At Right Angles

A Resource for School Mathematics

MATH PROJECTS
SCALE DRAWINGS

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SCALE DRAWINGS

Mathematical projects that involve practical work combined with essential mathematical concepts have the capacity to stimulate interest and create enjoyment of the discipline. In this article, I have shared a few project ideas that integrate the topics of ratio and design in a practical manner. They help students to figure out how enlargement and reduction in terms of size transformation work mathematically, and clarify the concepts and methods involved. Some of them are hands-on activities involving working with cardboard, and some involve the usage of graph paper, dot paper, etc.

Recipes are often used as a context while teaching ratios, and understandably so. Most students can relate to the context of cooking as it is such an essential aspect of our lives. However, scale drawings involving enlargement and reduction are my favourite while teaching ratio and proportion. The topic lends itself to practical activities and can be integrated with social sciences, technical drawing, and art work.

Discuss many examples of enlargement that the students have encountered. Discuss examples of reduction such as toy cars, etc. Increasingly, children play around with various apps and games on mobile phones or computers that expose them to picture enlargements, zooming in, zooming out, and the idea of scale factor.

They are likely to have seen overhead projectors, movie projectors, photocopy machines, TV sets, telescopes, zoom lenses of a camera, etc.

Discuss the kind of enlargements they produce. Are there instruments that reduce the size of an object?

What role does the bulb of a projector play in the projection? Let students ponder over this question as they may not have an understanding of its importance. At a later point, the teacher can show its connection to the centre of enlargement.

What do convex and concave mirrors do to the images?

Let them find out some facts about these instruments and the power of modern electron microscopes.

Many years ago, when I worked with grade 6 on ratios, the class worked on building a small-scale model of the junior school which involved plenty of measuring, estimating, drawing to scale, and maintaining the uniformity of the scale in the 3-D model that we built. It was a project that took four weeks and integrated various mathematical topics including fractions, decimals, and angles. Many drawings involving front view and side views had to be made. It posed challenges for which we needed to think hard! Producing a realistic model developed our ability to select appropriate waste materials for building it and achieving a reasonable likeness. It was great fun!

Figure 1

Keywords: *Real world, design, projects, scale, ratio, measurements, similarity*

PROJECT 1: DESIGNING A HOUSE FOR A PET

Concepts: Measurement, Ratio, Scaling factor
Student groups: 4 students per group.
This activity can be done by any number of groups. Each group can select a pet for which they would like to design a house.
Materials: Old cardboard sheets, metal ruler, scissors or knife, Fevicol and colours.

Children have a natural liking for animals and often have pets at home. They can bring their knowledge and awareness of the animal size, needs and preferences of the animal to the project work. The discussion could extend beyond the intended house design by talking about other attributes like height, weight, diet preferences, animal behaviour, animal care, cruelty to animals, etc.

Figure 2

Here is a specification for a cat house the students could use for their class work. Cats love snuggling up in boxes, especially the right sized ones!

A good sized shelter for a cat should be 2 feet by 3 feet and at least 18 inches high. A large house is not necessarily the best as the heat will disperse quickly and cats need a warm shelter during the monsoon and in cold weather.

1. Students can be instructed to represent a 3D drawing of the house they intend to make, on paper.

How will they represent 2 feet in the drawing? What scale will they use? (Different groups can work with different scales.)

How will they represent 3 feet in the drawing? (Are they using the same scale as before? Discuss what would happen if they use different scales for different dimensions of the model.)

They should record the scale used for the drawing as "The scale of this drawing is …. to …. inches."

How will the roof be? Flat or sloping?

What should the height of the house at its highest point be, if it is to have a sloping roof?

Which side should have the entrance? How big should the entrance be?

2. As a second step, they should work out and list the sizes of the pieces needed to make the house.

Figure 3

Piece 1: 24 inches by 18 inches

Piece 2: … …

If they were to make the cat house from plywood sheet, will one sheet be adequate? A plywood sheet is generally 8 feet by 4 feet in size.

Figure 4

They should make a drawing of the plywood sheet on paper using an appropriate scaling factor.

They should also make scale drawings of the six pieces needed on dot paper.

They can cut out the pieces that they have drawn and try to fit them together on the drawing of the plywood sheet. Do they all fit on it?

PROJECT 2: DESIGNER ROOM

Student groups: 4 students per group.
Material: Dot or graph paper
Concepts: Estimation, scaling factor, representation, layout design

Students can design their bedroom by listing out their needs and prioritising the furniture needed in their bedroom.

