Developing an Understanding of Probability

… Experience from the classroom

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DIET Bhopal and Azim Premji Foundation, Bhopal jointly organize an annual seminar on the teaching and learning of mathematics. This year, the team decided to document the teachers' work so that they may present the same in the district level seminar. In the preliminary discussions, teachers understood the processes and decided on topics and concepts that they would like to work on with the children. Manisha ji chose "probability" for this task, for the following reasons:

1. Probability has been newly included in the new NCERT Books for class 8 (Previously it was not there in the state textbooks).

2. It is interesting and related to daily life.

3. It is a topic which could engage children who are afraid of math.

She worked with 8 children enrolled in class VIII with the aim of developing the following abilities related to probability –

• Understanding the vocabulary and concept of probability and related ideas such as possible and impossible events.

• Developing the ability to evaluate all the possible outcomes for a given situation.

• Bringing a rational approach to social superstitions.

• Finding the probability of different events.

She initiated discussions with the children on a few statements such as:

• India may win the next match.

• If a circle is large, its radius is also large.

- It may rain today.
- If a dice is thrown, 6 may appear on the upper face.

The children discussed the statement "India may win the next match…" among themselves. Anikesh said, "It may or may not happen." When other children asked the reason, he said – "it depends on the playing." On the second statement, "the bigger the circle…" everyone seemed to agree that it is certain that if the circle is bigger then, the radius is also bigger. She had similar conversations regarding such statements related to possibility of occurrence of different events with the children. She asked some oral and written questions through which she concluded that:

- Initially only two children were able to have an idea about the probability of events.
- Only one child was able to list possible events.
- Children were not aware of the concepts/terminology related to probability; this was understandable, as they were learning these concepts for the first time.

Keeping the level of understanding of the children and the objectives of the work in mind, she planned some activities for her class -

1. Discussions and activities to understand possible and impossible events

2. Listing of outcomes

3. Introduction to related terminology through various games and activities – weighing the options/alternatives, working out possible events, understanding what equally-likely outcomes are, and relating probability with chance

4. Work on social superstitions

1. Discussion and activities to understand possible and impossible events

Children were asked to make statements related to possible and impossible events. Children wrote and read out some such statements:-

a. Chances of the sun rising during the rainy season (Anikesh)

b. Heavy rains followed by floods (Deepanshi)

c. Rainbow sighting after rains (Jyoti)

d. Sun rising in the East (Mohit)

e. 32 days in the month of March (Jyoti)

f. Madam may or may not come to school tomorrow (Saloni)

g. Sun rising at night (Anikesh)

When a child made her/his statement, other children gave their opinion on whether it is possible or not. For statements such as "the rising of the sun during the rainy season" they agreed that it may or may not happen, so these were included in the possible events.

2. Listing of outcomes

Nikita asked a question to the children, "If she tries to climb the tree, what are the possibilities?" The children described different possibilities such as – you may climb a tree, you may fall from the tree, you cannot climb a tree, you can climb a tree but will not be able to descend, you can climb a tree and descend. Similarly, Dipanshi said "What are the possibilities if I want to go to Bhopal from Khurachani village to buy clothes? " The views of the children were – petrol runs out on the way, the vehicle tyre may get punctured, shop may be closed, you do not like the clothes, and you like the clothes and buy them… After a number of such discussions, we moved on to those questions which have a fixed number of possibilities.

The examples discussed in the next part included: If 2 green and 4 red balls are kept in a box and 1 ball is drawn then what are the possible colours of the drawn ball? What are the possible outcomes if a coin is tossed? What are the outcomes if a die is thrown? What are the outcomes if two coins are tossed simultaneously or two dice are thrown simultaneously? The children also worked with ball, coins, and dice. In this way the outcomes were worked out.

3. Introduction of terminology with games and activities

A race of Head and Tail - In this game, groups of two children were formed. For each group a number line was drawn on the floor in which the numbers from +5 to –5 were recorded. Each player took a counter and placed it at 0. A coin was given to each group and the rules were explained. Players will take turns tossing a coin; if heads come up, move forward one number, i.e., from 0 to +1 and if tails come up, go back one number, i.e., from 0 to -1. The player who reaches +5 or -5 first, will be the winner.

In this game, the children also recorded (i) In how many turns/rounds the game ended and (ii) how many times head or tail appeared in total. These were also discussed in large groups at the end of the game. The children concluded that the more times the coin was tossed, the closer the number of heads and of tails were.

Editor's Note: This documentation is a very good beginning to understand the Law of Large Numbers: empirical probability converges to theoretical probability as the number of trials increases. In this case, each round is a trial, empirical probabilities are the proportions of heads (or tails) out of all tosses, theoretical probabilities are P(head) = P(tail) = ½. Since the number of heads became closer and closer to the number of tails, the empirical probabilities, i.e.,

$$\frac{\text{number (no.) of tails}}{\text{no.of heads + no.of tails}} \text{ and } \frac{\text{number (no.) of heads}}{\text{no.of heads + no.of tails}}$$

both became closer and closer i.e., to the theoretical probabilities of these outcomes.

