

A COLLABORATION OF THE MATHEMATICS COMMUNITY ATAZIM PREMJI UNIVERSITY, BENGALURU


## ISSUE 1 | OCTOBER 2023

 STATE EQUATE RELATEMATHEMATICS, YOU SEE, IS NOTASPECTATOR SPORT. TO UNDERSTAND MATHEMATICS MEANS TO BEABLE TO DO MATHEMATICS. AND WHAT DOESITMEANTOBEDOINGMATHEMATICS?"

## A LETTER FROM THE TEAM

Dear Reader,

As the October rains announce themselves outside our B2 FG windows, we would like to introduce the first-ever issue of Mathaapu!

What is this "mattāppu"/"math-uh-poo" we speak of, you ask? Pronounced however you'd like, Mathaapu is a collective effort by the students majoring in Mathematics and the faculty from the Mathematics Group of the School of Arts and Sciences, Azim Premji University, Bengaluru, materialized as a magazine.

It features articles about Math born out of fierce debates in class, stories of sudden eureka-like moments in late-night tutorials, ideas amidst the monotony of an alarmingly long problem set, or even impromptu chats over cups of coffee in the cafeteria.

We hope that these stories paint you a colorful picture of what it means to do Mathematics and that it sparks more conversations about this subject that seems to draw us together.

Grab that coveted couch at the library and ready yourselves as we take you through 'State-Equate-Relate', Issue \#1 of Mathaapu!

Happy reading,
Team Mathaapu.

## IN THIS ISSUE, WE BRING TO YOU...

Understanding

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Picturing Math: Doodles,
Diagrams, and Insights
The Math Shelf:
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GRACE HOPPER TEACHING

# மததாபபு 

pronounced mattāppu noun. firecracker

I shall let you in on a moderately well-kept secret...
*pause for suspense*

People associated with Mathematics at Azim Premji University, Bengaluru are die-hard fans of George Pólya! What's more, if you weren't one before, like me, you will soon be coaxed, convinced, and pulled in by the enthusiasts sooner than you expect.

> "Pólya was arguably the most influential mathematician of the 20th century. His basic research contributions span complex analysis, mathematical physics, probability theory, geometry, and combinatorics. He was a teacher par excellence who maintained a strong interest in pedagogical matters throughout his long career."
-H Taylor and L Taylor, George Pólya: Master of Discovery (Palo Alto, CA, 1993).

As a former skeptic, I invite the eyebrow-raisers, the on-thefencers, and the proud fans alike to join me in attempting to understand exactly what it is about Pólya that seems to have us charmed so. Run through these pages as different students majoring in Mathematics present interpretations of one of his most famous questions- just what does it mean to work on our subject?

Although we cover a range of perspectives, the articles, artwork, and accounts in this magazine can be captured in a three-word summary- state, equate, and relate. Learning Mathematics, in this issue, can be understood through the lens of stating, equating, and relating. Whether it is an explanation of algorithms in multiplication or thoughts on learning the subject, our contributors have deeply reflected on how "doing" Mathematics takes shape in different circumstances.


PERIODIC DRAWINGS BYESCHER INSPIREDBY PÓLYA

So, sit back and take a moment to appreciate the puns that we've been generous with, the resources we've snuck in, the raw sincerity thrumming through our magazine, and the thoughtful conclusions drawn.

It is our hope that this issue celebrates the comfort of learning our subject together and leaves you with a mattāppu-like zeal to chase the promise that this space holds in doing Mathematics.
03.

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# "THE PRACTICE OF MATH MEANS ASKING QUESTIONS, MAKING CONJECTURES, EXPERIMENTING, TALKING, COMING UP WITH WRONG ANSWERS, LEARNING FROM MISTAKES, COMING UP WITH A CORRECT ANSWER..." 

# PICTURING MATH: DOODLES, DIAGRAMS AND INSIGHTS 

Quite often, we find ourselves drawing doodles. It may be a boring class where we land up making a doodle of the teacher's head or it may be a long phone conversation with a friend and suddenly we find ourselves with a doodle of geometric patterns, leaves or just clouds. We also draw pictures and diagrams when we are listening to a lecture or trying to solve a problem. What is the difference between doodles and diagrams and is this difference important? Does it help us with learning and doing mathematics? Is a diagram itself a mathematical object?

A doodle is very often drawn spontaneously (without planning), is very rough, and is an expression of the mental thought-processes of the doodler. Many students and researchers doodle to help focus as well as keep track of their thought processes and make notes.


WHAT IS DOODLING (L)
OODLE BY VAISHNAVI KR DOODLE BY $\underset{(R)}{\text { VAISHNAVI KR }}$

Maryam Mirzhakhani was awarded the Fields Medal in 2014 for her deep contributions to geometry and dynamical systems. In particular, she looked at curves called geodesics (curves connecting two points that have the shortest distance) on surfaces that are curved and on which one can do complex analysis. Visualizing the curves and these surfaces is hard, so doodles and pictures help in getting insights. Maryam would use long reams of paper to make doodles, so much so that her young daughter thought that her mother was an artist.


DOODLE BYMIRZAKHANI
Mathematical representations have a wide variety: figures on the plane, two-dimensional graphs to represent functions and planar curves, representations of threedimensional objects on the plane to understand surfaces, representation of solutions of differential equations, dynamical systems, combinatorics, graph theory, and so on.

Each of these representations helps in different ways: to understand and represent change, break down complex phenomena into understandable bits, guess patterns, raise further questions, help in coming up with proof, etc.

Here is another beautiful example of a picture that shows that the sum of odd numbers is always a square of some number. This could be thought of as a visual proof or proof without words. Such "proofs" are very helpful in giving insight and as an educational tool.