Figure 5

Assuming that the child has to share the bedroom with a sibling they can make a list of necessary items. They can make reasonable estimates of the measurements for these pieces of furniture to design their room plan.

Heights can also be considered where required.

Here is a possible list:

- Cots: 3 × 6 feet (2 Nos)
- Cupboard: 2 × 3 feet (1)
- Book shelf:
- Desktop table:
- Computer chair: 1
- Normal chair: 1

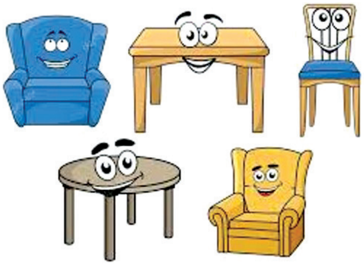

Figure 6

Let the groups work with a standard sized bedroom (10 x 10 x 10). The students can use dot or graph paper for their room plan.

What scale will they use?

Have they marked the doorway? Where will the windows be? Will that affect their arrangement?

The students should draw all the furniture pieces on another graph paper using the same scale. They can cut these pieces and try out different arrangements on their room plan to work out a good fit.

Discuss other possibilities.

What if they needed two study tables? What if they needed two cupboards?

Can the use of a bunk bed help to create a play area?

Students can work with a given design to work out the answers for the questions.

If this room in Figure 7 was 12 by 10 feet what is the size of the beds? What is the size of the play area?

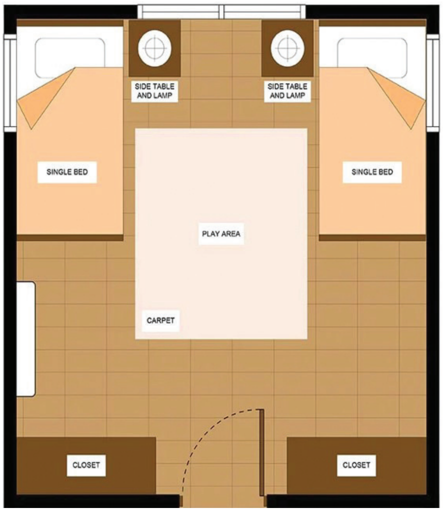

Figure 7

What is the actual length of the door?

PROJECT 3: PHOTOGRAPHS AND COPIES!

Student groups: 4 students per group.
Materials: Pairs of similar rectangles, dissimilar rectangles
Pairs of similar triangles and dissimilar triangles
Similar and dissimilar shapes of varied types.
(They can be printed by using appropriate software or apps.)
Concepts: Observation skills, Similarity, Scaling factor, usage of fractions in measurements

Figure 8

Things which are the same shape but a different size are said to be **similar**.

Talk about similar and dissimilar shapes using geometric materials. What attributes will be used?

Lengths? Angles? Any other attributes?

Which of the shapes in Figures 9-11 are similar to each other? Are there some shapes which are not similar to any other shape? Justify using math properties.

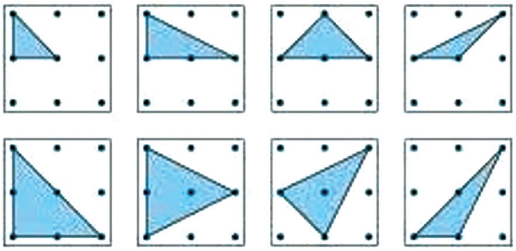
Figure 9

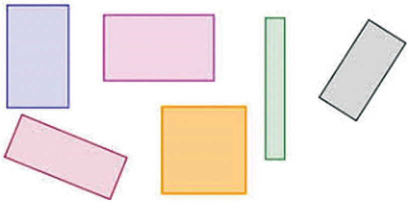
Figure 10

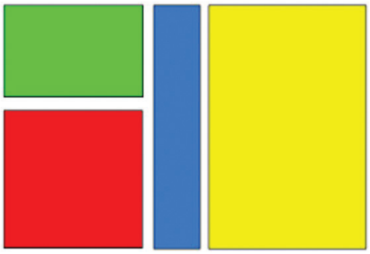
Figure 11

Use the classroom context and objects to talk about similarity. Are the windows in Figure 12 similar?

