TearOut

Playing with Symmetry

In this 7th TearOut, we will be exploring the symmetry of various regular polygons. This time, pages 1-3 are a worksheet for students while page 4 provides guidelines for the facilitator.

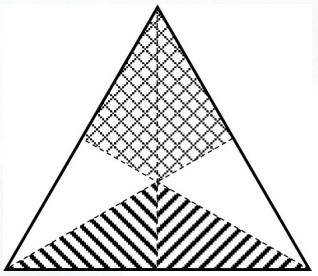
Let's warm up by filling this table. The column corresponding to the 'Square' has been done for you.

	Equilateral triangle	Square	Regular hexagon
No. of lines of symmetry		4	
Types of lines of symmetry		Diagonals, perpendicular bisectors of sides	
Order of rotational symmetry		4	

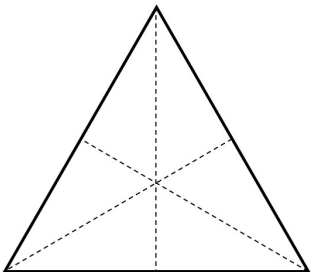
Colour or shade each polygon so that the resulting picture has the indicated symmetry. The first one is done for you.

Equilateral Triangle

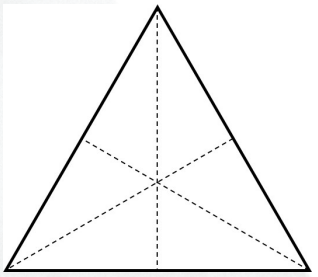
1. Exactly one line of symmetry and no rotational symmetry



2. Only rotational symmetry of order 3 and no line symmetry

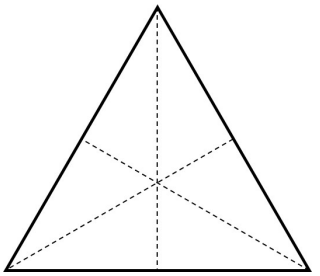


3. Two lines of symmetry



What is the angle between these lines? Does the coloured triangle have any rotational symmetry? What is the order?

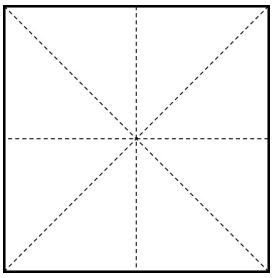
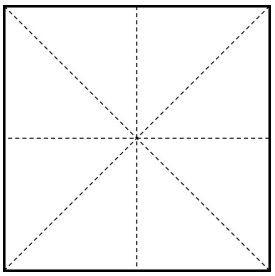
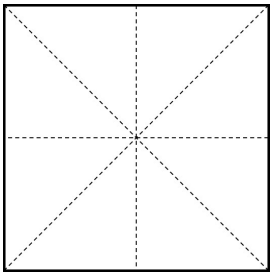
4. A line of symmetry and rotational symmetry



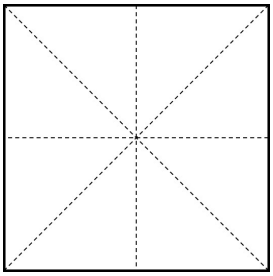
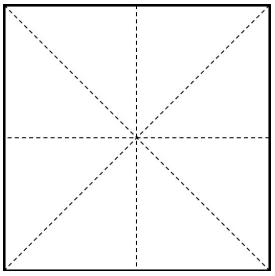
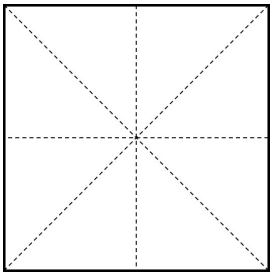
Does the coloured triangle have any other line of symmetry? What's the angle between them?

Square

1. Simple ones

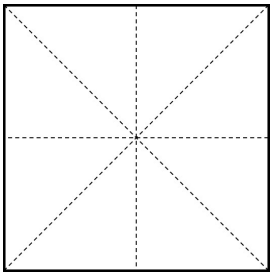
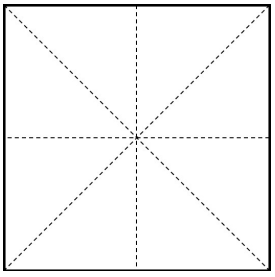
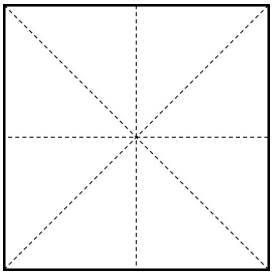
a. Exactly one line of symmetry and no rotational symmetry	b. Only rotational symmetry of order 2 and no line symmetry	c. Only rotational symmetry of order 4 and no line symmetry
		

2. Multiple lines of symmetry

a. Two perpendicular lines of symmetry	Can the same be done with other lines of symmetry? Try.	b. Two lines of symmetry at 45° angle
		

Does the coloured square have rotational symmetry? What is the order?