So visual representation can help not only get insights but also a visual way of understanding the result and a way of proving it.

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## Diagrammatic Representations of a Knot

While representations are useful in all branches of mathematics，the real power can be seen in geometry and topology．It is really fascinating to see how different visual representations of a knot lead to a rich understanding not only of knots but a variety of interconnected areas of mathematics．A knot is described by a function $f$ ：［0，1］to R－ three，such that $f(0)=f(1)$ and which does not intersect itself．It would be a natural question to make a list of knots．Obviously，we do not want to add a knot to the list if it has already appeared on the list．It seems daunting to find which of these are＂same＂，based on these abstract descriptions．This is where diagrams help．Here we give different ways of describing knots through diagrams and each of these is a rich area of study．The various diagrammatic representations help us build connections within Knot Theory as well as with other branches of mathematics．


PROJECTION OF TREFOIL KNOT（L）
CONVERTINGAAKNOTAIAGRAM INTOANOTHER KNOT DIAGRAM GRID DIAGRAM OF TREFOIL KNOT（R）


A GRAPH CORRESPONDINGTO A KNOT

To tie this all up，we posed a question as to whether doodling is different from drawing．Sunni Brown has explained this beautifully in ＇The Doodle Revolution：Unlock the Power to Think Differently＇，and we feel that both doodling and drawing diagrams would enhance our mathematical understanding in different and surprising ways．

Are diagrams mathematical objects？Yes，in the sense that when we start representing an idea／concept／process（in mathematics）it helps us understand things better，illustrate our thoughts，helps us prove theorems and calculate things and convey this to a fellow mathematician．

The world of visual representation is a rich source for not only understanding mathematics better but also opening up new directions for investigation and helping us understand the connections between branches of mathematics．They truly help us understand the richness of the mathematical tapestry．

## INTERESTED IN KNOWING MORE ABOUT MATHEMATICS AND REPRESENTATIONS？



## BE SURE TO CHECK OUT THIS ARTICLE IN QUANTA MAGAZINE AND SCAN THE OR CODE TO VISUALIZE PROBABILITY AND STATISTICS

WRITIENBYSHヘNiHへ BHUSHヘN
F＾CUITY



# "EVERYTHING <br> AROUND YOU IS MATHEMATICS" 



SHAKUNTALADEVI


## THERE ARE TWO MOTIVES FOR

 READING A BOOK; ONE, THAT YOU ENJOY IT; THE OTHER, THAT YOU CAN BOAST ABOUT IT.BERTRAND RUSSELL



Welcome to "The Math Shelf," where we recommend books for math patrons! We'll feature books about math, books that use math, and books that are just plain fun.

So whether you're a math whiz or just someone who's curious about the world of numbers, be sure to check out the reads we recommend! We promise you'll find something to your liking.


ALBERTEINSTEIN

Our top picks for the year include Charles Seife's, 'Zero: The Biography of a Dangerous Idea', 'Professor Stewart's Cabinet of Mathematical Curiosities' by Ian Stewart, and Eugenia Cheng's, 'How to Bake Pi'.

Our Math faculty have also suggested a few of their favorite reads, like 'Flatland', 'Mathematics and Its History', and 'Godel, Escher, and Bach: An Eternal Golden Braid' for the semester, be it for learning or leisure!

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SOBEAT THEMONSOONBLUESOR QUEST AFTER YOUR PERSONAL MATHEMATICAL BIBLE THIS YEAR BY PICKING ONE OF THESEREADS OFFOF THIS METAPHORIC SHELF!



PAULERDÖS


The Math（e）matics of Learning and Teaching：
A Character Study
कोई गणित से इत्तफाक ना रखे ऐसा नहीं होगा। गणित का साथ，जैसा भी रहा हो，साथ रहा जरूर है। कुरूप से लेके सुंदर तक，फिजूल से लेके महत्वपर्णू तक और मात्र औजार से लेके विश्वव्यापी दर्शन तक， सारे विशेषण गणित से जोड़ने वाले आपको मिल जाएंगे। मेरा और गणित का साथ उपरोक्त सभी विशेषणों से，गैर रेशीयता से，（नॉन－ लिनिअरली）गुजरता हुआ आज उपजीविका का साधन बनने तक पहुँच गया है।

ओशो अपने स्कूल के दिनों की एक कहानी में गणित का किस्सा सुनाता है। जिसमें यूक्लिड के एक्सिअम्स पर उसके विचार वो प्रकट करता है। उससे पहले वो गणित की उपयोगिता निःसंदेह कुबूल भी करता है। ओशो के शिक्षक ने जब बताया कि, "रेखा की सिर्फ लम्बाई होती है, मोटाई नही होती," तो ये बात बाल ओशो को हजम नहीं हुई। उसने पलट के प्रश्न किया के ये कैसे हो सकता है? शिक्षक ढंग का जवाब नहीं दे पाया और ओशो का गणित से साथ छूट गया। गणित से शायद इतना दिल टूटा के आगे चल कर ओशो यह भी स्थापित करने की कोशिश करता है के जो ज्ञान सच पर नहीं बल्कि हाइपोथिसिस पे टिका हो वो ज्ञान नहीं, बोझ है।
‘नोट्स फ्रॉम द अंडरग्राउंड’ में लेखक दोस्तोएवस्की, दो गुना दो चार होता है, इससे चिढ़ा हुआ है। वो गणित को यह कह कर धुतकारता है के भावनाओं और प्रेरणाओं का कोई समीकरण नहीं होता। वो गणित/रेशनलिटी को दीवार की उपमा देकर खदु को एक सांड के रूप में देखता है। वो सांड जो दीवार पर सर पटक रहा है, उसे तोड़ने के लिए।
‘दी सॅस ऑफ़ एन एंडिंग’ एक प्रसिद्ध उपन्यास है। इसमें लेखक, एड्रियन (किताब का एक किरदार), की आत्महत्या का जटिल भावनिक रहस्य बनाने और उसके बाद उसी रहस्य को खोलने के लिए एक समीकरण का इस्तेमाल करता है। इस दौरान लेखक गणित को बिलकुल तोड़-मरोड़ देता है, उसके समीकरण का गणितीय भाषा में कोई मतलब नहीं बनता, पर लेखक उस किरदार की पार्श्वभूमि और पूरे उपन्यास की रचना के जरिये उस समीकरण को अर्थबोध देने की भरसक कोशिश करता है।