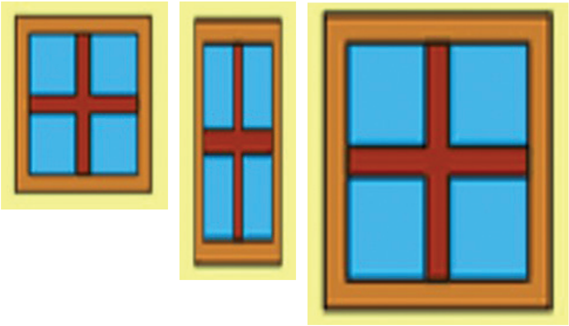
Figure 12

Use outdoor objects of nature like leaves to talk about similarity and dissimilarity in shape. Point out that while all leaves are not similar, functionally they are similar.

Are these dolls in Figure 13 similar? Justify your answer!


Figure 13

Are the pairs of triangles in Figure 14 similar? Use the given measurements to determine similarity.

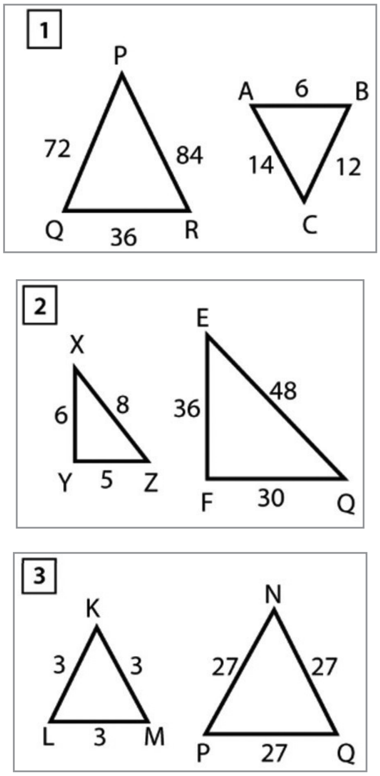

Figure 14

Challenge

Students can be given diagrams that are not drawn to scale and hence they cannot rely on what can be seen or measured. Can they now use mathematical reasoning to identify mathematically similar images?

Original Image:

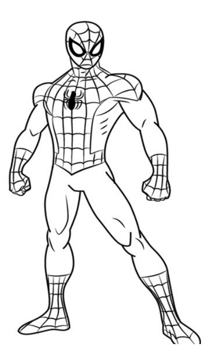

Figure 15

Which of the three images (Figures 16) is similar to the original image? Justify your answer. What methods can the students use?

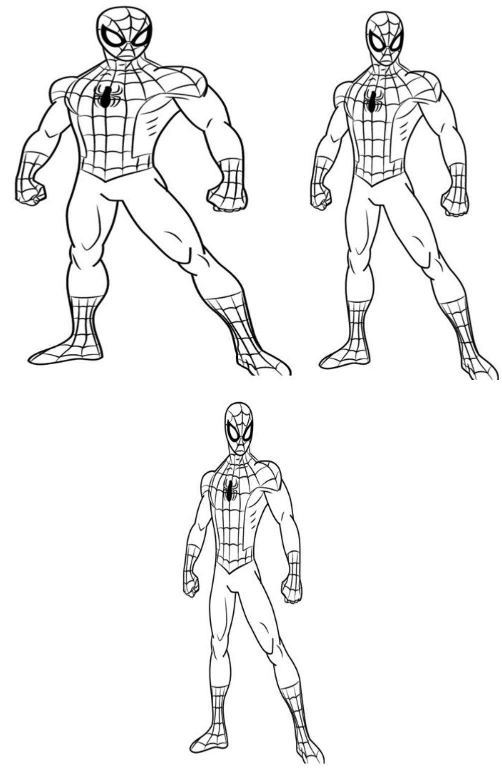

Figure 16

PROJECT 4: ENLARGEMENTS

Student groups: 4 students per group.
Materials: Square dot paper, similar mathematical shapes
Concepts: Enlargement, Scale factor, Similarity, Properties of similar figures

Students can be shown examples of enlarged figures to understand the principles of enlargement. Discuss the pictures (Figure 17) with them. Study the figures in terms of length to contrast and gain an understanding of the scale factor.

