Reinforcement and Practice in Mathematics

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There is a general agreement that learning maths requires practice essential to reinforce ideas that are to be absorbed. Though we know that learning needs to be understood as the ability to use what has been learnt in situations that require them, many people report that much of the maths that they learnt is no longer accessible to them when they need to use it. For example, unlike the shopkeeper who does the sale and purchase calculations quickly and efficiently, many of us are not confident of our ability to do so, or even write down and calculate as we did in school and prefer to use a calculator.

Our problem is that as teachers in classrooms, and more generally in interactions with children who are to learn these ideas, we find it difficult and perhaps unimportant to find ways to create situations that are comfortable and natural for the learner. We not only find it difficult to think of how to help a child learn but often do not know what and how this can be done.

The element of practice in formal mathematics teaching focusses on teaching specific methods to solve specific types of problems or even a particular problem, rather than on enabling students to find their own strategies to solve those.

> Learning maths has to be based on ensuring that basic concepts develop and evolve in the minds of the learners. As more complex ideas get introduced, the conceptual structure needs to be modified, requiring the readjustment of ideas that have been formed in the early stages.

The practice must force the learner to think about this constantly and be aware of the differences. It must enable the learner to automatically notice the mathematical objectives that have to be worked upon and interpret the task, choose the steps and arrive at the answer accordingly. The tasks given, whether in the classroom or as homework, should require the learner to think and engage with the mathematical objects along with the underlying concepts and not merely follow procedures mechanically.

Existing methods of teaching

The nature and form of practice in maths classrooms are generally very different from what has been described as essential here. Walking past a primary classroom, one is used to and still can in many schools, hear the class resounding with chorused voices, led by either the teacher or a student. The participation in these choruses is energetic and everyone seems to be actively involved in whatever is being recited repeatedly. Reciting number names in sequence or repeating them are considered useful and important elements of practice. Repetition chants for remembering the sequence of steps in an algorithm (set of rules in a process) are also composed. Schools use different devices aimed towards the same goal, some using gadgets and some, TLMs. All formulas for calculation of simple interest, addition, subtraction, division and multiplication of whole or fractional numbers or using decimal numbers have reinforcement items created that help in repeatedly doing the same steps or repeatedly recalling the same facts. The questions are: Is this the kind of practice that is relevant for the learning of maths? Does this help in the 'real' learning of maths?

Concepts and reinforcement

There are many ways to look at what is central to maths learning and how reinforcements can be planned – an aspect particularly important to maths. One evidence of maths learning is in the fact that the learner is able to solve problems, but the difficulty is that the range and kinds of problems expected to be attempted are limited to the exercises, which need to follow solved examples in the chapters. The chapter is taught by the teacher explaining the method by which the solutions have to be arrived at, detailing out the problems a little more. The teacher may solve some of the exercises from the book and students copy those solutions in their notebooks. The prescribed textbooks are also such that they give elaborate solved examples and provide techniques that are to be remembered and followed. A few textbooks that are written slightly differently are not used even if prescribed. The expectation is clearly that the kind of detailed examples and problems that are given in the books would be the kind of problems that would be asked in the assessment or even in competitions where the only expectation is 'cracking the test'.

Current methods of reinforcement

If we examine the ways in which reinforcements generally happen, we come across two frequently used directions; the key principle of both being arriving at the solution of the problems as the main object of the reinforcement. This is achieved by providing simple steps to solve problems that are likely to be included in assessments either by remembering the solution or having a simple, mechanically-followed procedure. This could mean either repeating the same questions or doing them repeatedly till they are mastered (in this case, remembered), again for testing.

For example, as mentioned, in many schools, pre-primary to class V have loud chanting and copying sessions in which children are given rhyming ways of reciting number names or multiplication tables and even addition tables. These tasks often appear engrossing and particularly interesting to children as these are routine and simple, and children develop their strategies to complete the copying tasks in ways that they have to do minimum work. There are classroom observations that suggest that while copying numbers, say from 1 to 100, children complete the task by writing single digits without even thinking about the two-digit numbers they are writing!

The second kind is the practising of a method, thereby reinforcing it. It is essentially an opportunity to repeat the task till the steps are memorised. When learners in the early stage of learning are given numerical sums of operations to solve, the practice focuses on making them work on numbers by writing these in columns place-value-wise and doing the operations column-wise by mechanically carrying over and borrowing. They are not looking at what the numbers that they are adding are or the results they are getting. This extends to multiplication and division as well, when they rigorously follow particular algorithms without any sense of the numbers that they are starting or ending with and goes even further to operations on fractional numbers and, later, to solve equations. The reinforcement tasks are so constituted that the learner can only follow the method being taught and explained through the examples. Very often, the mixed exercises even recommend the particular method that would be the most suitable for the problem. The objective of this type of reinforcement is to learn the steps to be followed in each of the specified methods.

Clearly, in the second kind of reinforcement, learners get problems that are identical in nature to the solved examples and, while these may have different numbers, there is no change in the method to be adopted. The method could be a standard procedure of solving or a shortcut created, as in the case of fractions. Each operation has different steps as a part of the solving procedure. However, these methods are not taught in the classrooms as concepts that lay the foundation for newer ideas that will come later but as standalone procedures or finished techniques that have to be followed like a manual. This means that the reinforcement does not help conceptually for further learning and interest in maths. Instead, it sets the expectation that all subsequent tasks, including that of assessment, would be similar. The learners would not have to think and/or use a mixture of methods or try to find a suitable method for the problem given. The expectation is that the question would clearly indicate what has to be done and the procedure and steps to be followed are made obvious.