गणित पढ़ानेवाले की हैसियत से मुझे बाल ओशो，दोस्तोएवस्की，एड्रिएन इन सभी की झलक विद्यार्धियों में दिखाई देती है और मैं अपने आप से पूछता हूँ कि उस बाल ओशो की निराशा का क्या उत्तर हो सकता है？क्या गणित का गणित के अलावा जीवन में कोई उपयोग नहीं？एड्रियन से की हुई गणित की तोड़ मरोड़ का क्या अर्थ लगाया जा सकता है？इन सब सवालों पर मैं गणित को चुप्पी साधे देखता हूँ। शायद इसलिए की ये सवाल गणित के नहीं बल्कि गणित पर लगे हैं।

यह तीनों उदाहरण，गणित सीखने के क्रम में आने वाले तीन मलुभतु बदलाव के चरण या ट्रांजीशन फेज़ेस को दर्शाते हैं। पहला，कॉन्क्रीट से एब्ट्ट्रैक्ट，दसूरा，यूजफुल से कंसिस्टेंट और तीसरा，मीनिंगफुल से फॉर्मल।

बाल ओशो जिस दुविधा में पड़ा है उसे आप ऐसे भी समझ सकते है कि किसी चीज़ का होना क्या सिर्फ भौतिक रूप में होना है？भौतिकता के परे क्या हम एक्सिस्टेंस की बात कर सकते हैं？यूक्लिड बिना मोटाई／चौड़ाई वाली रेखा का कोई अस्तित्व है？क्या वो अस्तित्व भौतिक हे？अगर नहीं तो किस स्पेस में अस्तित्व की बात हो रही है？ उसकी क्या वैधता हे？और उस स्पेस का रेयलिटी से क्या संबंध है？ये सवाल कोई बाल ओशो इस शब्दावली के साथ नहीं रख सकता पर यही वो सवाल है जो कॉन्क्रीट से एब्ट्ट्रैक्ट के ट्रांज़िशन के दौरान उभरते हैं और कुछ बाल ओशो इन सवालों से उभर नहीं पाते। यह ट्रांज़िशन फेज़ अरिथमैटिक से अलजेब्रा जाने के दौरान，ज्योमेट्री से एक्सिओमाटिक ज्योमेट्री तक जाने के दौरान，स्कूल मैथ में ही देखा जा सकता है। इस ट्रांज़िशन फेज़ के अलग अलग रूप अलग अलग स्तर पर देखे जा सकते है，जैसे कैलकुलस से एनालिसिस के दौरान，सिस्टम ऑफ़ इक्वेशन्स से वेक्टर स्पेस के दौरान，सेट ऑफ़ परमुटेशन्स से ग्रपु के दौरान। कई बार विद्यार्थी इस चरण को ＂गणित दिखना बंद हो जाने＂जैसे वाक्प्रचार से उद्धोषित करते हैं।

गणित की उपयोगिता के बल पर जो आगे बढ़ जाते हैं उनके लिए दूरी की एक चुनौती हैं ।

गणित जहां से शरू होता है वहां से जहां उसका उपयोग किया जाता है उसकी दूरी काफी होती है और कई मामलों में उपयोगिता से जोड़ मिलाना मुमकिन भी नहीं होता (बशर्ते आप गणित के बाहर उपयोगिता ढूंढ रहे हो)। ग्रेजुएशन तक आते आते गणित का नेचर बदलता जाता है। वो उपयोगिता का चोगा उतारकर अपने कंसिस्टेंट सिस्टम के मूल रूप में आने लगता है। जहां सैकड़ौ नोशन्स, दसों कन्वेंशंस एक दूसरे से लगभग अदृश्य धागे से जुड़े होते हैं। ऐसे में गणित की कंसिस्टेंसी की बाह जो थाम नहीं पाते वो यूजफुल सिस्टम से कंसिस्टेंट सिस्टम के गणित के स्वाभाव/नेचर के बदलाव से हात नहीं मिला पाते। इस फेज़ में फसे विद्यार्थियों की लिखावट में उनके सिम्बल्स मैनीपुलेशन में कुछ अर्थबोध भर देने की भरसक कोशिश दिखाई देती है। यह ट्रांज़िशन फेज़ आपको एब्ट्ट्रैक्ट अलजेत्रा या लीनियर अलजेत्रा के कोर्स में दिख जायेगा।

मैथमेटिक्स की कंसिस्टेंसी की खूबसूरती जो निहार पाते हैं, वो लम्बे रेस के घोड़े भी पीएचडी, पोस्ट डॉक करने के बाद एक समस्या से जूझते हैं जिसे मीनिंग क्राइसिस भी कहा जा सकता है। इस क्राइसिस में गणित के फॉर्मल सिस्टम की सुंदरता मीनिंग खोने लगती है, जैसे एक ही शब्द को बार बार पढ़ो तो वो शब्द मतलब खोने लगता है।

