Reinforcing the Learning of Integers

A Process

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VOICES

I decided to review the work we had done earlier before we began working on numbers with class VII students. The procedure for creating number groups of natural and whole numbers and integers, as well as the need for doing so, were discussed in class when the definitions of number groups were being explained to the students. Integers are used in situations of loss, debt, and deficit, so practice was done using integer addition and remainder operations.

Problems faced during work:

- Students were asked about their knowledge of natural, whole, and integer number groups. According to them, whatever is discovered in nature is natural; if one adds 0 to it, it becomes a whole number. Integers, including negative numbers, are examples of those that are not natural. Students did not include natural numbers in this. They were connecting number groups to both naturally occurring and man-made concepts. In reality, the existence of numbers is seen only on the number line. Whether they are natural numbers or any other kind, all numbers are abstract.
- Some students had a problem with just thinking of 0 (zero) as a whole number, not thinking of 0 (zero) as an integer, and only thinking of 2 as a natural number. Integers were being perceived by them more as negative numbers. Therefore, there was a need to clarify the number-group hierarchy.
- Students could use previously-learnt integer operation concepts to do basic addition and subtraction. A majority of the students erred by thinking in terms of borrowing to answer the straightforward statement with brackets on both sides, such as 18-4, 18-(-4), -18-4, -18-(-4): In each of these four issues, the borrowing context needs to be altered.

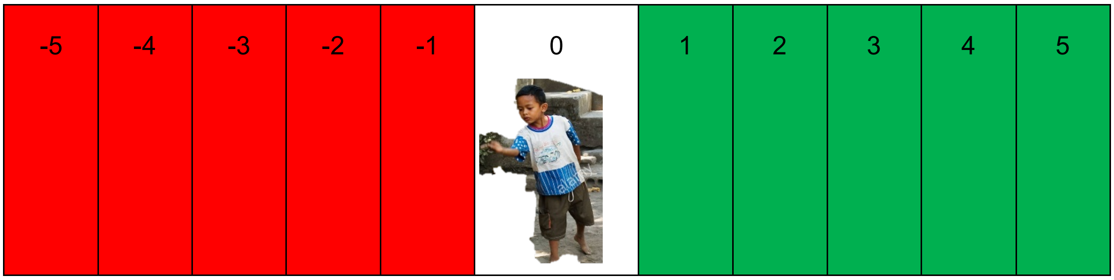
Given the aforementioned issues, it seemed that in not allowing students to learn the operation rules on their own, they had ended up memorising them and using them incorrectly. I felt the need to engage in additional activities with the students after considering the following:

- Everyone knows the rule that (-) and (-) become (+) when multiplied. There was a need to practice generating rules by manipulating the number line, spotting patterns, and paying close attention to the subtraction sign and the negative sign.
- Students were having trouble because they were solving the problem by comprehending it in terms of profit and loss rather than using the skill of looking at the number line and using the number line to solve the problem.
- The students were in a hurry to respond and there were certain instances where if they had taken some time to think, deliberate and engage in a dialogue, they would have been able to move ahead correctly.
- The questions were created by the students. But these were similar to the problems that had already been answered, such as 9-4, -9-4, 9-(-4), -9-(-4)
- Considerable thought should go into the context that will be taught to upper primary students. It must be ensured that their future learning is not hindered by this context. We must also take into account whether it will speed up or slow down the abstraction process.
- By demonstrating patterns, one should proceed from generalisation to rules. It is acceptable for students to formulate their own queries, but it is important that they comprehend the reasoning that went into their coming up with the question.
- Students frequently play the game of hopscotch. Could this game serve as practice for integer operations by building a house on the number line?

Game worksheet

Not every student was able to play the number-line game at home. However, it was evident that those who could, were able to see that the numbers were on either side of 0. Secondly, it was also clear that students could understand decreasing and increasing sequences, but more work was needed to help them understand what was happening mathematically as they moved from one point to the next.

Integer Game



Rules of the game:

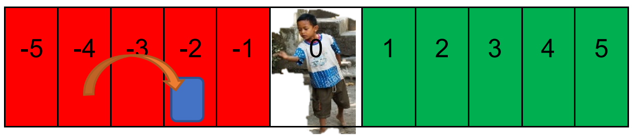
1. We must begin at zero, so we must stand at zero.
2. The player must toss the marker from zero but start with the negative numbers.
3. The marker must first be thrown at -1, returned to zero, and then recorded. The player of the team will be eliminated if the marker does not reach zero.
4. Players must record everything they do mathematically.
5. The + sign should be placed when moving from 0 to the right, and the - sign should be placed when moving from 0 to the left.