Developing conceptual ability

In conversations with learners who are given a set of random but level-appropriate problems from the type of exercises that they should be familiar with, what emerges is their anxiety about not knowing which operation is required and on which numbers. They have 'learned' how to do all operations in specific known situations, but do not know what each of these operations really mean. They have been given practice on using the procedures of operating on fractions but may not even know that the term 'fraction' denotes the denominator and numerator together. Or that the digits in decimal numbers (after the decimal point) are part of a number and are no longer independent numbers. This seems obvious but, in our hurry to make learners proceed with the operations, we do not allow them to engage with the underlying concepts and give them rules to follow mechanically and produce results of operations without having a sense of the numbers they are operating with or the result they are arriving at. This is made worse by an overload of a battery of methods, tips, shortcuts and procedures to follow, resulting in confusing procedures, with no idea of the range in which the result should lie. Thus, the learners are unable to review their results or think about the procedures followed.

What we need to remember is that practice and reinforcement are important, but only when they are not mechanical and repetitive. Tasks given to children must stretch their minds and give them the opportunity to develop the capability to read and understand texts related to maths and then devise processes for the solution of newer problems, making them feel confident when facing them. As the Position Paper on Mathematics (NCERT 2006) says, 'Children should be able to construct non-trivial problems to develop conceptual ability and confidence'.

> The point is that if we think of school maths as an activity that provides links between what is learned and life experiences, we have to adopt a different mode of teaching in which the ideas of spatial transformation, mathematical patterns, symmetry, ratio and proportion etc, do not remain confined to notebooks and classrooms but interact with the way learners organise things and their thoughts, communicate ideas, and the designs they make. These ideas need to become a part of the intuitive way in which a learner looks at and absorbs the world around them. This means that the learner becomes comfortable in dealing with mathematics and using it to look at the world. In order to be able to do this, it is essential that we underline the fact that learning involves understanding, otherwise it is not learning.

What practice in maths means

Practice must mean opportunities to struggle with problems that are new so that they are able to separate what is given from what has to be found. Practice and reinforcement must include tasks of reading related to mathematics, such as excerpts from textbooks and other sources. Texts chosen could be those that aim to build conceptual understanding, new types of problems to engage with, texts about mathematicians and the development of mathematical ideas among other things. The ability to visualise and find methods to explore and do mathematics retains interest and curiosity in and about mathematics. What is to be ensured is that just one way of solving does not get specified and the children who lag behind have the time to engage at their own pace even as the class tasks move at the pace of those who appear to be brighter. The purpose of learning to read excerpts from textbooks and other materials is to ensure that children become capable of reading and comprehending mathematical texts with competence, confidence and interest.

Another significant aid to retaining an interest in mathematics is to have practice and discussions among small groups on mathematical content. The effort spent to solve a problem or unpack a text as a team serves many purposes. It has, however, to be complemented with time for individual practice, then discussing the work with the teacher or presenting it to the whole class for its participation in improving the task done.

The other crucial issue that we must think about all the time is the nature of exploration tasks and the kind of problems assigned. From what we have discussed, some points are clear. For example, these tasks must not only help develop the ability to solve problems (not just the set given and pre-done but any new problem of appropriate level) but to also find steps to unpack any problem to some extent. Practice, therefore, must mean opportunities to struggle with problems that are new so that they are able to separate what is given from what has to be found.

Reflection, re-articulation and reformulation

As children explore maths, they can be encouraged to think of the numerous ways in which they can obtain any particular number. For example, the number 10 can be obtained from: 5x2, 10x1, 6+4, 15-5, etc. Or comprehend that all squares are rectangles and that they are also parallelograms and quadrilaterals as they have all the properties of a general quadrilateral and some special properties that all quadrilaterals do not have. Or that even though rectangles and squares have equal opposite sides and equal angles since all the sides of a rectangle are not necessarily equal, all rectangles are not squares.

This ability to visualise, generalise and particularise properties of sets helps in making such connections. If there is comfort and confidence with mathematical ideas more and more connections will continue to emerge for the learner thinking about them. For example, the idea of concrete fractional numbers from life experience (half a sweet, three-quarters of the class, etc) to fractions as division, needs modifications of previous understanding, which develops a sense of the properties of concepts. Practice and reinforcement must take cognisance of these and provide opportunities for reflection, re-articulation and reformulation. Such opportunities cannot come merely from solving problems based on a set of rules.

Understanding as reinforcement

One fascinating aspect of mathematics (like with words in a language) is that one can play with mathematical objects and create generalisations and formulations which are new for the learner who has just discovered them. Allowing a learner to articulate the manner in which they arrive at the knowledge gives them both confidence and clarity, leading to a more complex understanding of the basics. The logical reasons that they formulate for the connections made, help develop the notion of and ability to comprehend and develop mathematical proofs for themselves. Rather than having to remember proofs as tasks, understanding what proving something means and the logical, necessary formulae necessary for such proof is greater reinforcement than being given proofs of standard theorems, identities or statements.

This sort of reinforcement and practice must lead to finding exceptions to a mathematical statement or formulating mathematical statements for others to disprove. This practice can also help in the understanding of how a mathematical statement is made and why it needs to be specific.

Conclusion

To put it briefly, practice must be engaging, and it must require thinking. It must generate curiosity and exploration. Practice is not about remembering formulas, rules and definitions, but about a student being able to develop their own definitions, generalisations and methods of solving problems when looking at a problem, trying to understand, uncovering what has to be found and developing a way to move forward.

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