प्राथमिक गणित की उपयोगिता आज के वक्त में वादातीत भले ही हो पर ग्रेजुएट लेवल पर आते आते कई विद्यार्थी अलग अलग फेज़ेस में अटके दिखाई देते हैं। ग्रेजुएशन तक पहुँचे इन डोस्टोएवस्कियों，ओशो，एड्रियनों को आप भूले बिसरे कह सकते हैं，＂किसने कहा था ग्रेजएशन गणित में करने के लिए？＂＂गणित सबके लिए नहीं हैं＂ऐसा सर्वव्यापी उत्तर आप दे सकते हैं। और उस रोते हुए डोस्टोएवस्की के सामने＂यही जीवन की हार्ष रियलिटी है＂कह कर उसे फेल करते जा सकते हैं। क्या गणित सबके लिए नहीं है？क्या ट्रांज़िशन फेज़ेस को पार ना कर पाना सिर्फ और सिर्फ बौद्धिक क्षमता का सवाल हैं？क्या इन फेज़ेस का गैप पैडागोजिक इंटरवेंशन से कम किया जा सकता है？मेरे पास इन सवालों के जवाब नहीं है।

गणित से मेरा रिश्ता फिलहाल वैसा ही है जैसा जज के सामने नीचे बैठे हुए उस कर्मचारी का अदालत में चल रहे किसी केस से होता है। इल्जाम संगीन है और केस पुराना। हर साल नए ओशो，एड्रियन और डोस्टोएवस्की आते हैं। पराने इल्ज़ामात नए ढंग से पेश करते हैं। जज साहब हर बार अलग अलग तरह से दलीलों का अर्थ निकालते है और फ़ैसला सुनाते है।


PAULERDÖS
WRITIEN BY OMK RR DEVIEK R

F＾CUITY

## ARITH-ME-TICKLED

## HOW MATH EXPOSED MY ASSUMPTIONS



Let me introduce myself. I'm Patle Quatchita Ratna and I'm from Telangana. I did my BSc in Mathematics. I am currently doing a Master's in Education. I want to share my experience of learning maths.

During my school days, I was considered one of those students who were good at solving mathematical problems in little time. I used to score well on tests. I always read maths textbooks in my free time before the teacher taught them in class. I even tried reading my seniors' mathematics textbooks and spent time understanding them. I was amazed by the way topics get constructed in maths. For example, if I know the concept of factors and multiples, I can find common factors and multiples of given numbers. With these, I will also be able to find the least common multiples and the highest common factors. These can then be used to solve word problems and look at real-life applications of these concepts. Without knowing the factors, you cannot find the highest common factor.

Such basics in Mathematics are integral. Each concept always needs some kind of prerequisite knowledge. I enjoyed doing maths, by which I mean solving the problems given in the textbook.

When I came to college for my undergraduate, on the first day of class, our instructor gave me a worksheet that contained only three problems. I read the first question, 'Prove that the set of prime numbers is an infinite set.' I felt puzzled. I thought it was common sense that since natural numbers form an infinite set, prime numbers also form an infinite set. But how do we prove that? It puzzled me!

Before I knew it, I began feeling frustrated with myself for not being able to crack the very first question! The main issue was, 'Why would someone want to prove or even think about proving a fact that the prime numbers form an infinite set? Wasn't it a self-standing fact?'

When I thought about how I learned maths in school, I felt I took many things for granted. I was able to understand the terms in the given formula and was able to manipulate them. However, the manipulations that I did were just problem-solving. As I kept solving, I became faster but now, I don't see that as learning maths. It may be only one aspect of learning Maths amongst many others such as reasoning, conceptual understanding, and the ability to write mathematically which I learned at college.

Now, I find myself thinking about why we learn maths, rather than how we used to. Many think that it is because it has applications in our daily life. I don't feel that is correct. Even a person who hasn't gone to school might be able to apply mathematical concepts well. From my understanding, learning maths is connected to developing a way of thinking. We learn to reason logically, think analytically, and improve problem-solving skills. Most importantly, it pushes us outside the box to think creatively. My perspective on Mathematics has certainly changed. Somehow my love for Mathematics always encourages me to explore in every possible direction.

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# WHERE PAPYRUS MEETS PENCIL： ANCIENT EGYPTIAN MULTIPLICATION 



Every mathematical journey is begun by studying the basic operations of addition and multiplication．The algorithm for multiplication we learn in school is called long multiplication．Long multiplication is an excellent algorithm because it is easy，fast，and only requires us to byheart the multiplication tables up to 10 ．But there is another even ancient multiplication technique that requires almost no memorization！

The Algorithm－
Let us see how to multiply 13 and 47．Make two columns on paper and write 13 on the left and 47 on the right．Halve 13 （note the quotient 6 and discard the remainder）and keep doing this until you get 1 ．On the right column，repeatedly double the numbers as many times as you have to halve 13 to get 1 ．We will have the numbers $13,6,3$ ，and 1 on the left column．Correspondingly，we would have the numbers 47，94，188， and 376 in the right column．Then strike off all the lines with an even number in the left column－here it would be 6 on the left and 94 on the right．Add the numbers left in the right column to get the product． That is， $47+188+376=611$ ．This is equal to $47 \times 13$ ．

Now that we have understood the process using an example, let us consolidate the steps in this algorithm.
1.Given two numbers, make two columns on paper and write the smaller number in the left column and the larger one in the right column.
2. Halve the number in the left corner. If the number is odd, for example, 7 , write the quotient 3 below and discard the remainder. Keep doing this until you get 1 .
3. On the right column, you repeatedly double the numbers as many times as you have halved.
4. Strike off all the lines with an even number in the left column.
5. Add the numbers not struck off in the right column to get the product.

Why does it work?
The first observation is that if we simultaneously halve a number and double the other, the product does not change. More precisely, $\mathrm{x} \times \mathrm{y}=$ $\mathrm{x} / 2 \times(2 \mathrm{y})$.

