ClassRoom

Assessing Mathematical Proficiency at the Secondary Stage – I

MATH SPACE

The National Education Policy 2020 emphasizes that *"Mathematical thinking, problem solving, recreational mathematics, connecting classroom mathematics with 'real life mathematics' will be incorporated throughout the school curriculum …. in order to excite children about mathematics and develop the logical skills that are critical throughout school years and indeed throughout life."* The shift towards competency-based teaching and learning in the National Education Policy 2020 will be an important basis for curricular and pedagogical transformation in schools. In keeping with the thrust on competency-based teaching-learning proposed in the National Education Policy 2020, Azim Premji University has supported the Central Board of Secondary Education to develop a 'Learning Framework.' The learning framework is a comprehensive package which provides learning outcomes, assessment frameworks, samples of pedagogical processes, assessment items and marking schemes. Five such frameworks have been developed for English, Hindi, Science, Social Science and Mathematics at the secondary stage which may be accessed at https://cbseacademic.nic.in/cbe/learning-framework.html

This article describes the thinking behind the development of the Mathematics Learning Framework document, recounted with the objective of sharing the learning behind how the mathematics team studied the interface between the Class 9 and 10 NCERT Learning Outcomes, the CBSE mathematics syllabus, the textbooks, the Board examinations,

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and the consequent pedagogical practices of a secondary school mathematics teacher. The Curricular Expectations for the Secondary Stage, along with the Vision for Teaching School Mathematics (from the Position Paper on the Teaching of Mathematics, National Curriculum Framework 2005) were important factors in guiding our thinking, particularly, the mathematical processes described – Problem solving (both informal and formal), use of heuristics, estimation and approximation, optimisation, use of patterns, visualisation and representation, reasoning and proof, making connections, mathematical communication and the skills of Quantification, Abstraction and Modelling culled from the NCTM (National Council of Teachers of Mathematics, USA).

Nature of Mathematics

We began with the elephant in the room, the Nature of Mathematics. Mathematics is an intrinsic part of everyday life, from when we set our alarm for the morning routine to when we cook and eat our dinners. Relationships are made and broken on issues of fair shares or distances. But the relevance of mathematics is more than its utilitarian value. It helps us to think and reason about the world around us and take informed decisions, be it at the individual level to cope with life in various spheres of activity or at the societal level to contribute to technological and socio-economic development.

It is therefore inevitable that mathematics is a compulsory subject at the school level and, in fact, one of the first subjects encountered by the learner entering formal schooling. Sadly, however, the study of mathematics has been reduced to solving problems by remembering procedures and then arriving at 'correct answers.' The focus of school mathematics should be on developing the problem solving and reasoning skills needed to have evolved individuals who contribute to an organised and progressing society. For this, it is important for curriculum and curricular material developers to have a deep understanding of and engagement with, the nature of mathematics.

Mathematical objects and ideas are abstract – created by humans from the needs of science, economics, statistics, and any kind of quantitative analysis needed in daily life. They have no physical properties such as size, colour, sound, smell, etc. For example – A point which does not have breadth or length is represented as a dot by using a sharp pencil.

Mathematical ideas are formed by classifying similarly related and commonly noticed properties. This leads to the pedagogical challenge of making these ideas experiential. For example, Number, which is a root concept, is derived by providing experiences of collections of the same number of objects. The concept of addition is built on the concept of number, and it then becomes the pre-requisite concept for viewing multiplication as repeated addition. This in turn builds on to the understanding of higher concepts. Thus, mathematics builds up from the bottom, i.e., from axioms and definitions in a structured and hierarchical way as a vast network of interlinked concepts.

It is well recognized how rigid mathematics is. This is because mathematical truth, once established and consistent with existing results, lasts forever, and this 'rigid' structure is free from perspectives and subjectivity.

The language of Mathematics is also a part of the abstract nature of this discipline. Mathematics has its special vocabulary and symbols that are very specific to it, and which has been developed over the centuries to represent and communicate mathematical ideas.

We have mathematical propositions that are based on set of conventions, self-evident truths or axioms, definitions, and undefined terms. All the arguments, reasoning and procedures are followed by mathematical justification. This makes mathematics deductive by nature.

Children learn and apply mathematical knowledge by developing abilities through the mathematical processes described above, which, along with mathematical communication are important life skills.

Naturally, it is important to assess the attainment of these abilities. The nature of mathematics makes assessment in mathematics a delicate balance between understanding the difficulties faced by the student in handling abstract concepts (as well as the concise, precise language of mathematics) and the conceptual understanding gained by the student over the course of study. It is important to assess this through the lens of these mathematical abilities rather than simply through their recall of mathematical procedures. Designing assessment which reveals the student's ability to harness the power of mathematics requires that the spectrum of cognitive skills ranging from Knowing to Applying to Reasoning is tested. This requires a clear understanding of the learning outcomes expected from the student at a particular stage of education as well as the opportunities provided by the content of the subject in order to test the attainment of these.