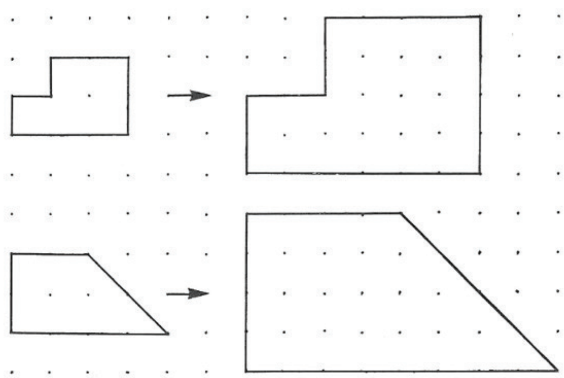

Figure 17

On a square dot paper, the students can make some letters of the alphabet (Figure 18) and make an enlargement by a scale factor of two or three.

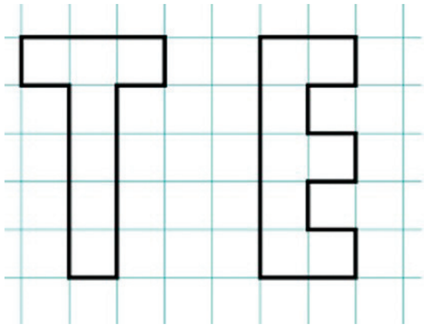

Figure 18

Students compare two given similar triangles (Figure 19) and note down their observations.

Triangle B is an enlargement of triangle A by a scale factor of …

The angles of triangle A and the angles of triangle B are …

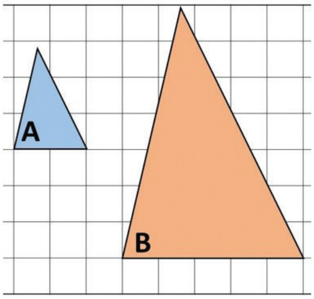

Figure 19

Students can compare two similar rectangles and verify if the scale factor holds for the diagonal.

Are these two rectangles (Figure 20) similar to each other? Justify.

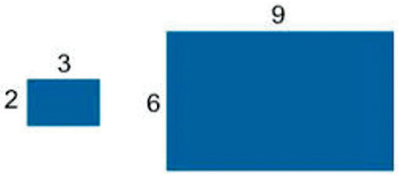

Figure 20

Are any of the shapes B, C, D an enlargement of shape A (Figure 21)?

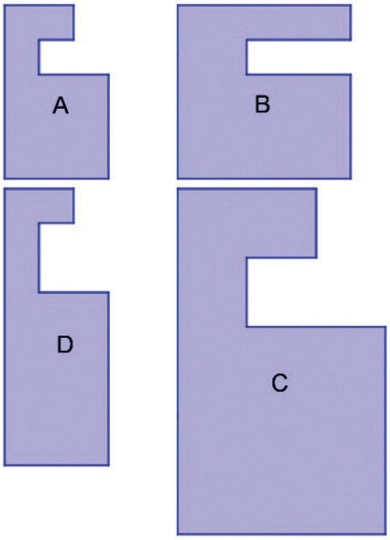

Figure 21

PROJECT 5: BLOW UP SHAPES

Student groups: 4 students per group.
Materials: Set of similar pictures

In Figure 22, measure the wingspan of the butterfly (wing tip to wing tip, measured horizontally) from one end to the other in picture *XY*.

Picture *X′Y′* is an enlargement of picture *XY.* Measure the wingspan in picture *X′Y′* from one end to the other.

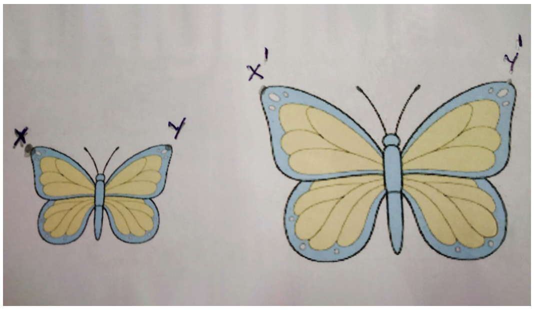

Figure 22

What is the scale factor of the enlargement?

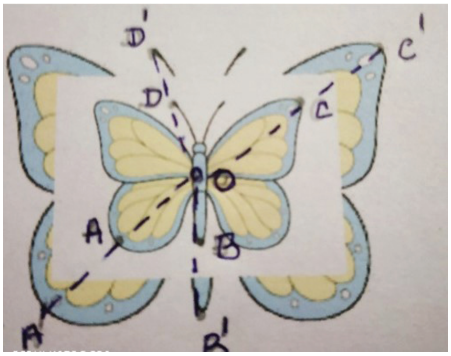

Figure 23: The dotted lines connecting corresponding points all meet at point O

The large butterfly is an enlargement of the small butterfly.