If instead of $13 \times 47$ if we were evaluating $16 \times 47$, we would then have only used this principle. More precisely, we are saying $16 \times 47=8 \times 94=4 \times$ $188=2 \times 376=1 \times 752$. Notice that the different lines in the two columns correspond to the pairs we are multiplying. Further, the only odd number in the left column is 1 in the last row, so the number in the last row in the right column is the product.

However, if we ever get an odd number on the left, we cannot divide it perfectly by 2 . We are in a sticky spot. But recall that any odd number can be written as $2 n+1$. And $(2 n+1) \times y=(2 n) \times y+y=n \times(2 y)+y$. In other words, when we throw away the remainder, we are missing out on y . So, we should remember to add this at the end. That is why we cannot cut the lines with an odd number in the left column.

## The Egyptian Algorithm

The Egyptians approached the problem in the opposite direction. They had large tables of powers of 2 . This allowed them to quickly express a number in terms of powers of 2 . For example, if we take the number 13, the largest power of 2 smaller than 13 is 8 . Express 13 and $8+5$. Now, the largest power of 2 less than 5 is 4 . Express 5 as $4+1$. Thus, we can express 13 as $8+4+1$ which is equal to the sum of 2 raised to 3 , 2 , and 0 . Therefore, the following is true-

$$
13 \times 47=\left(2^{3}+2^{2}+2^{0}\right) \times 47=2^{3} \times 47+2^{2} \times 47+2^{0} \times 47=376+188+47
$$

While the algorithm is easily understood, it is a bit tricky to write for a general pair of numbers because we first need an algorithm to express a number as the sum of powers of 2 . The idea that was used in the above example can be generalized.

Recursive Algorithm to Write a Number as a Sum of Powers of 2-

Suppose we want to express a number $n$ as a sum of powers of 2. Take the largest power of 2 that is less than or equal to $n$. More precisely, 2 raised to $k \leq n$ but 2 raised to $(k+1)>n$. Then, we can express $n=2$ raised to $k+m$. Thus, we have reduced the problem of writing $n$ as a sum of powers of 2 to the problem of writing $m$ (which is a smaller number) as a sum of powers of 2 . This idea is called recursion and is a central idea in many algorithms. If we keep repeating the process, the smaller number will eventually become zero.

Given this algorithm, it is easy to write the Egyptian algorithm to multiply two numbers.

1. Given two numbers, express the smaller number as a sum of powers of 2.
2. Compute the product of the larger number and each element in the above sum.
3. Add the results of the above products.

Binary Representation-
Representing a number as a sum of powers of 2 is extremely useful. We saw how 13 was represented in this manner (see next page).

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$$
13=2^{3}+2^{2}+2^{0}=1 \times 2^{3}+1 \times 2^{2}+0 \times 2^{1}+1 \times 2^{0}
$$

Thus, the number is determined by the coefficients of each power of 2 , which in this case was $1,1,0$, and 1 , we might just write 1101. This representation of the number is called a binary representation. You might have heard that numbers are represented in binary inside a computer.

If a number is represented in decimal form, multiplication by 10 amounts to just adding a 0 at the end. On the other hand, if we want to find the quotient when divided by 10 we just need to throw away the last digit. Similarly, if a number is represented in binary, multiplication by 2 amounts to just adding a 0 , and the quotient when divided by 2 is obtained by throwing away the last digit. Thus, halving and doubling becomes really easy if we represent the numbers in binary. So, if we had used binary representation (instead of decimal representation) then this is indeed the most natural algorithm for multiplication.

In fact, you should think of this algorithm as long multiplication for binary! Notice that 47 in binary is 10111 and 13 in binary is 1101 . We can multiply two numbers represented in binary the same way we would multiply two numbers represented in decimal. So, when we do long multiplication, we get-

| $\times$ |  |  | 1 | 0 | 1 | 1 |  | 0 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 0 | 1 | 1 |  | 1 | 1 |
|  |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |
|  | 1 | 0 | 1 | 1 | 1 | 1 |  | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |  | 0 | 0 |

We need to now add 101111 (which is 47 in decimal representation), 101110 (which is 98 in decimal representation), 00000000 (which is 0 in decimal representation), and 10111000 (which is 376 in decimal representation). In other words, we are doing something very similar to what we had done earlier!

# THE MATH OF LIFE M HOW EXPERIENCE <br>  SHAPES UNDERSTANDING 

Experiences and influences from people around us shape us into who we are. I had some interesting conversations with some of the Mathematics faculty members - Jam, Mohan, and Tulsi - about their experiences as a student, and their takes on teaching and imparting the language of Mathematics in their current profession.

While Mohan and Tulsi had vastly different schooling, they both had a similar approach to understanding Mathematics which led to them thoroughly enjoying and appreciating the subject.

Tulsi talked about her high school experience wherein students were given a hands-off approach and allowed to set their own pace. They were given worksheets to work out on their own. Mohan, on the other hand, went to a traditional school and discovered that Maths textbooks were written to be understood by students themselves. With the help and encouragement of a very influential teacher in his high school, he imposed self-paced learning for himself.

Mohan spoke about how it was drastically different to learn Mathematics in Tamil; a language that he was fluent in; as opposed to English, which he barely spoke at the time. Being taught Mathematics in Tamil made him connect and understand the concepts more clearly, making him realize that he was good at the subject.



