# An Algorithmic Approach to Multiplication using "Ekadhikena Purvena Sutra" 

## INTRODUCTION

Vedic mathematics is one of the contributions of India in the field of Mathematics. A saint of the Shankaracharya Order, Swami Bharati Krishna Tirthaji, happened to come across some Ganita Sutras or mathematical texts. He reconstructed them into 16 sutras and 13 upa-sutras. In the sutras he found some patterns of calculations related to topics such as subtraction, multiplication, squaring, square roots, cubes, cube roots, division, simple and quadratic equations, and much more.

The algorithms I am going to describe here are created with the help of the sutra named EKADHIKENA PURVENA. These algorithms stir up a lot of interest and enjoyment. Therefore, they come within the ambit of recreational mathematics.

In this article, we mention some limitations to the Ekadhikena Purvena sutra and describe modified algorithms to overcome the limitations. The limitations are taken as three different cases and the modified algorithms to overcome those limitations are given accordingly.

Keywords: Recreational mathematics, Multiplication of two
numbers, Algebraic expressions, Algorithms

The Ekadhikena purvena sutra. This sutra is used to multiply numbers when -

1. The working-base or base is the same in both the numbers.
2. The digits in the units' place add up to ten.

For example: (1) $77 \times 73$ (2) $81 \times 89 \quad$ (3) $63 \times 67$, etc.
Note: Bases are powers of 10 , for example $10,100,1000$, etc., whereas working bases are numbers which are scalar multiples of powers of 10 , for example $20,2300,4300$, etc. If the working base is a scalar multiple of 10 , for example $10 x$ where $x$ is a natural number, the base number will be $x$.
The process of calculation is as follows.

1. Two columns are made.
2. To get the left side of the answer, multiply the base number of the working base with one more than the same. In Example 1, the working base is 70, so multiply 7 ( 7 is the base number of 70) with 8 (which is one more than 7 ) to get 56.
3. To get the right side of the answer, multiply the digits in the units' place of the two numbers. The right-side column must contain two digits. If not, prefix a ' 0 ' to the product. In Example 1, the digits in the units' place are 7 and 3 and so the product is 21 .
4. After getting the products of the two columns, append the answer in the right column (21) to the answer in the left column (56) to get the required answer (5621).

## LIMITATIONS

Here, some questions arise.

1. What to do when the sum of the digits in the units' place is less than ten?
2. What to do when the sum of the digits in the units' place exceeds ten?
3. What to do when the working bases of the two numbers differ and the digits in the units' place of both the numbers is 5 ?

In response to the above, I place my answers below by using the following algorithms that I have created:
Consider the following examples in the case of Limitation 1: $42 \times 43 ; \quad 63 \times 64 ; \quad 75 \times 72 ; \quad 92 \times 96$; $223 \times 226$.

To explain the algorithm, take the example $42 \times 43$. Here, the working base is 40 , and the sum of the digits in the units' place is $2+3=5$, which is less than 10 .
Steps - Make two columns

| $42 \times 43$ |  |
| :--- | :--- |
| $\{40 \times(4+1)\}-\{4[10-(2+3)]\}$ | $2 \times 3$ |
| In the left column, first multiply the working base, i.e., 40 with 1 more <br> than the base number which is 4 and then from the product, take away 4 <br> times the difference of 10 and the sum of 2 and 3, as shown above. | In the right column <br> multiply the digits in the <br> units' place, i.e., 2 and 3. |
| $200-\{4 \times 5\}$ | 6 |
| 180 | 6 |

Now, after getting the products of the two columns, append them to get the answer.
And the answer is 1806.

Here we take another example, i.e., $63 \times 64$. Here the working base is 60 and the last two digits add up to 7 which is less than 10 . The steps are as follows

| $63 \times 64$ <br> $\{60 \times(6+1)\}-\{6[10-(3+4)]\}$ <br> $420-\{6 \times 3\}$ <br> $420-18$ <br> 402 |  |
| :--- | :--- |
| $402+1$ | $3 \times 4$ |
| Now after getting the products of the two <br> columns shift the tens place digit of the right-side <br> product to the left column and add it as <br> shown above. | 12 |

Consider the following examples in the case of Limitation 2: $53 \times 59,99 \times 98,55 \times 56,999 \times 997$, etc.
To explain the algorithm, let us take an example: $99 \times 98$. Here both the numbers are of the same working base that is 90 and when the units' digits are added up, it exceeds ten.