Example: Raju is playing while also filling out the table below.

Step 1: Raju is positioned at zero. He throws the marker at -4. His current position is 0-4=-4.

Names of students	**Position 1**		**Position**
	Situation on throwing stone	Attempt to bring the stone	Out or another turn
Raju	0-4 = -4	-4+2= -2	Out since did not come back to 0
Rashmi	0-2 = -2	-2-3=-5	Out since did not come back to 0
Anwar	0-2 = -2	-2-(-3)=1	Out since did not come back to 0

Step 2: Raju now moves to position -4 and strikes the marker with his foot, causing it to advance two positions to position -2. The mathematical representation of this will be: -4 + 2 = -2

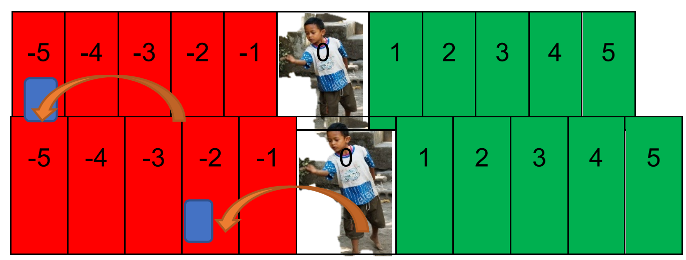


Rashmi took the stage next and performed the following moves:

Step 1: Rashmi is standing at zero. She throws the marker at -2, therefore 0-2=-2.

Step 2: The stone now advances three positions back, or to the left-5 position, after Rashmi strikes the marker by moving to position -2. This will be written mathematically in this way:

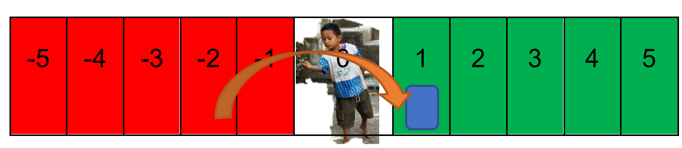
-2 -3 = -5



When Anwar got his chance, he played as follows:

Step 1: Anwar tosses the marker at -2 while standing at zero, so 0-2=-2.

Step 2: Now, when Anwar reaches -2 and strikes the marker, it advances three places, or to 1stplace. It will be expressed mathematically as follows.



