Problem Corner

Haras Numbers

HARAGOPAL R

Definition: A Haras number is a n-digit number $(n \geq 3)$ with the property that it is equal to sum of all the $(n-1)$-digit numbers whose digits are digits of the original number, *the circular wrap-around order being retained.*

For example, the 3-digit number $\boxed{abc}$ is a Haras number if $\boxed{abc} = \boxed{ab} + \boxed{bc} + \boxed{ca}$, i.e., if

$$100a + 10b + c = (10a + b) + (10b + c) + (10c + a),$$

which reduces to $100a + 10b + c = 11(a + b + c)$, and then to

$$89a = b + 10c.$$

Similarly, the 4-digit number $\boxed{abcd}$ is a Haras number if $\boxed{abcd} = \boxed{abc} + \boxed{bcd} + \boxed{cda} + \boxed{dab}$, i.e., if $10^3a + 10^2b + 10c + d$ is equal to

$$(100a + 10b + c) + (100b + 10c + d) + (100c + 10d + a) + (100d + 10a + b).$$

This reduces to $1000a + 100b + 10c + d = 111(a + b + c + d)$, and then to

$$889a = 11b + 101c + 110d.$$

Keywords: Circular wrap-around property

The 5-digit number $\boxed{abcde}$ is a Haras number if $\boxed{abcde} = \boxed{abcd} + \boxed{bcde} + \boxed{cdea} + \boxed{deab} + \boxed{eabc}$. This reduces to

$$10^4 a + 10^3 b + 10^2 c + 10d + e = 1111\,(a + b + c + d + e)\,,$$
$$\text{or: } 8889a = 111b + 1011c + 1101d + 1110e.$$

The general pattern may be seen from the above. The circular wrap-around property ensures that when we compute the sum of all the $(n-1)$-digit numbers whose digits are digits of the original number, each digit occurs with the same multiplicity.

More generally, the n-digit number $\boxed{a_{n-1}a_{n-2}\ldots a_2 a_1 a_0}$ is a Haras number if

$$10^{n-1}a_{n-1} + 10^{n-2}a_{n-2} + \ldots + 10a_1 + a_0 = 111\ldots 1\,(a_{n-1} + a_{n-2} + \ldots + a_1 + a_0)\,,$$

where $111\ldots 11$ has $(n-1)$ digits.

Let us now solve these equations and obtain some instances of such numbers. On solving the equations, we discover to our surprise that there only one such 3-digit number and only one such 4-digit number. Let us see why.

The case of 3-digit numbers.

The 3-digit number $\boxed{abc}$ is a Haras number if $89a = b + 10c$. We must find all possible solutions to this equation, a, b, c being digits with $a \neq 0$. Since b, c are digits, the quantity $b + 10c$ is at most 99, which implies that $a = 1$. Hence $b + 10c = 89$, and it is easy to deduce that $c = 8$ and $b = 9$. Hence the number is 198. (So, there is just one possibility.) We verify that it does satisfy the property:

$$\mathbf{198 = 19 + 98 + 81}.$$

The case of 4-digit numbers.

The 4-digit number $\boxed{abcd}$ is a Haras number if $1000a + 100b + 10c + d = 111(a + b + c + d)$. Note that this condition implies that $\boxed{abcd}$ is a multiple of 111 and therefore a multiple of 3. Hence the sum of the digits is a multiple of 3. This implies that $\boxed{abcd}$ is a multiple of 333 and therefore a multiple of 9. Hence the sum of the digits is a multiple of 9. This implies that $\boxed{abcd}$ is a multiple of 999. Hence $\boxed{abcd}$ is one of 1998, 2997, 3996, 4995, … The sum of the digits of all these possibilities is 27 (all four-digit multiples of 999 have this form), so it follows that $\boxed{abcd} = 111 \times 27 = 2997$. (So, there is just one possibility.) We verify that it does satisfy the property:

$$\mathbf{2997 = 299 + 997 + 972 + 729}.$$

The case when the number of digits is 5 or more.

For larger numbers of digits, we may use similar reasoning (as we did above), but it is simpler to resort to a computer-assisted search. Here we report only the results.

- 5-digit numbers: there are just three such numbers: 13332, 26664, and 39996. Thus, we have

  $$\mathbf{39996 = 3999 + 9996 + 9963 + 9639 + 6399},$$

  and similarly for the others.

- 6-digit numbers: there is just one such number: 499995. Thus, we have:

$$\mathbf{499995} = \mathbf{49999} + \mathbf{99995} + \mathbf{99954} + \mathbf{99549} + \mathbf{95499} + \mathbf{54999}.$$

- 7-digit numbers: there is just one such number: 5999994.
- 8-digit numbers: there are just three such numbers: 23333331, 46666662, and 69999993.
- 9-digit numbers: there is just one such number: 799999992.
- 10-digit numbers: there is just one such number: 8999999991.

HARA GOPAL R works as a mathematics teacher in Municipal High School, Kurnool, Andhra Pradesh. He is very interested in finding new properties and relationships between numbers and geometrical figures. He may be contacted at rharagopal@gmail.com