Cognitive domains in Mathematics

As the Math Space team at Azim Premji University studied the design of assessment in mathematics at the secondary level, it became very clear that if assessment was to be mapped to the cognitive levels described in the Revised Bloom's taxonomy of Anderson and Krawthol, then it was important to granularise these and develop a common understanding of the sense in which these granularised levels were used in designing assessment tasks in mathematics. Given below is our first attempt at mapping the questions from a Standard 10 Board examination paper, as we tried to granularise the well-known cognitive domains of Remember, Understand, Apply, Analyse, Evaluate and Create.

Remember		
Remember a specific problem from the textbook	Remember simple facts and formulas For example: Characteristics of a fraction which gives a terminating decimal	Remember a procedure For example: Putting a quadratic in standard form and finding the discriminant
Understand		
Usage of principle, method or a theorem For example: Given a trig ratio, interpret in terms of ratio of sides and find the unknown side and a new ratio using Pythagoras' theorem	Interpret a real-life experiment in mathematical terms. For example: Throwing a dice or picking a card and interpreting the result as favourable or unfavourable and calculate the probability.	To paraphrase and interpret. For example: Paraphrase given data to use relation between roots and coefficients of a quadratic equation
Apply		
Model a given situation and solve a problem. For example: Forms a pair of linear equations using given real-life information	Use given content to interpret a situation and solve a problem. For example: Finding a relation between the coordinates of three collinear points	Applying knowledge to new situation to solve a problem. For example: Taps filling a tank, boats going upstream/downstream
Analyse, Evaluate, Create		
Connect and integrate different pieces of information to solve a problem- analysis. For example: Given data about a real life situation, mathematize it, recognise the value to be calculated and then find it	Connect and integrate different pieces of information to deduce- synthesis. For example: Prove a geometrical result	Judge and/or justify the value or worth of a decision or outcome, or to predict outcomes based on values For example: Justify a conjecture with a valid proof OR Provide a contradiction to invalidate a conjecture OR Compare two proofs for validity

This exercise proved useful in many ways:

- We saw overlaps and fuzziness between the demarcation of the categories
- We realised the gaps in testing the cognitive domains
- Most of all, we realised that all of these categories could easily collapse into the Remember domain, if teachers exhaustively covered all possible problems and did not allow students to develop their muscle for problem solving.

The attempt to granularise also made us prefer three broader bands of Knowing, Applying and Reasoning for the six cognitive domains. Given below are our definitions of these bands and the granularisation of these cognitive domains.

1. Knowing – One of the key curricular expectations for mathematics is the consolidation and generalisation of the concepts learnt so far. This cognitive domain addresses the student's ability to recall, recognize, classify and state concepts, formulas, axioms, postulates and theorems which form the building blocks of the hierarchical structure of mathematics. Armed with these and with a set of heuristics, the student is able to push the levels of understanding and develop confidence in mathematical communication as well as exploration.

Recall	Recall definitions, terminology, properties of different sets of real numbers, rules and properties of arithmetic operations, units of measurement and their inter-conversion, algebraic identities, properties of geometric shapes, statements of theorems, rules and notations (For example: $\sin^2\theta = (\sin\theta)^2$, $a^{p/q} = q^{\text{th}}$ root of a^p), basic geometric constructions
Recognize	Recognize numbers, quantities, expressions, equations Recognize 2D and 3D shapes and their parts. Recognize different orientations of simple geometric figures Recognize special numbers (squares, prime factors, multiples of 2, 3, 5, etc.)
Classify / Order	Classify numbers, quantities, expressions, equations, and shapes by common properties
Compute	Carry out arithmetic operations (addition, subtraction, multiplication, division, exponentiation on real numbers), ratio-proportion and algebraic (substitution, manipulation, solving linear equations in one variable) procedures
Retrieve	Retrieve information from charts, graphs, tables, texts, or other sources Retrieve given and unknown elements of a problem
Measure	Use measuring instruments Choose appropriate units of measurement

2. Applying – This cognitive domain focuses on using knowledge to determine strategies and represent, model or construct objects. Here, students are required to engage in applying knowledge of facts, relationships, processes, concepts to solve problems in real life contexts and expand the dimensions of their acquired knowledge.