Measure OA and OA′. What is the scale factor?

Is it the same for OB and OB′?

Check the other extensions.

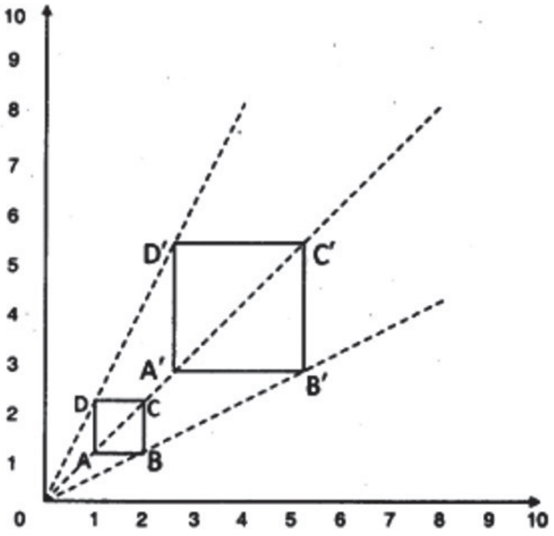

Figure 24

Can we say that a picture is an enlargement of another based on a single measurement?

Point O is called the centre of enlargement. In Figure 24, the larger square is an enlargement of the smaller square, the centre of enlargement being the origin.

The centre of enlargement need not be inside the picture. It can lie anywhere.

Draw any quadrilateral ABCD. Mark a point O inside it (Figure 25).

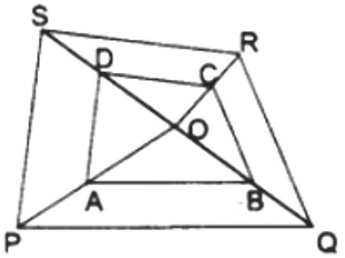

Figure 25

Draw lines from O to the four vertices A, B, C, D.

To enlarge the quadrilateral by a scale of 1.5, measure the length from O to A and extend the line to P by scaling the length by a factor of 1.5. Do the same for all the other lengths, OB, OC, OD.

The quadrilateral ABCD is now enlarged by a scale factor of 1.5.

The centre of enlargement can lie anywhere, as shown in Figure 26. The process of extending the lines is the same.

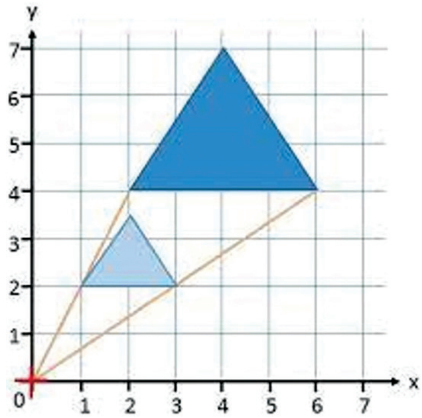

Figure 26

Enlargement can also be done by using coordinates. In Figure 26 the centre of enlargement is at the origin.

In Figure 27 the coordinates of the vertices of quadrilateral ABCD are A (1, 4), B (4,4), C (2,2), D (1,2).

The coordinates of quadrilateral PQRS are P (4,5), Q (10, 5), R (6,1), S (4,1).

Where is the centre of enlargement in Figure 27? What do you think is the scale factor?

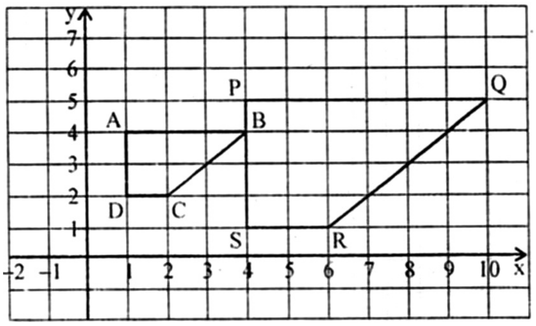

Figure 27

Figure 28 shows a small yellow triangle which has been enlarged.

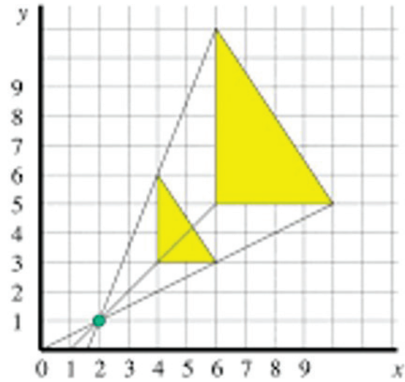

Figure 28

What is its scale factor?

