A Special Class of Strong Prime Numbers – Krishnan's Primes

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It is well-known that there are infinitely many prime numbers. The 'Twin Prime Conjecture' states that there are infinitely many primes p for which $p+2$ is prime. We define a special class of prime numbers as follows and state a conjecture about them:

Definition 1. A prime number p is a *Krishnan prime* if both $p+2$ and p^2+4 are prime numbers.

For example, 3 is such a prime, since $3+2=5$ and $3^2+4=13$ are both prime numbers. There are 22 Krishnan prime numbers below 10000:

3, 5, 17, 137, 347, 827, 2087, 2687,
3557, 3917, 4517, 4967, 5477, 5657, 5867, 6827,
7457, 7547, 7877, 8087, 8537, 8597.

Conjecture (Sasikumar)**.** *There are infinitely many Krishnan primes.*

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Discussion

Definition 2. A prime number p_n is said to be **strong** (see [1]) if it is greater than the arithmetic mean of its nearest prime neighbours, i.e., if $p_n > \frac{1}{2}(p_{n-1} + p_{n+1})$.

The first few strong primes are the following:

$$\begin{array}{cccccccccc} 11, & 17, & 29, & 37, & 41, & 59, & 67, & 71, & 79, & 97, \\ 101, & 107, & 127, & 137, & 149, & 163, & 179, & 191, & 197, & 223, \\ 227, & 239, & 251, & 269, & 277, & 281, & 307, & 311, & 331, & \ldots \end{array}$$

Our aim is to show that *all Krishnan primes* > 5 *are strong*. For that we start with a lemma:

Lemma. *Let p be a Krishnan prime greater than 5. Then the last digit of p is 7.*

Proof. Let p be a Krishnan prime greater than 5; then $p \geq 17$. We write

$$p = a_0 + 10a_1 + 10^2 a_2 + \cdots + 10^n a_n$$

for some natural number n. Since $p \geq 17$, $a_0 \neq 0, 2, 4, 5, 6, 8$, and $a_1 \geq 1$.

Since $p + 2$ is prime, $a_0 \neq 3$ (else $p + 2$ will be divisible by 5 and so cannot be a prime number since $p \geq 17$).

Next, $p^2 + 4 = a_n 10^{2n} + \cdots + a_0^2 + 4 \geq 293$ is a prime number (by definition), so $a_0 \neq 1, 9$. Hence $a_0 \neq 0, 1, 2, 3, 4, 5, 6, 8, 9$. Therefore $a_0 = 7$. That is, the last digit of a Krishnan prime beyond 5 is 7.

Now we prove our main theorem:

Theorem. A Krishnan prime greater than 5 is a strong prime number.

Proof. Let $p > 5$ be a Krishnan prime. By the lemma we know that $p - 3, p - 2, p - 1$ and $p + 1$ cannot be prime numbers. Let q and r be respectively the prime numbers just preceding and just succeeding p.

By the definition of p, we know that $p + 2$ is prime, so we conclude that $r = p + 2$.

Since $p - 3, p - 2$ and $p - 1$ are not prime, we conclude that $q < p - 2$. Hence $q + r < 2p$, i.e., $p > (q + r)/2$. Hence a Krishnan prime greater than 5 is a strong prime number. □

Conclusion. The set of Krishnan primes greater than 5 is a subset of the class of strong primes.

References

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