

Radius (त्रिज्या) and Sine (ज्या) – a study of the Names and their Relationship

DR. KOMAL ASRANI

India has had a long-standing relationship with mathematics, going back thousands of years. Starting with the invention of zero, computing the value of pi, defining the trigonometric functions and computing their values for various angles, solving quadratic equations, giving rules for operations with negative numbers, computing the square roots of numbers, ...; the breadth and depth covered are vast. Typically, results and formulas were expressed in compact, verse form.

In this short paper we look at a linguistic aspect of the work done in ancient India in trigonometry; namely, the names given to certain quantities. Specifically, we examine the relationship between Radius and Sine regarding their Hindi names. In Hindi, Radius is referred to as “त्रिज्या” and Sine is referred as “ज्या.” The objective of this paper is to examine whether if there is any relationship between the Radius and the Sine values of some specific angles, त्रिज्या and ज्या, i.e., does the condition of 3 times sine (ज्या) equal to the radius (त्रिज्या) have any significance?

The word ‘ज्या’ means *chord* or *rope*. The word ‘ज्या’ is used in *Brhatsamhitā*, an encyclopaedic Sanskrit work written by Varāhamihira. We find references to this word in the *Shulba*

Keywords: Indian mathematicians, linguistics, exploration, trigonometry.

Sutras, which are among the oldest works written by ancient Indian mathematicians on geometry [1].

The Indian contribution to trigonometry was significant during the Gupta period and the work had relevance to astronomy. Aryabhata discovered the sine function and described the same in *Surya Siddhanta*. The three trigonometric functions studied by Aryabhata were the ज्या (sine), कोटि ज्या (cosine), and उत्क्रमज्या (tan) [3]. The Sanskrit word ज्या went through numerous adaptations and variations. After a few centuries, it ended up as *Sine* (a Latin word), the term in use today.

Discussion from perspective of Radius and Sine.

In modern mathematics, 'त्रिज्या' is referred to as radius and 'व्यास' is referred as the diameter. व्यास means *disjoined*, which breaks or distributes the circle into two halves (two semicircles).

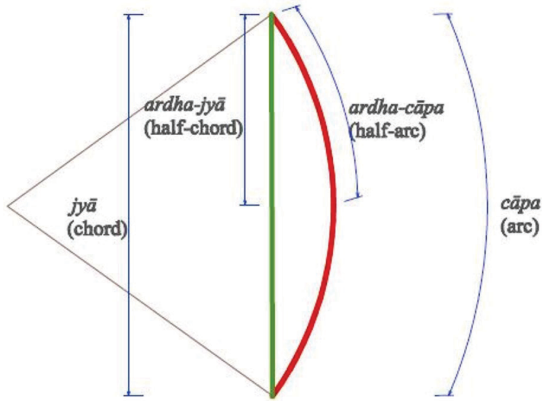


Figure 1

Referring to Figure 1, the arc of a circle is referred as धनु in Sanskrit. When the extremities of an arc are joined, like the string of the bow, a chord of the circle referred to as 'ज्या' is formed. Later, it was identified that the half chord or *ardha-jyā* is of greater relevance than the full chord [2]. Hence the qualifier *ardha* was omitted and 'ज्या' was used. Thus 'ज्या' gave reference of radius for computation purpose. The relationship between arc and the chord was defined by Bhaskara II as - "What is really the

arrow between the bow and bowstring is known as *Versed sine*."

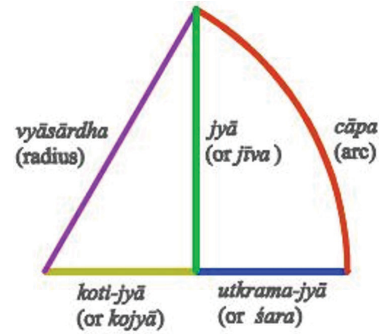


Figure 2

The arc of the circle looks like a bow and is called *dhanu*. As seen in Figure 2, when the arc of the circle 'धनु' subtends an angle of 90° at the center, it is called a quadrant of a circle or *vritta-pāda*. It is well known that there are 12 zodiac signs in astronomy; each zodiac sign defines an arc of 30° . According to Bhaskara I, "Three signs form a quadrant and these quadrants are distinguished as odd and even." The same was extended by Bhaskara II as - "Three signs form a quadrant and a circle is formed of four quadrants. These quadrants were again divided into odd (*ayugna, visama*) and even (*yugma, sama*)."

Interpreting the quotes of Bhaskara I and II, three consecutive zodiac signs define a quadrant or *vritta-pāda*. Further, the *ardha-jyā* of an arc of 90° in a circle is equal to the radius of the circle (since an arc of 180° corresponds to a semicircle, the corresponding chord is a diameter of the circle, which is twice the radius; hence the *ardha-jyā* corresponds to the radius). Keeping in mind that the qualifier *ardha* later got deleted, it makes sense that "the *jyā* of an arc of 90° in a circle is equal to the radius of the circle." Hence the term 'त्रिज्या' was coined by Hindu mathematicians to denote "the *jyā* of three signs," i.e., "the sine of three zodiac signs."

Note that this is only our conjectured explanation for the linguistic connection between these two terms. Though it seems very plausible, we may never know the full story.

References:

1. Satyanarayana, Dr Bhavanari “Geometrical Concepts in Indian Ancient Works,” 10.13140/2.1.3355.5529, 2013.
 2. K. Ramasubramanian, M. S. Sriram, M. D. Srinivas, “Ganita-Yukti-Bhasa (Rationales Mathematical Astronomy) of Jyesthadeva: Vol. 1 – Mathematics” Springer.
 3. https://hy.w3we.com/wiki/History_of_trigonometry
-



DR KOMAL ASRANI is currently serving as Professor in the Department of Computer Science and Engineering in BBDNIIT, Lucknow (India). She had obtained her Ph.D. in Computer Science Engineering. She also completed Diploma in Vedic Mathematics. Her areas of interest are Object Oriented Programming, Computer Graphics, Image Processing and Computation Analysis. She has published a number of papers in various International and National Journals. She has also conducted various Workshops for Vedic Mathematics. Currently, she is working to explore the application of Vedic Mathematics in Computer Science Engineering. She may be contacted at komalasrani74@gmail.com