A game of buttons

In this game 2 red and 2 blue buttons were taken. All four buttons were put in a small envelope. Two teams of the children of the class were formed. It was decided in the game that two buttons will be drawn simultaneously (which children will take out during their turn). Teams have two options – the first is to have both buttons of the same colour (both the buttons are either red buttons or blue buttons) and second is that the buttons are of different colours (i.e.,

one red button and one blue button). The first team chose the option different colours, i.e., if one blue and one red button came out, then their team would get 1 point and if both the buttons that came out were of the same colour, then the second team would get 1 point. The team that reaches 10 points first will win the game.

The game started and the first team won the game. The children expressed their desire to play one more time. They were asked – "Which option do you want to choose – same colour buttons or different colour buttons?" Both the teams were ready to take any option. This time also the team which had opted for different colour buttons won. Then the teacher asked a question - which option is more likely to win and why? A few children said, "different buttons," but they were not very confident and couldn't discuss the reason. The teacher encouraged them to think about the outcomes as they did in the previous activities. The children started working on the possible outcomes and weighed the options. That if one button is red, then what the options for the second button are or what kind of pairs shall be made.

The children found out that there are a total of 6 possibilities out of which 4 are in favour of those who choose different colour buttons and only 2 are in favour of those who choose the same colours. Therefore, the team that chooses the option with the different colour buttons has a double/higher chance of winning, whereas one feels that both teams have equal chances of winning. This helped a lot in understanding how
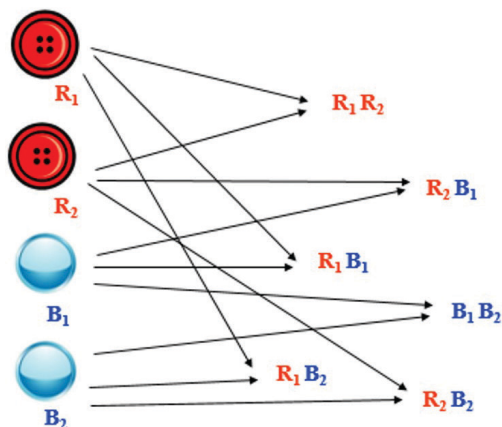
we should analyse and make careful decisions in choosing the options.

Editor's Note: The arrow diagram is a nice application of Cartesian Product and is a very useful tool in combinatorics to determine all possible cases. Such diagrams help children in understanding the situation much clearer compared to just textual explanation. It also enables them to draw similar diagrams to solve other problems they encounter.

Activity with balls in a bag

5 blue and 3 pink balls were put in a bag. The children were asked if 1 ball is drawn from these, what are the chances it will be blue (or pink)? Which colour ball is more likely to be drawn? And why? Such questions helped to arrive at the different outcomes and the probability of getting each outcome.

Taking this further, questions such as, if two balls are taken out of this bag at the same time, then what can be the possible outcomes were also asked. What is the probability that both are (i) blue/pink/ of same colour or (ii) of different colours?

Editor's Note: Such simple activities are important to illustrate that even when there are two possibilities, i.e., in this case blue or pink, the probabilities may not be half-half. It is a common misconception that arises from equally likely outcomes (from coin tossing and dice rolling – unless the dice is suitably modified) and this is a simple way to prevent that misconception.

Activity with cards and coins/dice to discuss related concepts

This activity consists of cards with words such as chance, outcome, impossible event, probability, equally likely outcomes, etc. Those cards were turned over and placed on the table. Along with cards, some materials like bag, balls, coins, and dice were kept. A child would come to the front and pick up a card and explain the concept written on that card in his own words with the help of the material placed on the table or on the board.

Editor's Note: Again, a simple idea allowing the teacher to assess the learning of the children while giving the latter the opportunity to articulate their understanding with suitable manipulatives which they are free to choose and use.
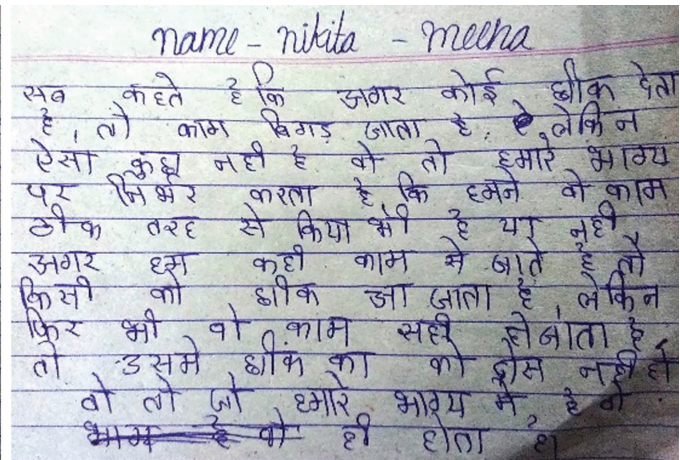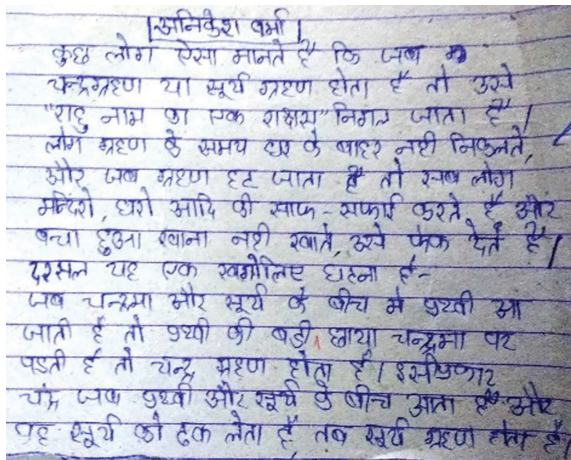
4. Work on social superstitions and prejudices