His experience with undergraduate Mathematics was heavily influenced by his time at MTTS（Mathematical Training and Talent Search Programme）which changed the way in which he perceived Mathematics．

Mohan also spoke about his experiences with＇bad＇teachers from high school，up until his post－graduation at IIT，and how that shifted his interest toward Mathematics education．Unfortunately，he was actively discouraged from taking up Mathematics education research because he felt no one took it seriously at the time as the focus was usually concentrated on pure Mathematics．He also spoke about how isolating PhD was for him，and how it changed his perspective about teaching Mathematics．He talked about the blatant caste－based discrimination prevalent in IIT Madras，and how only a few people resorted to discussions on the topic．

Jam spoke about his experiences with undergraduate Mathematics in regard to discovering that there is a lot more to the subject than what is usually prescribed in the syllabus．He drew similarities between Mathematics and literature．＂While we know that there is more beyond the text when it comes to poems，we aren＇t necessarily aware that it is the same for Mathematics＂，he reflects．He elaborated on this by saying that as part of a particular course，say Algebra，there is usually more content than just traditional algebraic mathematics which students may not realize immediately．

All three instructors emphasized the importance of being conscious of your audience when you are a teacher. Understanding your students and learning as you go. They spoke about how useful assessments are to teachers as it gives them an insight into how much their students have really understood. They also believe that it is supposed to help students understand where they stand. Tulsi highlighted the importance of peer discussions and casual conversations around Mathematics. Mohan specifically talked about how different teaching is in principle compared to how it is practiced: the make-up of the classroom and the involvement of students play a huge role in how you end up teaching. Not every student in the classroom is going to take up Mathematics seriously, however, it is important to give them an interesting and enriching experience in the subject while they are still here, in a Mathematics classroom.


FEYNMAM, A RENOWNED TEACHER...
WRITIEN BY SUNAY^N^R

## MY CAT－AND－MOUSE RELATIONSHIP S WITH MATHEMATICS

## STAY WITH US AS ASMA NARRATES THE UPS AND DOWNS OF HER JOURNEY WITH MATH．．．

As I look back on this journey，I feel like my relationship with Mathematics can be compared to that of a cat and mouse．Like a cat， forever in search of a mouse，the subject of Math has perpetually been after my soul．Since the beginning，at my homeschooled nursery，I used to love studying but was often reduced to tears because I wasn＇t able to understand Maths．It is embarrassing to share that I used to shed tears in front of the whole class even in the 5th and 6th grades because of my inability to understand Mathematical concepts．I used to feel so helpless while trying to make sense of it．This is how Maths became the cat to my mouse，the Tom to my Jerry，always hunting．


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A constant issue that I had with Maths was that while in other subjects I could have believed myself somewhat well prepared for the exam, in the subject of Math, one can never know which problem would arrogantly confront you - like a monster that one cannot fight even with the relevant weapons of formulas and concepts. But somehow I survived by resorting to learning concepts and memorizing the solutions to the questions. Through this, I got good scores. Then, in 9th grade, I moved to a new school, and the "ratta-fication" or the rote-learning method did not work anymore. At that time, I was struggling with almost everything in the new school, be it language, culture, assessments, friends, or my ability to understand the content. Fortunately, I was granted a benevolent teacher who led me through the Mathematic arena. Maths became my only comfort.


THEPOWER OFA GOOD


TEACHINGGEOMETRY
In any relationship, as the bond grows stronger one learns the other side of the coin. Even as my interest in Maths increased, the concepts of geometry plagued me. Geometry provided a window to the world of proofs but I found that world to be an alien. The proofs by construction were always a source of confusion to me, primarily because of their counterintuitiveness.

However, while carrying the fear of geometry in my heart, eliminating other options led me to choose Maths in llth grade. And that was the time when differentiation and integration entered my life.

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I liked the concept of differentiation to a great extent and preferred it to the mysterious enigma that is integration. I was frequently overwhelmed by the many strategies that were used in each question to get an answer.

In 12th grade, I practiced a lot of integration questions and the secret finally made itself known. I liked solving those questions so much that I used to lose track of time. Keeping this in mind, and after pondering over many options, I decided to do a degree in Maths. I was completely unaware of what l'll be learning in a B.Sc. B.Ed. Mathematics degree before joining college. With a curious mind and a sneaky feeling that I would be doing more complex questions, I entered university - amidst a pandemic.

I was late for my first math class at university, a foundational course called Introduction to Mathematical Thinking (Part 1). It was an online class and when I entered, I was already in a breakout room with three peers and a worksheet visible on the screen. I was surprised to see that it contained simple questions of permutation and combination that just required the use of direct formulas. However, the instructor then told us that we should try solving those questions without using any previously learned formulas. In this way, in the first few classes, I realized that we are expected to deal with Mathematical problems based on how we approach them and think about them rather than what we know through formulas.

Later, I was working with a new group on a problem that led to the moment when we derived permutation and combination formulas while solving problems. We made use of logic involving making a choice and considering possible options. Then in the next class, we got the reasoning behind the number of possible subsets using functions and the logic of possible options. That was my "A-ha" moment. I can't express in words how amazed and overwhelmed I felt at that moment. Realizing the connection between set theory, functions, and permutation/combination was the highlight of my Mathematical life.



The second year of college started with the course Calculus 2 . In the first year, it was all about exploration. The real rigor of Mathematics was introduced to us through this course. Understanding theorems and applying them to prove other theorems was a difficult task. It was frustrating to prove everything in the most rigorous and settled manner. It used to become more frustrating to discover the counterintuitive nature of proofs after spending days on it. The distributions introduced in the probability theory added fuel to the fire. At that point, I had lost all enthusiasm and my only motive was to pass the courses and to just finish the degree.