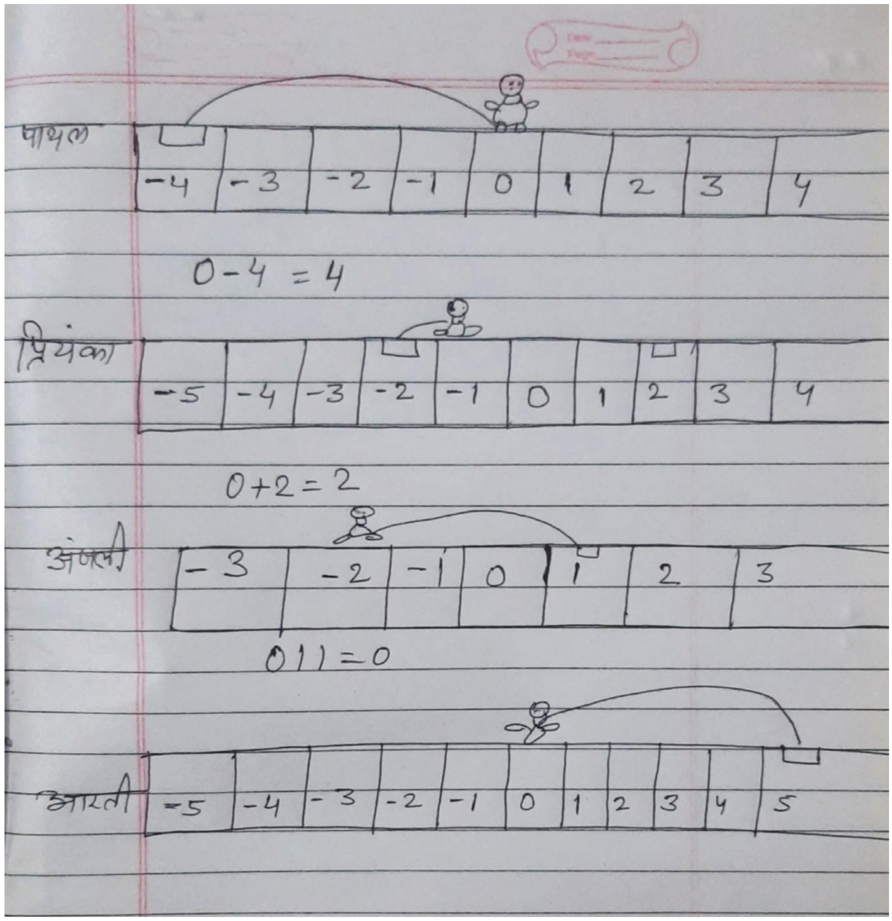
Anwar writes it as: -2-(-3) =1

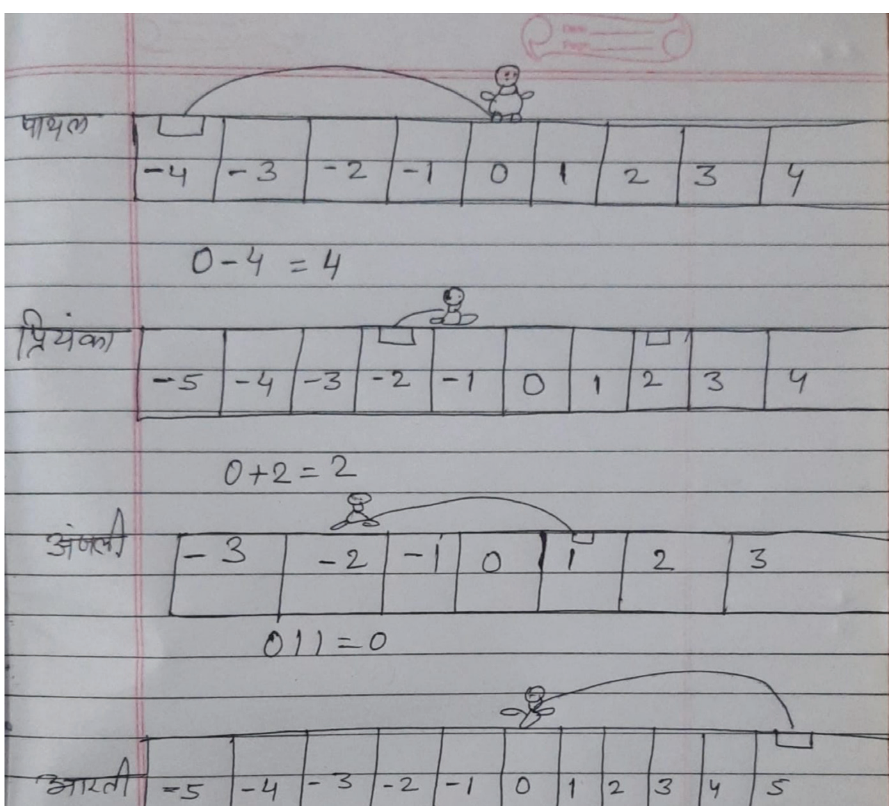
He wrote this because when Rashmi went to the left when she got -3, its opposite could be -(-3).

Similar to this, you can set up a table, play the game, and involve others.

Figure 1 . The integer game as it was played in the classroom.

This game was played in pairs in the classroom. One student was asked to write the mathematical representation while the other (their partner) was given the task of playing it.





Figures 2 - 3. This is how the students were representing on paper the game that their partners were playing.

All the other students were instructed to watch both of these procedures and get ready for their turn. The students' confusion and mistakes were primarily because of the following:

1. The location of where they were standing had to be written first using symbols. (For this, they were given chances to begin at 0 the first time and at a different house the second time.)
2. It is necessary to use the appropriate symbol to indicate whether to move the token in the direction of ascending order or the direction of descending order. (For ascending order use + sign, for descending order use - sign.)
3. They had to understand: What is the absolute value of how many houses we move? For this, whichever direction they went, they had to count the number of houses.
4. Finally, telling their position on the number line, would be the correct answer.

In fact, it was the students who came up with these four stages. Initially, if the solution to the problems they were doing was identified, they were given a chance to try again. So, after about 45 minutes of practice, keeping this understanding of the game in mind, several addition and subtraction drills were done using the number line on the blackboard.