Can you enlarge it by a scale factor of 3?

Is the red outlined figure in Figure 29 an enlargement of the black outlined figure?

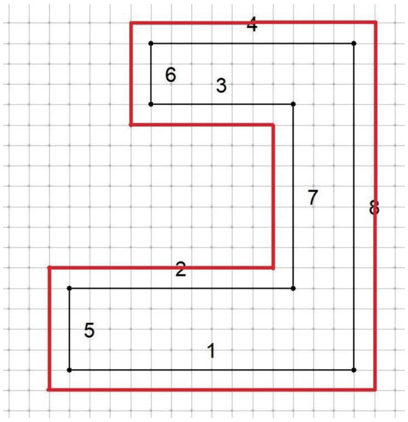

Figure 29

Can you enlarge the black figure by a scale factor of 2?

Students can be helped to articulate what enlargement does to the points in an object by asking leading questions.

When an object is enlarged what happens to the vertices of the object? How do they move? Do they move in a straight line?

What happens to the other points on it and in it? Do they also move away in straight lines?

What happens to the points that are furthest away? How does a bigger scale factor influence the movement?

Do the students see that to enlarge a shape, we only need to know how the crucial points behave?

It will be appropriate to discuss how the centre of enlargement behaves, using different examples.

PROJECT 6: SHRINKING SHAPES

Student groups: 4 students per group.
Materials: Set of similar pictures

Reduction or shrinking is the opposite of enlargement.

Students can experiment with drawing reduced copies of a figure on graph paper, using a scale factor of 1/2 or 1/3.

Figure 30

Can they reduce the given figure by a factor of 1/2?

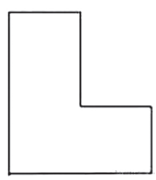

Figure 31

By what scale factor has the original stamp design been reduced to the actual size?

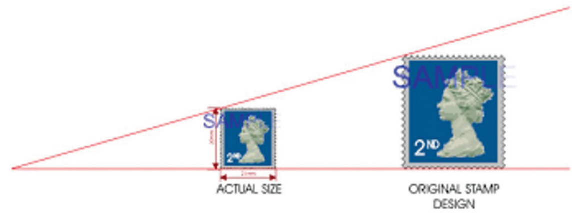

Figure 32

By what scale factor is each cuboid shrinking in height?

Figure 33

PROJECT 7: GROWTH PATTERNS

Student groups: 4 students per group.
Materials: Set of similar pictures of a shape in different sizes

Here is a trapezium that has been enlarged several times.

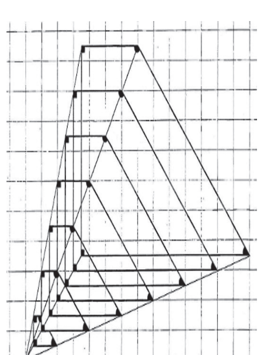

Figure 34

Create a table for the growing trapezia in Figure 34.

Scale factor	Original length	Corresponding new length	Original area	New area	Ratio of original area to new area

What do you notice about the ratios in columns 1 and 6?

Investigate how changing the side of a square by a scale factor of 3 affects its area.

Materials: Set of 3D objects in increasing size

Here is a set of cubes in increasing size.

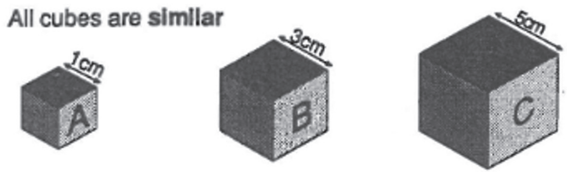

Figure 35

What do you notice about the ratios?

Investigate how changing the side of a cube by a scale factor of 2 affects its volume.

Investigate a rectangle which has been enlarged by a scale factor of 3. How are the perimeter and the area of the new rectangle affected?

Create a table for the growing cubes.

Scale factor	Original length	Corre-sponding new length	Ratio of the original surface area to the new surface area	Ratio of the original volume to the new volume

How does doubling the height and base of a triangle affect the area of the triangle?

What happens to the proportions of the new shape?

