ClassRoom

Integers: Extending the Number line with Coloured Counters

This article is intended for students – as hands-on play with integers by extending the number line and combining it with coloured counters. For this activity, rectangular dot sheets are better than square grid sheets since the dots can be joined by horizontal lines to form number lines and will not get mixed up with existing lines. We also recommend sketch pen/crayon/colour pencil of two contrasting colours to draw the counters.

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Take some rectangular dot sheets, sketch-pens of two contrasting colours (for example, red and green), a ruler and a pencil, and you are good to go. Join the dots along the second line from the top. Then do the same for the 5th, 8th, 11th, … lines so that there are 2 lines of dots followed by a line connecting them. Pick a dot somewhere in the middle and label it 0. Then label the dots on its right successively 1, 2, 3, etc. Label each horizontal line similarly. Make sure all the zeros are along the same vertical line (Figure 1).

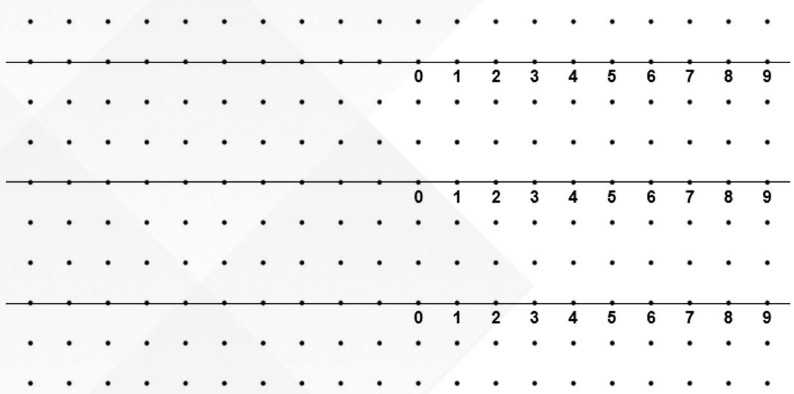


Figure 1

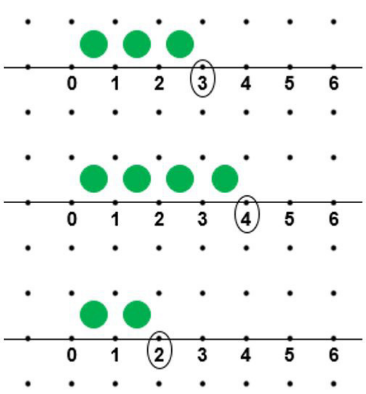


Figure 2

Keywords: integers, addition, subtraction, number line, modeling, word problems

1. Successors and Predecessors

(a) Pick any number between 2 and 5. Circle the corresponding dot. Draw as many counters as the number represents, equally spaced as shown (Figure 2 - chosen number here is 3).

(b) In the next line, draw the successor of your number and circle the corresponding dot.

(c) In the next line, draw the predecessor of your number and circle the corresponding dot.

(d) Draw the predecessor of 1. How many counters did you draw? Why?

(e) What would be the predecessor of 0? Where and how can we draw it?

For predecessor, we move one step ___________ (right/left).

> To find the dot corresponding to the predecessor of 0, we extend the number line to the left. Since this is one unit away from zero, we must mark it as 1, but since it is on the left side, we mark it as "-1" to distinguish it from the 1 on the right. As before, we draw one counter, but since it corresponds to movement towards the left, we use the other colour and we draw it below the number line (Figure 3). We call counters of this second colour **negative** and those of the first colour, **positive**. Accordingly, the numbers on the right of zero are called positive and those on its left are called negative.

2. Newer numbers with predecessors

(a) Draw the predecessor of -1. Using the same pattern, how do we write this number?

(b) Now, mark the part of the number line which is to the left of 0 accordingly, i.e., -2, -3, -4, etc.

(c) Draw -4. How many ________ (red/green) counters did you use __________ (above/below) the line to the ________ (left/right) of 0?

> To draw the predecessor of any negative number, we have to add a negative counter as we shift one step left.

Try these:

I. Draw the predecessor of -3

II. The predecessor of -19 is _____. It is shown with _____ (18/20) _________ (red/green) counters. It is to the ________ (left/right) of -19.

Think:

A. Which city is colder? Shimla with a temperature of -7°C or Leh at -8°C.

B. If sea level is at 0m, and a pole of height 5m is shown by 5, then a hole of depth 5m is shown by ____. Does -6 show a hole deeper than a hole of depth 5m?