During the process, we noticed that some students' focus shifted from calculating -3 +5 on the number line to solving the integers on the number line by looking at both numbers separately. Instead, the focus should have been on how -3 to 5 are occurring in one process. Therefore, I removed the integers on the number line and made the students do exercises on the number line classified only with 0.

A majority of the students in the class were able to understand these simple equations using the number line. After that, the worksheet on integers was explained, and the students were asked to solve it on their own.

Here is an example of students' completed work.

The worksheet shows that students were able to compare whole numbers accurately, but when they saw negative numbers, they were confused. I realised that the students were only focusing on the numbers. So, their attention was drawn again to the number line and more practice was done for how the value of a number increases when it moves to the right from its original place, while it decreases when it moves to the left.

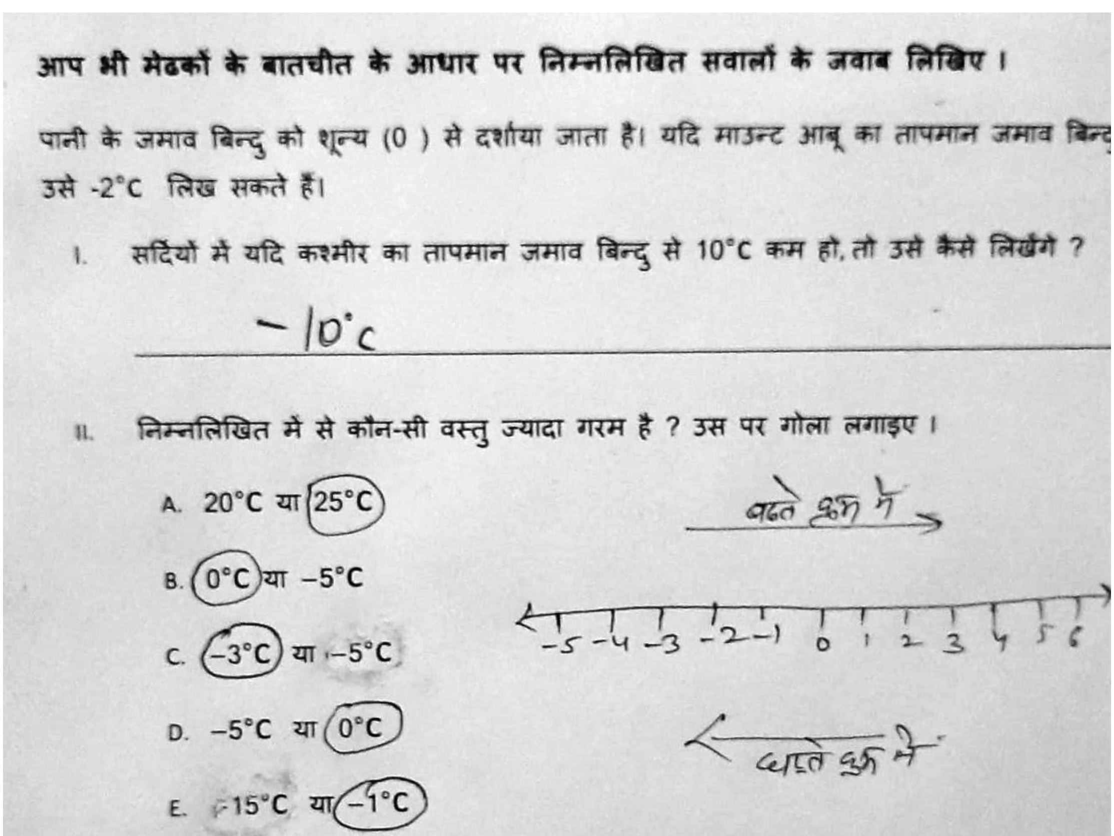


Figure 4. When children were given problems of comparison, they were able to solve some problems correctly but were not able to visualise these on the number line.

Following this, the students checked their own work and attempted to fix it. They discussed a lot among themselves; gathered around the board, and decided which number would be bigger and which number smaller. They came to the conclusion that if a number is on the right side of the number line, it will be bigger and if it is on the left, it will be smaller.

Two students began to discuss that they would not be drawing the number line every time they had to make such mathematical calculations. So, they requested some additional work that could be done without the use of the number line. Everyone agreed.

Figure 5. Students doing problems by drawing the number line on the board.