As before, make two columns.

| $99 \times 98$ |  |
| :--- | :--- |
| $\{90 \times(9+1)\}+\{9 \times[(8+9)-10]\}$ | $9 \times 8$ |
| $\begin{array}{l}\text { To get the answer of the left column, multiply the working base, i.e., } \\ 90 \text { with one more than the base number which is } 9 \text { and add it with } 9 \\ \text { times the difference of } 10 \text { from the sum of } 8 \text { and } 9 .\end{array}$ | $\begin{array}{l}\text { To get the answer in the right } \\ \text { column multiply } 9 \text { with } 8 . \\ 9 \times 8=72\end{array}$ |
| $900+\{9 \times 7\}$ | 72 |
| $900+63$ | 72 |
| 963 |  |$]$| $963+7$ | 2 |
| :--- | :--- |
| 970 | 2 |
| The right column should consist of 1 digit; if not then bring the tens <br> digit of the right product to the left column and add it with the <br> product as shown. |  |
| After getting the results of the two columns, combine them to get the answer 9702. |  |

Let us take another example: $222 \times 229$.
Here we will take the working base as 220 because the two numbers fall under the same working base, i.e., 220 and the units' digits when added up, exceed 10 .

| $222 \times 229$ |  |
| :--- | :--- |
| $\{220(22+1)\}+\{22[(9+2)-10]\}$ | $2 \times 9$ |
| $5060+22$ | 18 |
| 5082 | 18 |
| $5082+1$ | 8 |
| 5083 | 8 |
| The final answer is 50838. |  |

Consider the following examples in the case of Limitation 3: The pattern of the numbers would be of the following nature: $25 \times 35,35 \times 95,325 \times 465,105 \times 115,25 \times 65$, etc.

To view the algorithm let us take the example $25 \times 35$. Here the two numbers have different working base and the units' digits are 5 .

The steps are given below. Make two columns

| $25 \times 35$ |  |
| :--- | :--- |
| $\left\{[2 \times(1+3)]+\left[(3-2) \times \frac{1}{2}\right]\right\} \times 100$ | $5 \times 5$ |
| On the left side multiply the smaller number, i.e., 2 by one more |  |
| than the greater number, i.e., 3 and add the result to half of the the answer of the right |  |
| difference of the two numbers, i.e., 2 and 3 as shown above. | Tolumn multiply 5 by 5 to get 25. <br> $(8+0.5) \times 100$ |

Now multiply 8.5 by 100 to get 850 and bring the last two digits of the answer, i.e., 50 to the right side and add it to 25

| 8 | $25+50=75$ |
| :--- | :--- |

Now combine the results to get the answer 875 .
Let us take another example: $105 \times 125$.

| $105 \times 125$ |  |
| :--- | :--- |
| $\left\{[10 \times(12+1)]+\left[(12-10) \times \frac{1}{2}\right]\right\} \times 100$ | $5 \times 5$ |
| $(130+1) \times 100$ | 25 |
| $131 \times 100$ | 25 |
| 131 | $25+00$ |
| The answer is 13125. |  |

These are the new algorithms that overcome the limitations mentioned.

## Understanding The Algebra Behind The Third Algorithm

Let $(10 x+5)$ and $(10 y+5)$ be positive integers, where $x$ and $y$ are two single digit natural numbers with $y>x$. Then, the same multiplication can be done using the third algorithm with a twist of algebra in it which is shown below.

| $(10 x+5) \times(10 y+5)=$ | $\left\{x(y+1)+\frac{(y-x)}{2}\right\} 100$ | $5 \times 5$ |
| :---: | :---: | :---: |
| $(10 x+5) \times(10 y+5)=$ | $\left\{(x y+x)+\frac{(y-x)}{2}\right\} 100$ | $5 \times 5$ |
| $(10 x+5) \times(10 y+5)=$ | $\left\{\frac{(2 x y+2 x+y-x)}{2}\right\} 100$ | $5 \times 5$ |
| $(10 x+5) \times(10 y+5)=$ | $\left\{\frac{(2 x y+x+y)}{2}\right\} 100$ | $5 \times 5$ |
| $(10 x+5) \times(10 y+5)=$ | $\left\{\frac{(200 x y+100 x+100 y)}{2}\right\}$ | $5 \times 5$ |
| $(10 x+5) \times(10 y+5)=$ | $50(2 x y+x+y)$ | 25 |

Here, at the end we shift the two last digits of the product of the left column to the right column and add it. Hence, we can also write the same as -

IDENTITY 1. $(10 x+5) \times(10 y+5)=50(2 x y+x+y)+25$
This identity is applicable for all natural values of $x$ and $y$.

## CONCLUSION

The appearance of the illustrated algorithms may give an impression that it is complicated and mind-boggling, but once the algorithm is understood and the calculations are done mentally, it becomes simple and less time consuming. We encourage the readers to explore the algebra behind them. These algorithms make calculations easier for students as they inspire them to explore the manipulative power of numbers. Besides, it takes them away from the boredom of stereotyped calculations. It also increases their interest in mathematics and develops a zeal for it. Further it promotes creativity, curiosity and an explorative frame of mind.

## References

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