C. Titir with a debt of ₹300 is _______ (richer/poorer) than Tinku who has a debt of ₹200.

3. Successor of -1

For successor, we take one step to the _______ (right/left).

(a) So, which number is the successor of -1?

(b) How many counters (and of which colour), are used to show the successor of -1?

(c) For successor (and therefore to step right), we have always added a positive counter. When we do that here, we get a _____________ (red counter/green counter/red-green pair)?

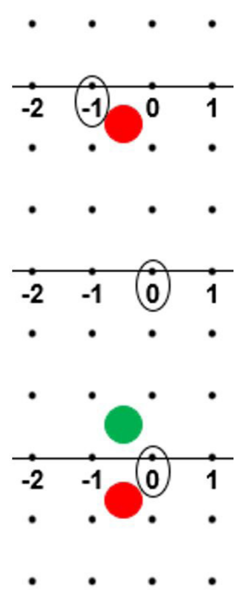


Figure 3

This positive-negative counter pair is equivalent to zero (Figure 3). Therefore, we will call such a pair **zero pair** from now on. Since they are equivalent to zero, they can be removed (or added) as and when needed.

Try this:

III. 0 = ___ + ___ = ___ + ___ = ___ + ___

4. Adding positive numbers

When we add positive numbers, we take not one, but several steps to the _____ (right/left).

(a) Add 3 to 4 (b) Add 2 to -5 (c) Add 5 to -3

(a) 4 + 3: Take ___ steps to the _______ (left/right) of 4, adding 1 _______ (red/green) counter at each step.

(b) (-5) + 2: Take ___ steps to the ______ (left/right) of -5, adding 1 ______ (red/green) counter at each step.

(c) (-3) + 5: Take ___ steps to the ______ (left/right) of -3, adding 1 ______ (red/green) counter at each step.

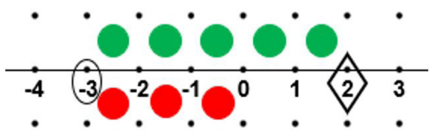


Figure 4

You used the same strategy for all three sums. Did you get zero pair(s) for any sum?

Is adding a green counter the same as removing a red counter? Try the same sums by doing this (when you can). Do you get the same answers?

So, **adding a positive number** is equivalent to:

(i) **Adding** positive counters (and removing zero pairs if any) OR

(ii) **Removing** negative counters

Try these:

IV. (-5) + 5 = ____

V. (-8) + ___ = 0

VI. ____ + 4 = 0

Think:

D. Titir got ₹100 as birthday gift and paid off some of her debt of ₹300.

a. How much debt remains?

b. So, how much debt was paid off, i.e., subtracted?

c. If the original debt of ₹300 is written as -3, then the gift is ___ (1/-1) and the remaining debt is ___.

d. Debt remaining = Original debt + gift = original debt – debt paid off, i.e., ____ = (-3) + ___ = (-3) – ____

e. What would have happened if the gift was ₹500?

E. Tintin had a debt of ₹200 but made a profit of ₹600.

a. How much did Tintin have after making the profit?

b. If a debt of ₹200 is -2, then a profit of ₹600 is ___ (6/-6) and the amount Tintin had after profit is ___.

c. So, (-2) + ___ = ___

5. Subtracting positive numbers

When we subtract positive numbers, we take not one, but several steps to the __________ (right/left) and you remove __________ (red/green) counters.

(a) Subtract 2 from 6 (b) Subtract 3 from -2
(c) Subtract 5 from 3

(a) 6 – 2: Take ___ steps to the __________ (left/right) of 6, removing 1 counter at each step.

(b) (-2) – 3: Take ___ steps to the ______ (left/right) of -2, removing 1 counter at each step.

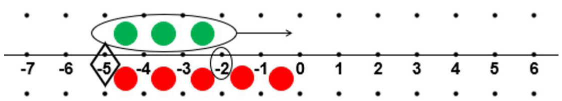
What did you do when you ran out of counters?

(c) 3 – 5: Take ___ steps to the __________ (left/right) of 3, removing 1 counter at each step.

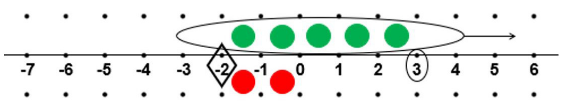
What did you do when you ran out of counters? See Figure 5 for some ideas.