3. Mixing it up

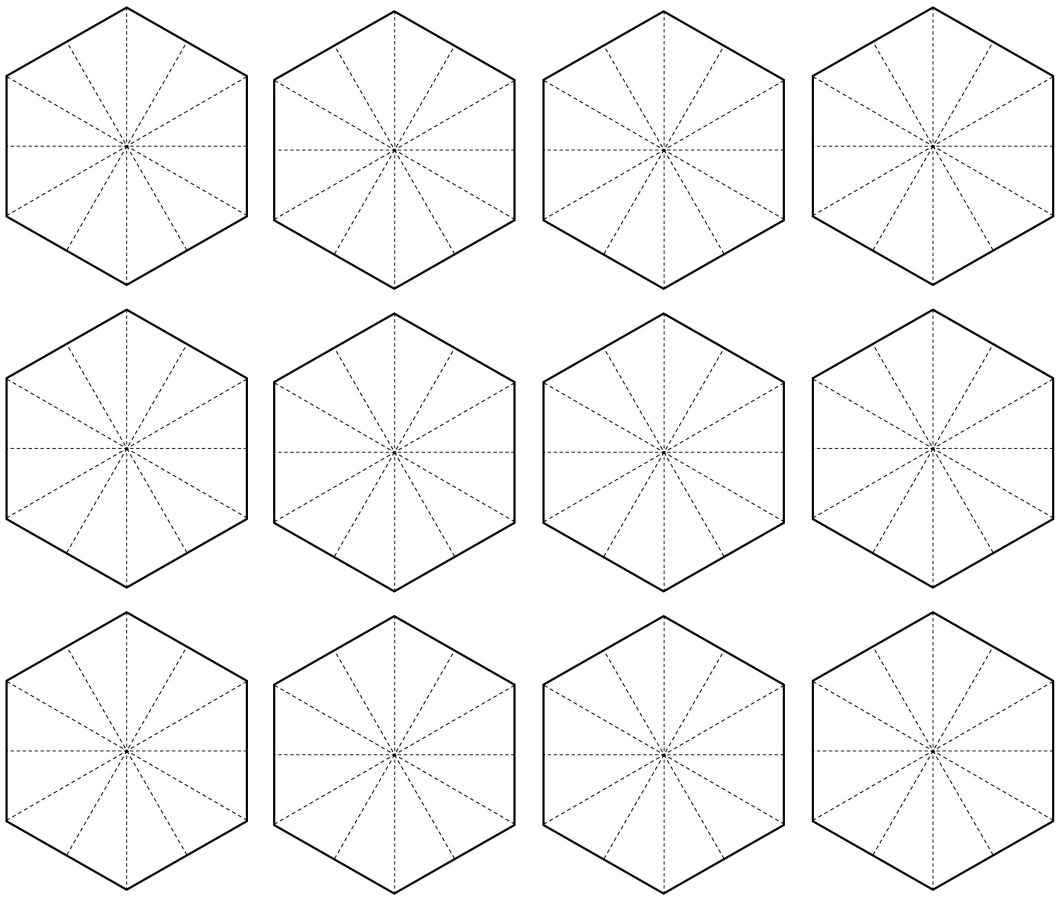
a. A line of symmetry and rotational symmetry of order 2	What if a different type of line of symmetry is selected?	b. A line of symmetry and rotational symmetry of order 4
		

Does the coloured square have other line(s) of symmetry? What's the smallest angle between two such lines?

