Some more Divisibility Rules

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Divisibility rules for a divisor p are algorithms that tell us whether a given number is divisible by p or not. From the lower grades we are familiar with divisibility rules for the divisors 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12, but when it comes to prime divisors greater than 10, we are clueless. In this article, I show how one can find tests of divisibility for divisors such as 13, 17, 19, 23, 29, …. I focus on divisibility by 13, 17 and 19, and give examples. These methods are not unique; readers may come up with other ways to check divisibility by these numbers.

Let p be a given divisor (e.g., $p = 13$). We start by expressing p or a multiple of p using integer multiples of its digits and addition or subtraction. We then use this relation iteratively to check divisibility. How we do this is illustrated below.

Divisibility by 13

Let us express 13 in terms of its digits. Here is one possibility: $13 = 1(4) + 3(3) = 13$, so 13 can be expressed as the sum of four times its ten's digit and three times its one's digit.

Now, to check divisibility of a given number N by 13, we apply the same operation to N. That is, if $N = 10a + b$, we replace N by $4a + 3b$. Then we do this replacement operation repeatedly. At any stage, if the resulting number is a multiple of 13, we conclude that the given number N is a multiple of 13.

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Example 1

Take the number 91. We have, $91 \longmapsto 9(4) + 1(3) = 39$, where 39 is divisible by 13 so 91 is also divisible by 13.

Example 2

Consider the four-digit number 1547

$1547 \longmapsto 154(4) + 7(3) = 616 + 21 = 637$. As we are not sure whether 637 is divisible by 13 or not, we repeat same procedure for 637.

$637 \longmapsto 63(4) + 7(3) = 252 + 21 = 273$. As we are not sure whether 273 is divisible by 13 or not, we repeat same procedure for 273.

$273 \longmapsto 27(4) + 3(3) = 108 + 9 = 117$. As we are not sure whether 117 is divisible by 13 or not, we repeat same procedure for 117.

$117 \longmapsto 11(4) + 7(3) = 44 + 21 = 65$. Here we know that 65 is divisible by 13.

Hence 273, 637 and 1547 are also divisible by 13.

Example 3

Take the number 79.

$79 \longmapsto 7(4) + 9(3) = 28 + 27 = 55$, which is not divisible by 13, so 79 is also not divisible by 13.

Divisibility by 17

Now we try to express 17 in terms of its digits. We have, $17 = 1(3) + 7(2) = 3 + 14 = 17$, so 17 can be expressed as the sum of three times the ten's digit and two times the one's digit.

Example 4

Let us experiment with another number and check whether the number is divisible by 17 or not.

$34 \longmapsto 3(3) + 4(2) = 9 + 8 = 17$, as 17 is a multiple of 17, hence 34 is divisible by 17.

Example 5

Let us try for slightly bigger numbers! Take the number 119.

$119 \longmapsto 11(3) + 9(2) = 33 + 18 = 51$, here 51 is a multiple of 17, hence 119 is divisible by 17.

Example 6

Let us consider another number which is none other than the current year 2023.

$2023 \longmapsto 202(3) + 3(2) = 606 + 6 = 612$, here we do not know whether 612 is divisible by 17 or not, so, we repeat the procedure for 612.

$612 \longmapsto 61(3) + 2(2) = 183 + 4 = 187$, here we do not know whether 187 is divisible by 17 or not, so, we repeat the procedure for 187.

$187 \longmapsto 18(3) + 7(2) = 54 + 14 = 68$, here we know that 68 is multiple of 17, hence 187 is divisible by 17.

Therefore 612 and 2023 too are divisible by 17.

Example 7

Let us check for 152.

$152 \longmapsto 15(3) + 2(2) = 45 + 4 = 49$, which is not divisible by 17. Hence 152 too is not divisible by 17.

Divisibility by 19

Let us try to express 19 in terms of its digits.

$19 = 1(1) + 9(2) = 19$, here 19 can be expressed as sum of its ten's digit and twice the one's digit.

Let us check for few numbers!

Example 8

Take the number 57.

$57 \longmapsto 5(1) + 7(2) = 5 + 14 = 19$, as 19 is a multiple of 19, hence 57 is divisible by 19.

Example 9

Let us try another number, 114.

$114 \longmapsto 11(1) + 4(2) = 11 + 8 = 19$, as 19 is a multiple of 19, hence 114 is divisible by 19.

Example 10

Let us try a bigger number, 2166.

$2166 \longmapsto 216(1) + 6(2) = 216 + 12 = 228$, as we are not sure whether 228 is divisible by 19 or not, so, we repeat the procedure for 228.

$228 \longmapsto 22(1) + 8(2) = 22 + 16 = 38$, as 38 is divisible by 19, 2166 too is divisible by 19.

Example 11

Let us try another number 2024.

$2024 \longmapsto 202(1) + 4(2) = 202 + 8 = 210$, we are not sure whether the number is divisible by 19 or not, so we repeat the procedure.

$210 \longmapsto 21(1) + 0(2) = 21 + 0 = 21$, we know that 21 is not divisible by 19 so 210 is also not divisible by 19, hence 2024 is also not divisible by 19.

Hope you all have enjoyed reading this article.

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Closing note by the editors

Divisibility by $k = 10x + y$ is checked by writing $k = ux + vy$ for integers u, v. Then, to check divisibility of any given number n by k, we write $n = 10a + b$, compute $au + bv$, and we iterate this operation till we can check mentally if the resulting number is divisible by k. If it is, then n is divisible by k.

Here are some questions for exploration we pose to the reader.

1. What is the logic behind the test? How can we be sure that $10a + b$ is divisible by k if and only if $ap + bq$ is divisible by k?
2. Different choices may be available for the integers u, v. How should we choose them so that the test proceeds swiftly? For example, to check the divisibility of 85 by 13 using $(u, v) = (4, 3)$, i.e., the test described above, the iteration proceeds very slowly:

$$85 \longmapsto 32 + 15 = 47 \longmapsto 16 + 21 = 37 \longmapsto 12 + 21 = 33 \longmapsto 21 \longmapsto \cdots$$