With that sour ending of year two, the third year of university life started. At that point, I realized that my friendship with Math is almost broken. So, I decided to give it one more chance. I thought about doing a mini-project under the guidance of a faculty member, which is a modified version of the Josephus Problem - I will explain it here in brief. Say, you have 5 people, which you refer to as a, b, c, d, and e. You count in each person in order and eliminate or 'kill' whoever lands on the number 5 . So you start counting, $1 \rightarrow a, 2 \rightarrow b, 3 \rightarrow c, 4 \rightarrow d, 5 \rightarrow e$, and so e gets 'killed'. Now, we will start counting from 1 again but this time ' $e$ ' is no longer in the game. So, $1 \rightarrow a, 2 \rightarrow b, 3 \rightarrow c, 4 \rightarrow d, 5 \rightarrow a$. Hence, ' $a$ ' gets killed at the end of this turn. In the next turn we have, $1 \rightarrow b, 2 \rightarrow c, 3 \rightarrow d$, $4 \rightarrow b, 5 \rightarrow c$ (since both ' $a$ ' and ' $e$ ' are out). Hence, $c$ gets killed here. Following this we have, $1 \rightarrow \mathrm{~b}, 2 \rightarrow \mathrm{~d}, 3 \rightarrow \mathrm{~b}, 4 \rightarrow \mathrm{~d}, 5 \rightarrow \mathrm{~b}$, and hence, b gets killed.

## TO KNOW MORE ABOUT THE JOSEPHUS PROBLEM, CLICK HERE!

I talk about Maths now and then to my peers. I explore interconnections between Mathematical concepts and come to know different perspectives to look at the concepts, for instance, through Maths education courses.

In this context, the cat and mouse's bond is not as strong as it was in the first year but not as weak as it was in the initial schooling years. I can say, unlike the high school me, I don't restrict Mathematics to just problem-solving. I have moved beyond my initial collegiate years in that I don't see Maths as a playground to simply explore. I think it's a rich domain, an interconnected and infinite web of ideas, skills, techniques, explorations, and much more. To do Mathematics is to enter the web fully aware that you will never be able to fully remove the spider silk from your skin even though you leave with more confusion than you began with. I know it's vague but that's the fun of it; Maths is always like this, a filmy piece of lace with no beginning or end, an abstract art.


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PAINTING BY ARPITA SINGH

# THE MATHEMATICS OF HINDUSTANI CLASSICAL MUSIC <br> WITH JIOO NIMKAR 

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SCAN THIS QR CODE TO LISTEN！

## THE VENN, THE WHAT, THE HOW, AND THE WHY

For all the future (and present) Math teachers out there: Check out our review of Gola Gola, a story that introduces the concept of Venn diagrams.


When Mariyu, a school-going young girl decides to eat gola at the mela, her friends ask her to bring golas of different colors. Yohan, a friend of Mariyu, sees her struggling to remember this and comes up with an idea to help her with all the colors.

The book Gola Gola written by Aithihya Ashok Kumar intends to introduce the idea of Venn diagrams to children. However instead of using mathematical definitions and terms, the book uses illustrations and colorful golas to reach its goal. Creating secondary colors with the help of primary colors is something most of us might have learned in an art class. The author uses this overlap of colors to explain the concepts of intersection and union.

Although the intersection between domains like mathematics and art shown in the book is intriguing and possibly helps expand the audience base, the question that prompts Yohan to help Mariyu, "How will she remember everyone's golas?" does not completely capture the motivation behind using Venn Diagrams. They themselves don't help us in counting and it is only a method to represent the colors of the golas that Mariyu's friends wanted. This being an example, the book does come off at some point as one written by someone not necessarily familiar with such concepts. If Mariyu wanted to count, she would have to end up using formulas that we learn in higher secondary schools which might be difficult to understand for the intended young readers.

Aside from this, the story seems like a good introduction to the concept for young schoolgoers of around 1st to 3rd grade or even to middle school children as a simple example of the somewhat difficult concept of Venn diagrams. It was interesting to note that the illustrations were trying to reinforce the presence of circles in the children's surroundings, including a giant wheel, bubbles, and a moving pinwheel. I recommend this book as it not only includes captivating and colorful illustrations but also helps children comprehend the idea behind the story which makes it interactive and educational at the same time.


## SCAN THIS QR CODE TO ACCESS 'GOLA GOLA' ON STORYWEAVER.ORG




यहाँ पर मैंने साथ में गणित पढ़ने की बात कही। हम अकेले शुरुआत कर सकते हैं，किन्तु ऐसे समूह में होने से，जहाँ सभी गणित की बातें करते हो，वह अच्छा होता ह। केवल एक बात का हमें ध्यान रखना होगा कि हमारी वो प्रश्नों को हल करने तथा अवधारणाओं को समझने की जो वेग प्रारंभ में बनायीं थी，वो आदत नहीं जानी चाहिए，हमें हमेशा काम करते रहना होगा। गणित को अच्छा करने में एक और महत्वपूर्ण पहलू है और वो है भाषा। आप जिस भी भाषा में पढ़ते हो，उसमें आपकी अच्छी पकड़ होनी चाहिए। किसी भी क्षेत्र में अच्छे होने के लिए हमें भाषा की आवश्यक्ता है，गणित में भी।


अभ्यास करें एक समूह के साथ
इस छोटे से लेख से आप इन सब बातों को अपने साथ लेजा सकते हैं। गणित में अच्छा होने के लिए अवधारणाओं को समझ कर प्रश्नों का हल करना होगा। कोई एक ऐसा समूह ढूढ़ना होगा जहाँ गणित की बातें होती हो और अपनी भाषा को और अच्छा करना होगा，चाहे वो हिन्दी हो या अंग्रेजी， बांग्ला हो या कोई अन्य भाषा।


Read on as Dhanush explains his experiments with Origami and circular paper in the Math Lab!