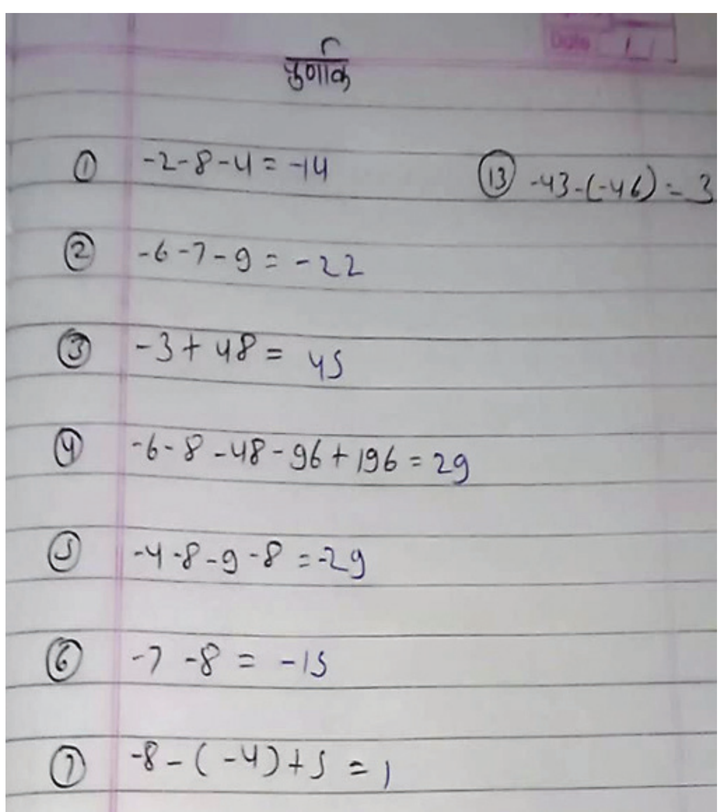


Figure 6. Students attempted these problems without using the number line.

In the same way, the students also tried to solve the question from their textbooks, which they did correctly to a great extent; along with this, they also tried to solve the magic square.

There were some conversations as the children solved the problems, sharing how they were approaching the rules, for example:

- Numbers with identical signs should be added to indicate the direction because they either increase to the right or decrease to the left.
- When a question was given as - 43 - (- 46) then the children got confused - there were two signs here. In response, one child answered, 'one - (minus) means going backwards, another - (minus) means 'reverse', so considering the fact that the reverse of backwards is forward, therefore, we will go forward.

Multiplication with integers

After this, we proceeded with problems involving jumping numbers.

In working with children on multiplication of integers, I thought of the following types of questions:

4×3

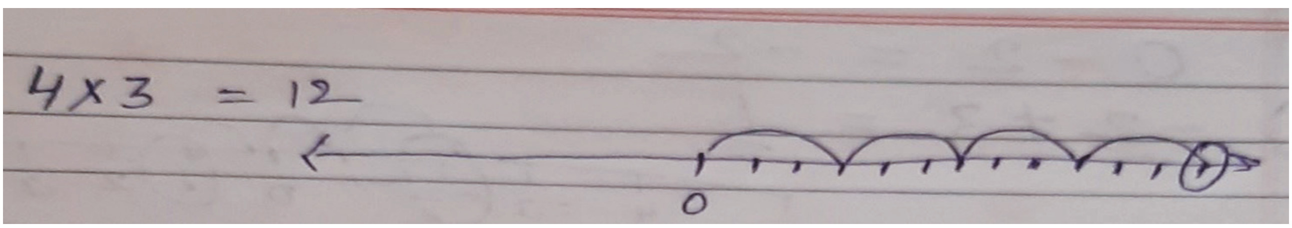


Figure 7. Here, it was clear on the number line that one has to jump 3 houses 4 times in the positive direction.

4×-3

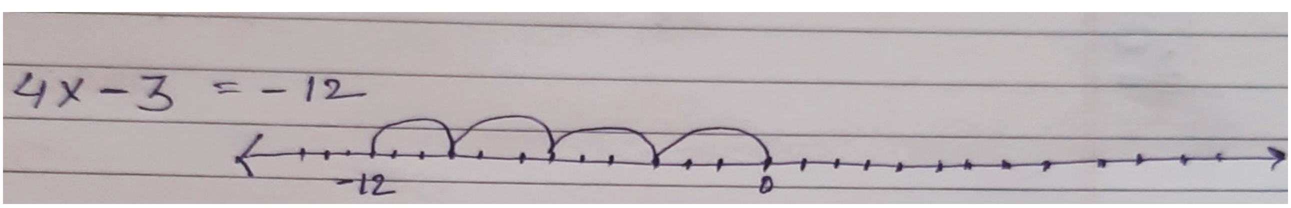


Figure 8. Here, one has to jump 4 times in the negative direction.

