 + a_2 \times 2$ $\qquad$ $S_2 = a_3 \times 1 + a_4 \times 3 + a_5 \times 2$

$M = S_1 - S_2$

Our rule says that if M is a multiple of 7, then N is also a multiple of 7.

Assume that M is a multiple of 7, i.e. $M = 7k$ for some whole number k.

$7k = (a_0 \times 1 + a_1 \times 3 + a_2 \times 2) - (a_3 \times 1 + a_4 \times 3 + a_5 \times 2)$

$\quad = (-2a_5 - 3a_4 - a_3 + 2a_2 + 3a_1 + a_0)$

$N = (100002a_5 - 2a_5) + (10003a_4 - 3a_4) + (1001a_3 - a_3) + (98a_2 + 2a_2) + (7a_1 + 3a_1) + a_0$

$N = 7(14286a_5 + 1428a_4 + 143a_3 + 14a_2 + a_1) + (-2a_5 - 3a_4 - a_3 + 2a_2 + 3a_1 + a_0)$

$N = 7(14286a_5 + 1428a_4 + 143a_3 + 14a_2 + a_1) + 7k$

So, if M is a multiple of 7, then so is N.

This can easily be generalized to any number of digits.

Note: Rule – 1-3-2 can be used for any number with 2 or more digits. It can help us to find the divisibility of any number by 7 easily and quickly as well.

Comparison

Method	Operations needed	Remark
Doubling the unit digit	×, –	Useful for 2 or 3-digit numbers.
Unit digit is multiplied by 5	×, +	Useful for 2 or 3-digit numbers.
Rule 132	×, +, –, grouping	Useful for more than 3-digit numbers.

Explorations such as this help teachers plan lessons in which students *develop capacities for problem-solving, logical reasoning, and computational thinking. Students become comfortable in working with abstractions and other core techniques of Mathematics and Computational Thinking, such as the mathematical modelling of phenomena and the development of algorithms to solve problems.* (NCF-SE 2023).

If the teaching of divisibility rules stops at practising number skills, then we are severely limiting the potential of such a rich topic. Asking why the rule works, trying to generalise it, comparing different rules and then trying to make their own rules will not just develop mathematical minds but also impart an understanding of the joy and beauty of the subject.

Reference

1. http://publications.azimpremjifoundation.org/2306/1/3_Chika%27s_test_for_divisibility_by_7.pdf
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Math is a cake walk!

At Right Angles November 2023 Challenge

We divided a delicious chocolate cake into 12 pieces and served each piece in a half plate!

Here is your challenge!

How many math questions can you make from this situation?

Send in your questions to AtRiA.editor@apu.edu.in

Response from reader
Rohini Khaparde
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1. How many plates will be needed to serve two-thirds of the cake?
2. If out of 12 pieces only 8 are to be served, then what is the ratio of the number of plates needed to the total number of plates needed to serve the full cake?

Response from reader
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1. If the radius of the original circular cake before cutting is 'r' and the cake is cut into 12 equal pieces, each served on a half plate with a radius of 'p', express the ratio of the area of one cake piece to the area of one half plate in terms of 'r' and 'p'.
2. If someone ate one-third of the cake, what fraction of cake is remaining?
3. What percentage of the cake is <u>on each plate</u> if you consider the whole cake as 100%?
4. If someone eats 3 pieces of cake, what fraction of the whole cake has he consumed?
5. If we want to share the cake equally among 4 people, how many pieces would each person get?