In the course "Experimental Methods and Mathematics", as part of the Mathematics program at APU, we tried to explore and understand mathematics using Origami - the Japanese art of making decorative shapes using paper, where cutting paper is not allowed.

In the most traditional sense, shapes are made using a square piece of paper. For constructing the shapes we wanted, like equilateral triangles, pentagons, hexagons, etc., we had some rules to follow. There were some folds we could do and some that we couldn't. These operations are what we called the Basic Origami Operations, also known as B.O.O. We looked at the geometric arguments behind Origami shapes, and also learned to construct three-dimensional polyhedra. Inspired by this, we started experimenting with circular paper. It initially took a lot of time to even understand where to start the process. We finally started off by using our explorations with the square paper to direct us in the right way with circular paper.


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The Origami Operations:
The first markers that we arrived at naturally were the center and the diameter. So we started thinking about whether the existing BOOs for square papers were enough to locate the center or if there was a necessity for new B.O.Os. The explorations led us to the addition of two more B.O.Os, one as a result of the realization that identifying an arc on the boundary of the paper inevitably creates another arc which is what is remaining of the circumference. We called this action, boundary partitioning. The operations used were as follows:

- Given a boundary partition into two arcs, we can make a fold such that one arc is taken completely onto the other arc on the boundary.
- Given a line and the boundary, we can locate their point of intersection.

Using the above two, we locate the center and draw two diameters that are perpendicular to each other (the axes).

In the figure below, the blue arc is the image of the red arc that falls on the circumference which in the process results in the construction of the diameter. Two such operations and we locate the center. Now bisect a diameter and we have two perpendicular axes.


Constructions using single circular paper:

We started off with constructing regular polygons with one circular piece of paper. We also constructed simple objects that were also made using square paper so as to compare the crease patterns.

Regular $N$-Gons:
In order to get a square, we need to fold the paper along the blue dotted lines as shown in the image as we already have the perpendicular diameters.

From these diameters，we can also construct the largest equilateral triangle on a circular paper and prove this using geometrical arguments．Using this triangle that we just made and with the help of angle bisectors，we can make a regular hexagon as shown in the image．


Now that we made some basic shapes，we wanted to construct more polygons from here，and using angle trisection，we theorized all possible constructions．By trisecting the angles in a regular hexagon， we constructed an 18－gon（an 18－sided polygon）．


Once we made enough polygons to find patterns，we moved on to try some other objects making them with both square and circular paper． Some examples of what we made are given below．


Modular Origami:

Interestingly, while working on this project, Pavan came up with a modular unit of circular paper which we named SECTOR-16 UNIT. Using twenty-four of these units, we constructed an object shown in the picture below that stays together due to friction between the units.


There were times when we were in the face of hurdles and did not know how to proceed further. But the questions that this process of mathematical struggle led us to think about, encouraged us to keep trying. We thoroughly enjoyed working on this project and are satisfied with the end result. We worked on creating something in Mathematics which was fun!


## FOR ENTHUSIASTS TO EXPLORE!

Observing the crease patterns of the crane, the flower, and the flash vortex, we came up with a hypothesis- objects with radial symmetry can be created with both circular and square paper giving close results. While objects with bilateral symmetry are not so close.




## THE "AVERAGE" MATH STUDENT AT APU...

Ever wondered how the Math majors seem to be busy as bees on campus? Or perhaps where how we seem to know a little bit about nearly everything?

Check out Kavana's comic about the busiest bodies on campus!

"RUMOR HAS IT,
THIS COMIC IS
COMPLETELY TRUE,
EXCEPT FOR THE
PARTS THAT ARE
COMPLETELY MADE
UP"

- NOT FROM INVENTING KAV(ANNA)


M ADE BYKAVANAKIRAN
"OH, I GET BY WITH A LITTLE HELP FROM MY FRIENDS."

\author{

- THEBEATLES
}
A S S I S T A N i
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> BERTRAND RUSSELL

DEAR READER,

WE WOULD LIKE TO THANK YOU FOR MAKING IT ALL THE WAY THROUGH OUR MAGAZINE! FEEL FREE TO REACH OUT TO THE AUTHORS AND ARTISTS OF THIS ISSUE TO FURTHER EXPLORE THE CONCEPTS SPOKEN ABOUT, HERE.

IF OUR THOUGHTS, IDEAS, AND STORIES HAVE INSPIRED SOME OF YOUR OWN, CONSIDER SHARING THEM WITH OUR LARGER COMMUNITY AT AZIM PREMJI UNIVERSITY BY CONTRIBUTING TO OUR NEXT ISSUE.

INTERESTED FOLKS CAN REACH OUT TO OUR FACULTY COORDINATORS, SHANTHA BHUSHAN AND AJAYKUMAR K, AT THE MATH DEPARTMENT!

WE STRIVE TO BE AN INCLUSIVE TEAM AND ENCOURAGE MULTILINGUAL CONTRIBUTIONS.

UNTIL THEN,
TEAM MATHAAPU.



## ADDITIONAL CREDITS

COVER PAGE PHOTOGRAPH - SUDIPTA MONDAL

ARTWORK ON PAGES 12 AND 14 - PEN AND INK BY ARPITA SINGH

## DIAGRAMS ON PAGE 6:-

1.KNOTS, LINKS, AND LASSOS BY P.D. TUMANSKI.
2. EQUIVALENCE OF DUAL GRAPHS BY M. AZRAM.

3. GRID DIAGRAMS AS TOOLS TO INVESTIGATE KNOT SPACES AND TOPOISOMERASE-MEDIATED SIMPLIFICATION OF DNA TOPOLOGY IN 'SCIENCE ADVANCES'.

ARTWORK ON PAGE 39-ADOLF ADLER'S 'JEWISH MAN STUDYING'


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