Construction of a Tunnel Through a Mountain

RADHAKRISHNAMURTY PADYALA

This article describes a method for the construction of a straight tunnel through a mountain, given the two openings at the two ends of the tunnel.

Constructing a tunnel through a mountain sounds a herculean task and appears almost impossible. Yet we see that such tunnels are constructed for defence, for shortening the distance between two places for trade, etc. It involves great engineering design skills and civil construction skills. It also needs tools, implements and instruments such as the dioptra. This article describes a method to construct a tunnel through a mountain. I felt that this simple geometric method would be of interest to students at high school and college levels as well as to a general reader who would appreciate the simplicity of the solution to what looks to be an unsolvable problem. The problem is stated in simple terms thus: To make a straight tunnel through a mountain, given the locations of the openings at each end.

We have taken the material presented here from M. R. Cohen and I. E. Drabkin [1]. A point to note is that Drabkin's paper contains a serious conceptual problem with the diagram, which gives a wrong conceptual perception with respect to the magnitudes of the proportions dealt with.

The problem

To construct a straight tunnel through a mountain, given the openings at each end.

It may appear puzzling, 'What is the problem in drawing a straight line when two points are given?' The catch, however, is that the two points are not directly accessible from one another. We cannot see the line connecting the points or even see one point from the other – a mountain is there between the two points! Therefore, an indirect method is to be sought.

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The method of solution described below is simple and easy. The strategy consists of two steps: **1.** Treating the line joining the two given points as the hypotenuse of a right triangle, we construct the remaining two sides. **2.** We construct two triangles similar to this right triangle. Our job is now almost done. We use the properties of similar triangles to get at the solution.

Geometrical Method of Solution

Let us imagine a mountain with base ABCD (Figure 1). Let B and D be the given openings at the two ends of the tunnel.

Draw a horizontal line BJ through B. From any point, say E, on BJ draw a perpendicular EF to BJ. Again, from any point, say Z, on EF draw a perpendicular ZG to EF. Now from any point, say H, on ZG draw a perpendicular HQ. From any point, say T, on HQ draw a perpendicular TK and similarly KL, perpendicular to TK. This process is repeated till we reach the level of point D.

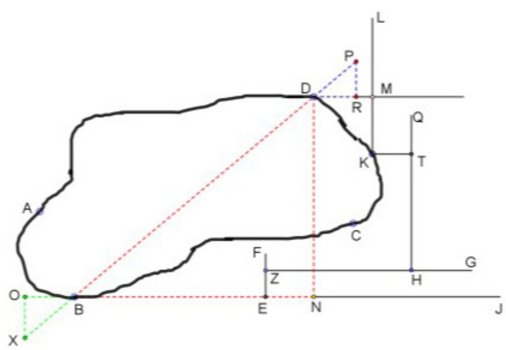

Figure 1. ABCD is the base of a mountain.
B and D are the given end points of the tunnel.

We now move the dioptra [2] along line KL (keeping the line of sight always perpendicular to KL) until point D comes into sight in a direction perpendicular to KL at point M. Draw a perpendicular from D to BE (or its extension) to meet it at N.

With the geometrical construction thus done, we can find length DN from lengths EZ, HT and KM (see Eq. (1)). Similarly, we can find length BN from lengths BE, ZH, TK and MD (see Eq. (2)).

$$DN = EZ + HT + KM \quad (1)$$

$$BN = BE + ZH - TK - MD \quad (2)$$

We now have two sides of the right-angled triangle whose hypotenuse is the line joining the two given points, the three sides together making the right triangle BDN (dotted line red triangle). Let $BN{:}DN = \nu{:}1$ (we shall use this ratio in constructing two more right triangles).

Our next step is to construct two more right triangles that are similar to the right triangle *BDN*.

Now imagine that DB is drawn and extended to a point X and XO drawn perpendicular to the horizontal BE. We get a right triangle BXO (dotted line green triangle). Similarly, suppose that BD is extended to a point P and PR drawn perpendicular to DM. We get a right triangle DPR (dotted line blue triangle). It is easy to see that triangles BND and BOX are similar:

$\angle BND = \angle BOX = 90°$,

$\angle NBD = \angle OBX$ ("vertically opposite angles"),

$\angle OXB = \angle NDB$.

