Suma Numbers

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Definition 1. *N* is called a 'Suma' Number if it is the smallest positive integer for which the number of factors of *N* is equal to the units digit of *N*.[a]

For Example:

(1) Consider **14**
Number of Factors of 14 = 4 [1, 2, 7 & 14]
Units digit of 14 is 4

(2) Consider **76**
Number of Factors of 76 = 6 [1, 2, 4, 19, 38, 76]
Units digit of 76 is 6

Before finding 'Suma' numbers, we must know how to calculate the number of factors of any given number.

To find the number of factors of N, we first need to express N as the product of prime numbers in exponential form, i.e.,

$$N = {p_1}^{\alpha_1} \times {p_2}^{\alpha_2} \times {p_3}^{\alpha_3} \times \ldots . \times {p_n}^{\alpha_n},$$

where $p_1, p_2, p_3, \ldots . \, p_n$ are distinct prime numbers and α_1, $\alpha_2, \alpha_3, \ldots \, \alpha_n$ are positive integers.

Then the number of factors of $N = (\alpha_1 + 1) \times (\alpha_2 + 1) \times (\alpha_3 + 1) \times \ldots . \times (\alpha_n + 1)$.

[a] The author usually names his findings with a specific name that reflects the property of that number or work. If no such name suits, then he uses the name of a person who motivates his mathematical observations. Mrs. Suma is one such person.

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For Example: To find the number of factors of 72.

$$72 = 2^3 \times 3^2$$

$$\therefore \text{Number of factors of } 72 = (3+1) \times (2+1)$$

$$= 4 \times 3 = 12.$$

Now let us find some 'Suma' Numbers, proceeding systematically as the units digit changes from 1 to 9. Remember the definition of a Suma Number: The smallest positive integer for which the units digits is equal to the number of factors.

Units Digit	Reasoning	Suma Number
1	There exists one and only one number with only one factor and that is '1'	The 'Suma' number with units digit '1' is 1
2	The only numbers which have exactly 2 factors are the prime numbers and of these, 2 is the only prime with units digit 2	The 'Suma' number with units digit '2' is 2
3	The only numbers which have exactly 3 factors are the numbers of the form p^2 where p is a prime number. No square number has units digit 3, so N (with units digit 3) $= p^2$ is impossible.	There is no 'Suma' number with units digit 3
4	Since $4 = 4 \times 1$ or 2×2, using the above statement regarding the number of factors of a number, the only numbers which have exactly 4 factors are numbers which are either in the form: p^3 where p is a prime number; OR	There exists no prime p such that the units digit of its cube is 4.
	$p_1{}^1 \times p_2{}^1$ where p_1, p_2 are distinct prime numbers. If $\text{N} = p_1{}^1 \times p_2{}^1$ Since $4 = 2 \times 2$ does not fit the constraints, consider the next positive integer with units digit 4, i.e., 14. $14 = 2 \times 7$ and has 4 factors namely 1, 2, 7, 14. [**Note:** for $p_1 = 2$ & $p_2 = 17$ also, we get units digit of $\text{N} = 4$. For any prime p_2 whose units digit is 7, $\text{N} = 2 \times p_2$ gets unit digit $= 4$; however, the Suma number, being the smallest positive integer satisfying this property, is 14]	The 'Suma' number with units digit '4' is 14.
5	To get the number of factors of 'N' equal to 5, N should be in the form of p^4 where p is a prime number and the units digit of p has to be 5 for the units digit of N to be 5. $5^4 = 625$	The 'Suma' number with units digit '5' is 625.

Units Digit	Reasoning	Suma Number
6	To get the number of factors of 'N' equal to 6, we consider that $6 = 1 \times 6$ or $2 \times 3 = (\alpha_1 + 1) \times (\alpha_2 + 1)$ So, $\alpha_1 = 0$ and $\alpha_2 = 5$ which gives $N = p^5$ where p is a prime number Or $\alpha_1 = 1$ and $\alpha_2 = 2$, which gives $N = p_1{}^1$ x $p_2{}^2$, where p_1, p_2 are distinct prime numbers. If $N = p^5$ there exists no prime p such that the units digit of N is 6 If $N = p_1{}^1 \times p_2{}^2$ we proceed systematically through 6, 16, 26, 36, … until 76, when we get $p_1 = 19$ & $p_2 = 2$ $19 \times 2^2 = 19 \times 4 = 76$	The 'Suma' number with units digit '6' is 76
7	To get the number of factors of 'N' equal to 7, N should be in the form of p^6 where p is a prime number. But no prime exists whose sixth power ends with 7.	So there exists no 'Suma' number with the units digit '7'
8	To get the number of factors of 'N' equal to 8, N should be in the form of p^7 where p is a prime number OR	If $N = p^7$ there exists no prime p such that 'N' ends with 8
	$p_1{}^1 \times p_2{}^3$ where p_1, p_2 are prime numbers. If $N = p_1{}^1 \times p_2{}^3$ for $p_1 = 11$ & $p_2 = 2$ we get units digit of $N = 8$ $[11 \times 2^3 = 11 \times 8 = 88]$ [**Note:** for $p_1 = 31$ & $p_2 = 2$ also we get units digit of $N = 8$ & for any prime p_1 whose units digit is 1, above 'N' gets units digit $= 8$]	'88' is the 'Suma' number with units digit '8'
9	To get the number of factors of 'N' equal to 9, N should be in the form of p^8 where p is a prime number or $p_1{}^2 \times p_2{}^2$ where p_1, p_2 are prime numbers.	If $N = p^8$ there exists no prime p such that 'N' ends with 9
	If $N = p_1{}^2 \times p_2{}^2$ for $p_1 = 3$ & $p_2 = 11$ we get units digit of $N = 9$ $[3^2 \times 11^2 = 9 \times 121 = 1089]$ [**Note:** for $p_1 = 31$ & $p_2 = 3$ also we get units digit of $N = 9$ & for any prime p_1 whose units digit is 3, p_2 whose units digit is 1, above 'N' gets units digit $= 9$]	So '1089' is the 'Suma' number with units digit '9'

Here is the list of 'Suma' Numbers with different units digits

For the Units Digit	'Suma' Number	Number of Factors
1	1	1
2	2	2
3	No such number exists	–
4	14	4
5	625	5
6	76	6
7	No such number exists	–
8	88	8
9	1089	9

There is a chance of extending this for the last two or more digits of given number:

Observe the number 2312.

To find the number of factors for 2312 …

Let us write 2312 as $2^3 \times 17^2$

$\therefore$ No of factors of 2312 $= (3 + 1) \times (2 + 1) = 12$

We get exactly 12 factors which means numbers of factors is equal to last two digits of 2312.

So 2312 is the 'Suma' number whose last two digits (12) is equal to number of factors of 2312, provided it is the smallest number which satisfies this property. Can you verify this? Which clearly indicates that there will be a chance for the existence of 'Suma' numbers with higher number of factors which are equal to last group of digits instead of units digit alone.

Closing Remarks: Such investigations improve students' observation skills. By understanding constraints, doing a systematic search and documenting their work, they find that even small connections between numbers are to be noted and they have beautiful patterns. This will encourage them to try and find such interesting relations and connections between the numbers, so that with the help of their logical & reasoning skills they may develop research abilities also.

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