-4×3

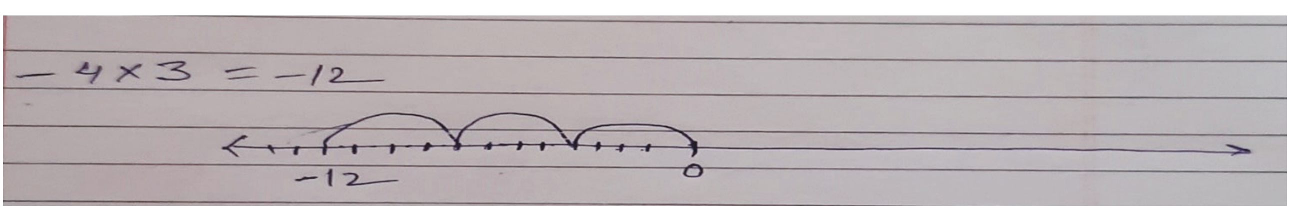


Figure 9. In this, one has to jump 4 houses, three times in the negative direction.

-4×-3

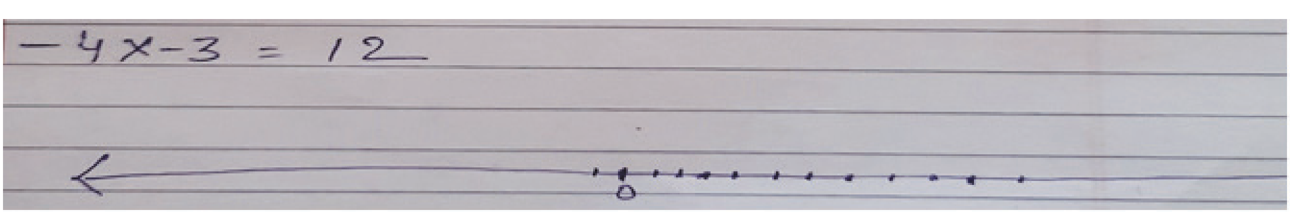


Figure 10. Here, I explained that 4 jumps have to be made in which 3 houses each have to be jumped in the negative direction. But because there is a negative sign in front of 4, it means that it is the opposite of what is being indicated. Instead of jumping towards the negative, we have to jump towards the positive, thus, the answer would be: -4×-3 = 12.

Jumping numbers

While we were discussing this, I realised that similar to jumps, students had already mastered the idea of repeated addition in previous classes. Could we use the concept of repeated addition to help students understand this better?

To do this, I planned an activity with dice. I am sharing it here. It would be great to know the views of other teachers – which is better explaining with dice or using the number line?

When the multiplier is negative, it is not possible to 'say it that many times', so we interpret it in our activity as follows:

When the multiplier is negative, multiply a counter equal to the multiplier by the value of the multiplier a given number of times, then invert all resulting counters because of the negative sign of the multiplier.

For example, 4 (-3) = -12

So, if 4 green (positive) dice are taken 3 times and 3 has a negative sign, all the dice are flipped over, turning them all red, and the result is -12.

Likewise, -4 x (-3) = 12

Our learnings

If the following considerations are made while teaching integers in the classroom, then students advance with understanding and their conceptual knowledge is also better.

- Considerable thought should go into the references used in upper primary classes to ensure that the context set at this level does not interfere with students' learning in the future. We should also consider if the examples used and the understanding they impart will speed up or slow down the abstraction process. For example, when whole numbers, natural numbers and integers were first introduced to the students on the number line, they were only considering 0 as a whole number and all negative numbers as integers. Later, I used different contexts, and these numbers were identified in different ways to clarify this. More such questions should be discussed at the beginning so that the children can develop their understanding.
- By demonstrating patterns, teachers should proceed from generalisation to rules. Students should develop their own inquiries and comprehend the reasoning behind them. For example, when the children were solving integer-related problems, they themselves grasped the pattern of what steps to take in each situation. For example, one student said that they could add all the negative numbers and positive numbers separately and then subtract these two. For the same sign, another student observed that 'it is again going in a positive direction', that is, the multiplication of negative numbers is positive. This is how the children began to arrive at inferences on their own.

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