Problems on Number Bases

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We normally use the base 10 or decimal system of writing numbers. One could also use other bases to write down numbers. If n is the number base, then the place values are (from right to left) 1, n, n^2, n^3,…. For example, take the number 124, which is 'one hundred and twenty four' in the base 10 system.

That is, $1 \times 10^2 + 2 \times 10 + 4 \times 1 = 124$.

When 124 is read in base 5, its value is $1 \times 5^2 + 2 \times 5 + 4 \times 1 = 39$ in base ten.

To write a number in base n we need n numerals: 0, 1, 2,…., $(n - 1)$. In the problems that follow, you are required to move between different number bases and make connections. We denote the numerals by lower case letters and the base used is indicated by a subscript at the end of the number. For instance, abc_5 represents a 3-digit number in base 5. The letters a,b,c need not represent different digits – two different letters could stand for the same digit. The first digit in any string of numerals is never zero. There may be more than one solution. Please find all solutions.

Hint: In solving these problems, first form an equation reflecting the values in the given base systems. Find values for a,b,c,d that satisfy this. The numerals to be considered will depend upon the bases involved. Consider only those that are less than the lower of the bases used. For instance, in problem 3, the bases used are 6 and 9; consider only the numerals 0-5.

Problems

1. If $2(abc_2) = abc_3$, find (a,b,c).

2. If $abc_6 = 2(abc_4)$, find (a,b,c).

3. If $abc_9 = 2(abc_6)$, find (a,b,c).

4. If $abc_6 = cba_5$, find (a,b,c).

5. If $abc_{12} = cba_{11}$, find (a,b,c).

6. If $abcd_{10} = 6(abcd_5)$, find (a,b,c,d).

Solutions to the problems are on page 59