Regular Hexagon

1. Simple ones	2. Multiple lines of symmetry	3. Mixing it up
a. Exactly one line of symmetry and no rotational symmetry	a. Two perpendicular lines of symmetry	a. A line of symmetry and rotational symmetry of order 2
b. Only rotational symmetry of order 2 and no line symmetry	b. Two lines of symmetry with 60° angle between them Can the above be done with other lines of symmetry? Try.	b. A line of symmetry and rotational symmetry of order 3 What if a different type of line of symmetry is selected?
c. Only rotational symmetry of order 3 and no line symmetry	c. Two lines of symmetry with 30° angle between them	c. A line of symmetry and rotational symmetry of order 6
d. Only rotational symmetry of order 6 and no line symmetry	Does the coloured hexagon have rotational symmetry? What is the order?	Does the coloured hexagon have other line(s) of symmetry? What's the smallest angle between two such lines?

[Optional] Now that you have explored three regular polygons, what do you think are the possibilities for a regular pentagon? What about a regular octagon? Try these on your own.



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This worksheet can be used to assess the understanding of symmetry. Note that most questions are open ended, i.e., have multiple correct options. All foster a sense of aesthetics and allow creativity. The first part for each regular polygon involves only one type of symmetry and avoids the rest. However, it may happen that other kinds of symmetry may creep in. The student should be guided to eliminate them. For example, Figure 1 has both line symmetry and rotational symmetry whereas Figure 2 has only line symmetry and no rotational one. So, Figure 1 is correct for Square 2.a, but not for Square 1.a.

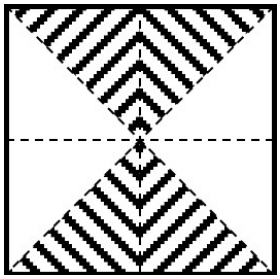


Figure 1

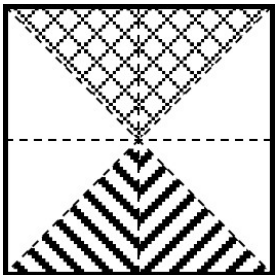


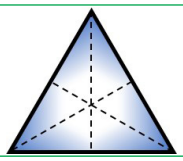
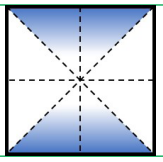
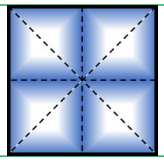
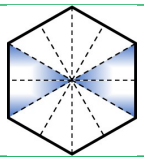
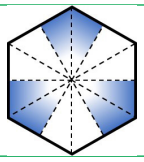
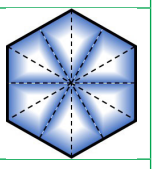
Figure 2

The order of rotational symmetry and its factors also play a crucial role. The order is 4 for square and it can be coloured to have order 2 or 4, which are factors of 4. Similarly, the hexagon can be coloured to have order 2, 3 or 6 – the factors of 6. Students can be encouraged to explore the possible orders for other coloured regular polygons.

The next set of questions with multiple lines of symmetry hints at the following:

1. Multiple lines of symmetry imply rotational symmetry
2. Order of the rotational symmetry = 360° ÷ (2 × the smallest angle between the lines of symmetry)
   Or, the angle of rotational symmetry = 2 × the smallest angle between the lines of symmetry

Students can be asked to tabulate their results of Triangle 3, Square 2 (a, b) and Hexagon 2 (a, b, c) to arrive at this relation.

	Equilateral triangle	Square		Regular hexagon		
Angle between lines of symmetry	60°	90°	45°	90°	60°	30°
Example of coloured polygons						
Angle of rotation	120°					
Order	3					

Note that, in the above example for Square 90°, the lines of symmetry pass through the midpoints of the sides. We could have chosen the diagonals to be the lines of symmetry to get another possibility. Similarly, in the above example for Hexagon 60°, the lines of symmetry are the diagonals. Instead, they could have passed through the midpoints of the sides. However, for Hexagon 90° such alternative choices are not available. One may wonder how this could be linked with the angle in question (second row of the above table).

The last set is the reverse of 1, illustrating

3. Line symmetry and rotational symmetry combined implies more than one lines of symmetry as well as the earlier relation between the order of rotational symmetry and the angle between the lines of symmetry. The above table can be utilized to cross reference the findings from this last set.

   In the optional part, students should realise that regular polygons with odd number of sides have only one kind of line of symmetry – each starting from a vertex and ending at the midpoint of the opposite side. On the other hand, regular polygons with even number of sides have two kinds of lines of symmetry – the diagonals and the lines connecting the midpoints of opposite sides.

This TearOut is based on the project of Keshvi, MA Education student at Azim Premji University as part of her Curricular Material Development – Mathematics course.