Children discussed their social superstitions and prejudices. Each child described the prejudice of his/her society, some of which were:

- If you see your face in the mirror as soon as you wake up in the morning, then the whole day gets spoiled.

- Money comes when the hand is itchy.

- If we sneeze, we should stop for a while and only then leave for work.

- If the crow caws near the house, then guests may be expected that day.

- If the cat crosses one's path, one should stop.

Children narrated their experiences related to each superstition and shared them in writing. Some said that this is really true, while some were not able to decide. It was decided that we should observe carefully for a few days, write them down and then discuss again. One day, their experiences were discussed in the class.

The children tried to note observations and discussed in group that allowed them to verify their beliefs, prejudices, and social superstitions. To a large extent it helped the children understand that it is not good to associate results with superstitions. E.g., most of them could

अनिकेश वर्मा

कुछ लोग ऐसा मानते है कि जब भी चन्द्रग्रहण या सूर्य ग्रहण होता है तो उसे "राहु नाम का एक शक्स" निगल जाता है। लोग ग्रहण के समय घर के बाहर नही निकलते और जब ग्रहण हट जाता है तो सब लोग मन्दिरो, घरो आदि की साफ-सफाई करते है और बचा हुआ खाना नही खाते, उसे फेक देते है। दरसल यह एक खगोलीय घटना है- जब चन्द्रमा और सूर्य के बीच में पृथ्वी आ जाती है तो पृथ्वी की बड़ी छाया चन्द्रमा पर पड़ती है तो चन्द्र ग्रहण होता है। इसीप्रकार जब पृथ्वी और सूर्य के बीच आता है और यह सूर्य को ढक लेता है तब सूर्य ग्रहण होता है।

name - nikita - meena

सब कहते है कि अगर कोई छींक देता है, तो काम बिगड़ जाता है। लेकिन ऐसा कुछ नही है। वो तो हमारे भाग्य पर निर्भर करता है कि हमने वो काम ठीक तरह से किया भी है या नही। अगर इस कही काम में बात है तो किसी वो ठीक जा जाता है, लेकिन फिर भी वो काम सही हो जाता है तो उसमे छींक का को रोल नही है। वो तो जो हमारे भाग्य में है वो ही होता है।

understand that sneezing has nothing to do with getting success or failure. At the same time a few of them were also associating the success / outcome with "luck".

Assessment

During this whole process, we kept working on the related questions of the textbook. Manisha did oral, written and activity-based assignments again with the children for the assessment. From this assessment it emerged that –

- **Understanding of chance:** Initially only 2 out of 8 children were able to demonstrate this understanding, now 6 children understood the concept of chance and they could even explain with examples.

- **Thinking about possible and impossible events:** 5 children were able to describe possible and impossible outcomes, one more child was able to discuss after a few hints.

- **Working on the ability to make the right decisions considering all possible alternatives to an action/event:** 6 children not only described all possible outcomes, but they were also able to evaluate their choices based on this. While playing different games with buttons/counters, they were asked to choose their option(s). They explored all possible outcomes and then chose their option. For example, in a game we played with balls in a bag: 2 blue balls and 3 red balls were put in a bag and one ball was drawn at random. The children were asked to choose the colour of the ball. They chose red, with the logic that the chance of getting a red ball is more.

- **Motivating for thinking about social prejudices:** Intense and frequent discussions with children enabled them to reason out their views on some social prejudices and superstitions.

- **Understanding the terminology/concepts related to probability:** 5 children developed a good understanding of experiment, event, chance, probability, equally likely events, etc. They could find the probabilities of different events.

Learnings for the teacher

- Teaching through activities not only makes it easy for the children to understand but also makes it easier for the teacher.

- Repeated discussion of various issues with the children enabled them to think and start expressing their views. They could think, express, and write their thoughts. Such opportunities helped them to assess and reflect on what we generally think.

- Children's self-confidence increased.

- The teacher also had problem with some concepts but during this work her understanding also increased by studying and teaching.

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