How Much or Till What: When and Why?

MATH SPACE

One of the decisions policy makers, syllabus and textbook writers need to consider for mathematics at the primary level (Class 1-5) or foundational and preparatory stages (pre-school to Class 5 or 3-11yrs), is what is the biggest number children should be introduced to at various classes. The National Curricular Framework for Foundational Stage (NCF-FS) has made its recommendations. Here is another take at that along with some justification and interlinkages with other topics within the subject.

Numbers have essentially 3 aspects – (i) the quantity each represents, usually the cardinality of a set for whole numbers, (ii) the number name (e.g., thirteen or hundred and five) and (iii) the numeral (e.g., 13 or 105). Learners are supposed to establish a 3-way mapping linking these three aspects together. It may be helpful to separate out (i) and (ii) before connecting (iii) with the same, at least initially.

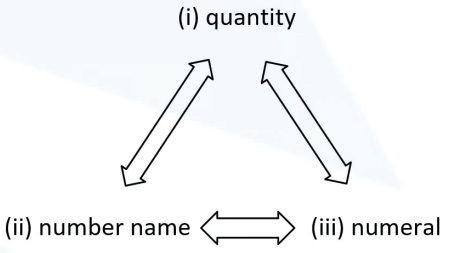

Figure 1

Keywords: Number sense, numerals, number names, math pedagogy

Usually, the sequence is as follows:

Class 1	• 1-5 or 1-9 and possibly some addition-subtraction • 0 • 0-20 along with addition-subtraction with these numbers • Exposure to 0-100
Class 2	Extensive engagement with 0-100 – place value and addition-subtraction
Class 3	Playing with 0-999 or 0-1000
Class 4	0-9999
Class 5	0-99999
Class 6+	even bigger numbers

The usual tussles include whether to stop at 9, 99, 999 etc. or to go till 10, 100, 1000 etc. Here are our suggestions and the justifications for the same:

One to Ten: This is connecting (i) the quantities with (ii) the number names. Numerals don't come into this part. Children are essentially learning to count. We would say that a child can count till ten if,

1. S/he remembers the number name sequence – one, two… ten
2. S/he correctly counts a given collection of ten or fewer objects
3. Given a number ≤ ten, s/he picks up exactly that many objects from a larger collection

Fingers are very useful in counting. Most of us have 10 fingers and that is considered to be the reason that we have a base-10 system (place value) of writing numbers. So, it feels artificial to stop at nine.

The main reason for excluding ten is that it is a 2-digit number. But if we consider connecting quantity with number name (and leave out the numeral aspect) then that reason is no longer relevant.

1-9: Based on the knowledge children have acquired, they can be nudged to extend their understanding of numbers to the numerals for 1-9. This can be achieved using double sided cards, one side showing the numeral while the other show the quantity as dots.

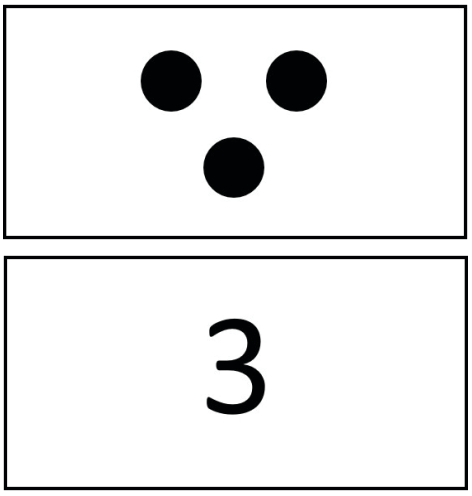

Figure 2

It is advisable to initiate addition-subtraction after this as is done in several textbooks.

Zero (0): Introduction or zero as nothing and the numeral 0 representing the same. This can be done as showing a quantity decrease by one gradually till nothing is left (as done in several textbooks). And then the numeral can be introduced using a similar card with 0 on one side and nothing, i.e., no dot on the other.

0-20: This should be done in several steps as follows:

a. Connecting quantity with number names for eleven to fifteen (say)

b. Initiate bundling, first in twos, threes, etc., and then in tens

c. Discuss numeral of 11-15 as number of bundles followed by number of loose sticks outside the bundle, ganitmala can be used to discuss why the number of tens, i.e., the ten's digit is on the left of the number of ones, i.e., the one's digit (see reference for further details)

d. Discuss what the numeral for ten should be by problematizing and drawing upon children's knowledge of zero

e. Extend understanding of numbers 0-20

It is advisable to continue addition-subtraction with 0-20 after this. It is possible to introduce doubling and halving (as divide equally in two groups) at this point as well.

0-100: Extend numbers further with bundles and loose sticks. It makes sense to stop at 10 tens, i.e., hundred and give children a glimpse at how bigger bundles must come in when we have 10 of a kind. Hundred is a number that is encountered quite often in day-to-day lives as currency and as century (in a cricket loving country in particular). Including 100 provides an opportunity to understand how the numerals are obtained for bigger and bigger numbers – one of the basic ideas of place value: if there are ten of a kind (loose sticks or bundles), make a new (and bigger) bundle.

The chart of 1-100 with ten rows and ten columns is also a widely used teaching aid. Stopping at 99 would create a hole in that.

0-1000: Thousand is the next bigger bundle, i.e., 10 hundreds. And is used in the standard units with respect to kilo and milli (1km = 1000m, 1kg = 1000g, 1m = 1000mm, 1g = 1000mg and 1l = 1000ml). Therefore, thousand is also a number associated deeply with day-to-day life. So, instead of stopping at 999, it makes sense to go till 1000 especially if standard units are discussed in the same class.

Inclusion of 1000 also allows children to observe the above-mentioned basic idea of place value play out with another set of bundles, viz., hundreds.

This is where subtraction poses newer difficulties for problems like 500 – 162. Also, where all four operations get introduced.

Beyond thousands, the next bundle name is lakh. It comes after bundling thousands twice;

(10 thousands = 1 ten-thousand, 10 ten-thousand = 1 lakh) and is not used that often in day-to-day lives. So, we feel it is ok to restrict to 4-digit (i.e., up to 9999) and 5-digit (till 99,999) numbers. This is also because unless lakh is introduced, the main implication of these restriction is limited to the numbers used in the four operations, specifically, sums, products and dividends should not exceed 9999 or be within 4-digit. Same restriction applies to minuend or the first (or bigger) number in a subtraction. The need for sums, minuends, products or dividends beyond 4-digits does not occur that naturally in the contexts of primary mathematics.

One should also keep in mind that these restrictions (till 999 or till 1000) are possibly needed at the policy level to provide guidance to curriculum/syllabus/textbook developers, teachers and to track progress of children's learning levels in a large system. These do NOT come from mathematics. In fact, mathematically speaking, numbers naturally grow especially as children start playing with them, and explore further (e.g., palindromic numbers). So, if children are able to engage with multi-digit numbers with ease, i.e., add-subtract and possibly also multiply-divide, then a teacher should not put an artificial restriction on the largest number they encounter.

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References:

Swati Sircar: 'Why TLM – its aims and uses', *Learning Curve*, Issue 3, December 2018, p.56

MATH SPACE is a mathematics laboratory at Azim Premji University that caters to schools, teachers, parents, children, NGOs working in school education and teacher educators. It explores various teaching-learning materials for mathematics [mat(h)erials] – their scope as well as the possibility of low-cost versions that can be made from waste. It tries to address people at both ends of the spectrum, those who fear or even hate mathematics as well as those who love engaging with it. It is a space where ideas generate and evolve thanks to interactions with many people. Math Space can be reached at mathspace@apu.edu.in

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