Similarly, triangles BND and DRP are similar:

$\angle BND = \angle DRP = 90°$,

$\angle DBN = \angle PDR$ (since $BN \parallel DR$ and PB is a transversal),

$\angle BDN = \angle DPR$.

We now see that the big red right triangle BDN is similar to the two small right triangles BXO and DPR. Consequently, all the three right triangles BDN, BXO and DPR are similar. Therefore, the corresponding sides are proportional. Therefore, we get,

$$BD : DN : BN = BX : XO : BO = DP : PR : DR \quad (3)$$

$$\frac{BN}{DN} = \frac{BO}{XO} = \frac{DR}{PR} = \nu \quad (4)$$

Having thus completed the mathematical portion of the problem, we come to implementation. Produce the horizontal through B. Select an arbitrary point O, on it. Draw a perpendicular to the horizontal at O. With O as center and radius equal to (BO/ν) draw a circular arc to intersect the perpendicular, at X. It follows that line XB when extended passes through the point D which is on the other side of the mountain. Similar analysis on the side where D lies shows us that the line PD, if extended, goes through the point

B which is on the opposite side of the mountain. Let us do the analysis: On the horizontal through D select an arbitrary point R. Draw a perpendicular to the horizontal at R. With R as center and radius equal to (DR/ν) draw a circular arc to intersect the perpendicular at P. It follows that PD if extended passes through the point B which is on the other side of the mountain.

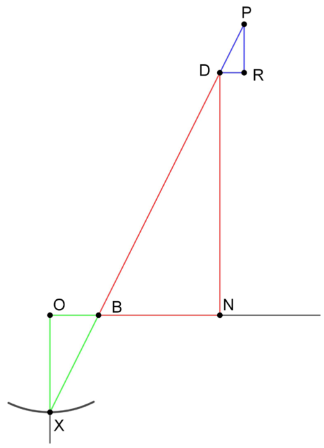

Figure 2.

We shall now commence our tunneling operations along the line XB from B and along line PD from D. For the rest of the tunnel, we proceed by setting our direction line along the determined lines XB and PD. If the tunnel is dug in this way, the workers (working from opposite ends) will meet at some point on the line BD [3].

In essence what was done to solve the problem of constructing a tunnel through a mountain is this: given two points B, D on the ground, on opposite sides of the mountain, we located two more points X and P so that all the four points lie on a straight line.

Another such problem, equally interesting, is the following:

> *Given two points on one side of a ditch or tank, determine the line on the other side of the ditch or tank that forms an extension of the line segment joining the two given points. In other words, given the points X, B (in the problem solved above) on one side locate points D, P on the other side such that all the four points lie on the same line. No dioptra to be used! You will be provided with marker pegs and assistants for completing the job.*

Interested readers could work on it. We shall present a solution in our next article.

Acknowledgement

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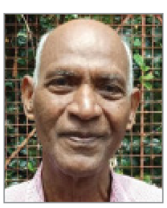
References and notes

[1] Morris R. Cohen and I. E. Drabkin, *'A Source Book in Greek Science'* Harvard Univ. Press, Cambridge, (1948) 341-342.

[2] The Dioptra in its simplest form is a sighting tube. In the form described by Hero of Alexandria, it is an instrument or pair of instruments for surveying and leveling, for the measurement of heights and distances of inaccessible places, and for the determination of angles in both terrestrial and astronomical problems. With it is combined a hydraulic level for the determination of the horizontal plane. The tilting of the line of sight to any plane between the horizontal and the vertical and to any position in the plane is made possible by sets of screws and geared wheels. The precise angles involved could, presumably, be read off when the position of the sighting piece was determined. In finding the relative height of two widely separated points a series of intermediate sightings was made with the help of leveling staffs.

[3] It appears that some time before 500 B. C. the engineer, Eupalinus of Megara tunneled through a hill in Samas for the purpose of carrying water, through pipes to the city. The tunnel, about 1000 yards long, is described by Herodotus; it was discovered in 1882. The tunneling operation was conducted from both ends with remarkable accuracy! Imagine the engineering skills the engineers had long before Christ.



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