Determine	Determine efficient/appropriate operations, strategies, and tools for solving problems Determine variables and their relationships
Represent	Represent situations and relationships with appropriately labeled diagrams, figures, tables, expressions and/or equations
Model	Display data in tables, graphs, geometric figures, or diagrams that model problem situations; identify the variables involved; and create equation(s) based on their relationship(s)
Solve	Solve problems involving familiar mathematical concepts and procedures.
Construct	Construct geometrical figures based on given specifications

3. Reasoning – In this domain, students are engaged in reasoning to analyse data and other information, draw conclusions, and extend their understanding to new situations. In contrast to the more direct applications of mathematical facts and concepts exemplified in the applying domain, learning outcomes in the reasoning domain involve unfamiliar or more complicated contexts. Mathematical reasoning also encompasses pattern recognition, conjecture and proof.

Analyse	Determine, describe, or use relationships among numbers, expressions, quantities, and shapes. Determine steps for a geometric construction
Differentiate	Identifies objects or situations falling under different categories Distinguishes objects using common characteristics Contrasts objects based on their characteristics
Connect	Relates two representations or models or ideas Relates two quantities (measurements), properties, identities or formulas Connects reasoning to properties of an object or to mathematical statements
Integrate/Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems
Evaluate	Evaluate alternative problem-solving strategies and solutions Evaluate validity of an argument or a solution
Draw Conclusions	Make valid inferences on the basis of information and evidence
Generalize	Make statements that represent relationships in more general and more widely applicable terms
Justify	Provide mathematical arguments to support a strategy or a solution or a geometric construction
Create	Pose problems given an equation, graph or other stimulus Find alternative proofs or solutions to problems

The granularisation did not merely give verbs mapped to the three levels – rather, they explicitly indicated what was expected at that level. For example, the verb 'Prove' is used at all three levels, but questions mapped to the three levels can be easily differentiated as:

Remembering: Recalls proofs

Applying: Proves an unfamiliar statement using one or two known facts

Reasoning: Proves using logical reasoning and understanding of properties, laws and theorems

Now that we had granularised the cognitive levels (we, of course, understood that this was a work-in-progress and that our granular levels would evolve as we worked), we had to complete the loop back to connect these to the content of secondary math. It became very apparent that we would have to define learning indicators for each learning outcome; these would specify the content domain and the cognitive domain and would provide a reference point for teachers as they planned their pedagogy and designed their assessment.

In the Learning Framework, the NCERT Learning Outcomes at the Secondary stage as defined by NCF 2005 were the primary point of reference. The nature of mathematics shaped these, and both constrains and gives scope to the Learning Outcomes. Learning Outcomes provide a benchmark on which learning progress can be tracked both quantitatively and qualitatively. The learning outcomes for each subject are expressed in terms of the cognitive skill to be demonstrated and the content to be acquired by the students. In most of the secondary mathematics Learning Outcomes, the content is explicitly delineated. However, the link to the cognitive domain is far more subtle. This has had a deleterious effect on the teaching and assessment of mathematics at the secondary level. Most of the large amount of content to be taught is addressed in the classroom, resulting in assessment being reduced to recalling not just theorems and formulas but actual problems and constructions too.

Our first task was to break down the overarching Learning Outcomes into Content Linked learning Outcomes (CLOs) and then into learning indicators focused on subject specific skills that students need to attain through different concepts addressed in the CBSE curriculum. A clear understanding of the scope of these learning outcomes for each concept dealt within a textbook chapter will be immensely helpful for both teachers and students to plan their teaching and learning better. Having attempted this task for the Learning Framework, we are eagerly anticipating the National Curriculum Framework 2023 which will be released later this year. Once the revised syllabus and learning outcomes are released, we will apply our understanding to the task of granularising these learning outcomes into learning indicators. The next part of this article will describe these.

Conclusion

Understanding this taxonomy helps in enabling teachers to design instruction activities and assessments aligned to learning outcomes, thereby ensuring the attainment of these learning outcomes. Linking the NCERT Learning Outcomes with the curricular expectations and granularising them into learning indicators provides clear direction to teachers to determine pedagogical processes which develop these learning indicators in their students. Further, it guides teachers to develop assessment which gauges how their students are developing. Competency-based learning encourages students to not only acquire knowledge but also apply this knowledge and the skills they have developed to successfully perform tasks in real-life situations. The students of the 21st century must be equipped to handle new and unfamiliar problems and while teachers cannot predict every such problem, they can certainly help their students to face and handle these with confidence and resilience. This goes far beyond 'completing the syllabus' by doing every problem in the textbook in class. It means ensuring that students develop the prescribed competencies and go beyond the scope of the textbook and the classroom.

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[i]**MATH SPACE** is a mathematics laboratory at Azim Premji University that caters to schools, teachers, parents, children, NGOs working in school education and teacher educators. It explores various teaching-learning materials for mathematics -their scope as well as the possibility of low-cost versions that can be made from waste. It tries to address both ends of the spectrum, those who fear or even hate mathematics as well as those who love engaging with it. It is a space where ideas generate and evolve thanks to interactions with many people. Math Space may be reached at mathspace@apu.edu.in