PROJECT 8: NEGATIVE ENLARGEMENT

Student groups: 4 students per group.
Materials: Set of pictures with negative enlargement

Discuss with the students about what is happening in the negative enlargements.

Do they notice that the figure is on the other side of the centre of enlargement?

When we draw the lines in the opposite direction, it is called a negative enlargement (Figure 36).

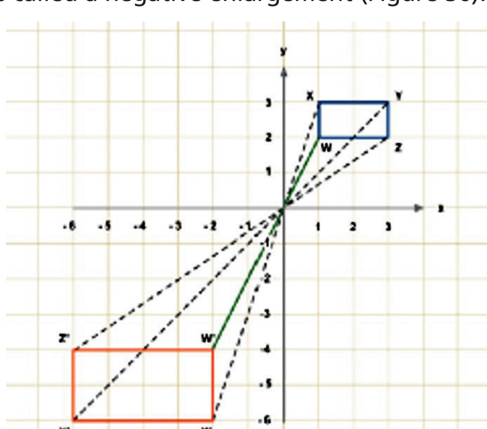

Figure 36

Discuss with students the meaning of a negative scale factor.

A **negative scale** factor -2 indicates double the distance from the centre of enlargement, but in the opposite direction. Here is one such example.

Let the students study the following diagrams (Figures 37-39) carefully.

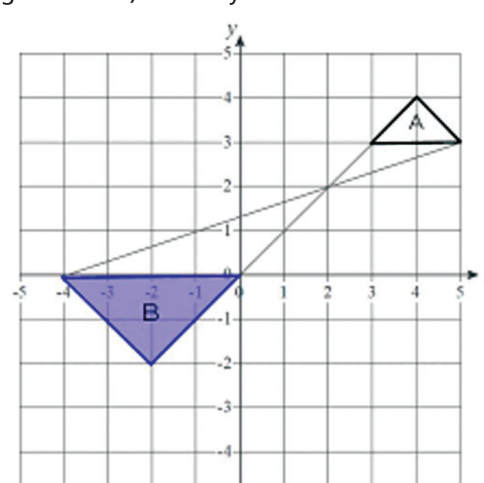

Figure 37

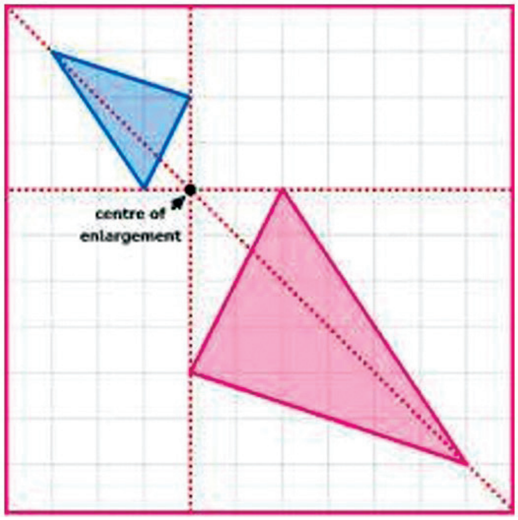

Figure 38

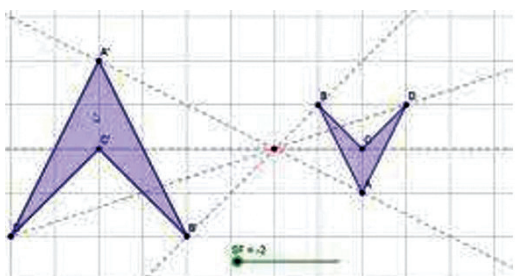

Figure 39

Do the students notice that a negative enlargement causes the image to be inverted or turned 'upside down'?

The students can make a figure like a kite and make a negative enlargement, say by a factor of -3.

What happens when the scale factor is greater than 1?

What happens when the scale factor is equal to 1?

What happens when the scale factor is between -1 and 1?

Notice that the shape reduces in size.

What happens when the scale factor is negative?

Investigation

Students can select any shape and experiment with different centres (one lying outside, one lying inside, one lying on the shape) to draw enlargements of uniform positive scale to observe what difference it makes.

What will remain the same? Size or shape?

What changes? Their location.

They can also experiment with negative scale factors.

Acknowledgements

School Mathematics project (SMP) https://generic.wordpress.soton.ac.uk/smp2/access-to-smp/

Smile: https://mypages.iit.edu/~smile/mathinde.html

https://ng.siyavula.com/read/maths/jss3/similar-shapes/09-similar-shapes

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