Solutions to Problems on Number Bases

Problems may be found on page 37

1. Since one of the bases is 2, we need to consider only the numerals 0 and 1.

   We have $2(4a + 2b + c) = 9a + 3b + c$. On simplification this yields $b + c = a$. Since $a = 1$, one of b,c is 1 and the other is zero. If $b = 1$, $c = 0$, we have the solution $2(110_2) = 110_3 = 12_{10}$. If $c = 1$, $b = 0$, we have $2(101_2) = 101_3 = 10_{10}$.

2. We need to consider only the numerals 0-3. We have $36a + 6b + c = 2(16a + 4b + c)$. This leads to $4a = 2b + c$. The maximum value of RHS is 9 and so a can be only 1 or 2. The possible solutions are $112_6 = 2(112_4) = 44_{10}$, $120_6 = 2(120_4) = 48_{10}$ and $232_6 = 2(232_4) = 92_{10}$.

3. We need to consider only the numerals 0-5. We have $81a + 9b + c = 2(36a + 6b + c)$. This leads to $9a = 3b + c$. Maximum value of RHS is 20 and so a can be only 1 or 2. The possible solutions are (1,2,3), (1,3,0) and (2,5,3). That is, $123_9 = 2(123_6) = 102_{10}$, $130_9 = 2(130_6) = 108_{10}$ and $253_9 = 2(253_6) = 210_{10}$.

4. We need to consider only the numerals 0-4. We have $36a + 6b + c = 25c + 5b + a$ leading to $35a + b = 24c$. The only solution to this is (2,2,3). That is, $223_6 = 322_5 = 87_{10}$.

5. The relation given in the problem yields $144a + 12b + c = 121c + 11b + a$ leading to $143a + b = 120c$. A bit of trial and error yields the following as the only solution: (5,5,6). That is, $556_{12} = 655_{11} = 786_{10}$.

   (For bases higher than 10, we may sometimes need new numerals: for base 11, one numeral for ten, generally represented by 'A', and for base 12, one more for eleven, generally represented by 'B'. As an illustration consider the number $2A3B_{12}$. This stands for $2 \times 12^3 + 10 \times 12^2 + 3 \times 12 + 11 \times 1 = 4943$ in base 10.)

6. We need to consider only the numerals 0-4. We have $1000a + 100b + 10c + d = 6(125a + 25b + 5c + d)$ leading to $50a = 10b + 4c + d$. The maximum value of RHS is 60. So $a = 1$. Then we can have $b = 4$, $c = 2$, $d = 2$ and $b = 3$, $c = 4$, $d = 4$. That is, $1422_{10} = 6(1422_5)$ and $1344_{10} = 6(1344_5)$.

It is hoped that the reader sees that the common aspect of different number bases is that of place value, a key idea that enables one to write down large numbers in a compact form and using only a few symbols. Students develop greater flexibility in their problem solving skills with such activities.