You used the same strategy for all three sums. Did you find that removing a green counter is the same as adding a red counter? Try the same sums by doing this, when you can. Do you get the same answers?

> So, **subtracting a positive number** is equivalent to:
>
> (i) **Removing** positive counters (and removing zero pairs if any) OR
>
> (ii) **Adding** negative counters
>
> Did you try the zero-pair strategy? What happens if we add zero-pairs till there are sufficient positive counters and then remove them?



(b) (-2) – 3 by adding zero pairs



(c) 3 – 5 by adding zero pairs OR

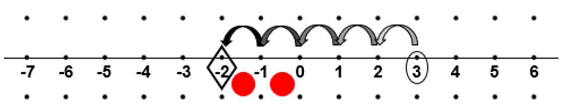


Figure 5

Try these:

VII. (-5) – 7:

a. Do you reach/cross 0 on the number line for this?

b. How many zero pairs do you need to add?

c. Try this one step at a time. How many negative counters do you add?

d. To which number do you add these negative counters?

e. So, adding ___ negative counters to _____ is ______ + (-_____). How is this related to (-5) – 7?

f. How are the number of zero pairs and the number of negative counters related?

VIII. 10 – 14:

a. Do you reach/cross 0 on the number line for this?

b. How many (minimum) zero-pairs do you need?

c. Try this one step at a time. How many negative counters do you add?

d. To which number do you add these negative counters?

e. So, adding ___ negative counters to _____ is _____ + (-____). How is this related to 10 – 14?

f. How are the number of negative counters and the minimum number of zero pairs related?

6. Adding negative numbers

When we add negative numbers, we take not one, but several steps to the ______ (right/left).

(a) Add -2 to -5 (b) Add -3 to 4

(c) Add -5 to 3

(a) (-5) + (-2): Take ___ steps to the _______ (left/right) of -5, adding 1 ______ (red/green) counter at each step.

(b) 4 + (-3): Take ___ steps to the _______ (left/right) of 4, adding 1 _______ (red/green) counter at each step.

(c) 3 + (-5) Take ___ steps to the _______ (left/right) of 3, adding 1 _______ (red/green) counter at each step.

Did you get zero pair(s) for any of the sums?

Is adding a red counter the same as removing a green counter?

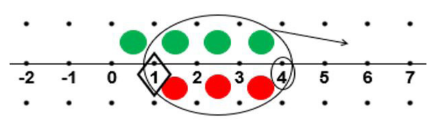


Figure 6

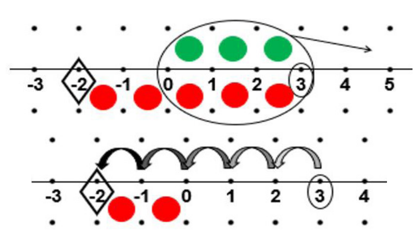


Figure 7

The direction of steps comes from point 2 above if we consider that adding -3 can be as adding -1 three times. Another way of looking at it would be that it is different from adding 3. So, it can't be stepping in the same direction as adding a positive number! Observe that if we add negative counters, then the resulting zero-pairs should be removed (Figure 6). What are the different strategies you can use for (c) Do you get the same result as proceeding one step at a time? (Figure 7)

So, **adding a negative number** is equivalent to:

(i) ______________ (adding/removing) negative counters

(ii) ______________ (adding/removing) positive counters

Is this similar to anything we have already done?

So, **adding a negative number** is equivalent to ______________ (adding/subtracting) **a positive number**.

How are the two numbers related?[1]

Try these:

IX. 7 + (-7) = ___

X. 3 + ___ = 0

XI. ____ + (-6) = 0

Think:

F. Tinku had a debt of ₹200 and had to borrow ₹500 more.

a. How much is the total debt now?

b. If debt of ₹200 is -2, what are the new debt and the total debt?

c. So, (-2) + ___ = ____ = (-2) – _____

G. Tintin had ₹400 and made a loss of ₹100.

a. If ₹400 is 4, then a loss of ₹100 is ___ (1/-1).

b. How much did Tintin have after the loss? And the loss of ₹___ is ___.

c. So, 4 + ____ = ____

d. If the loss was of ₹900, then what would Tintin have?

H. Tulu also had ₹400 and spent ₹100.

a. Using the same notation, is this 4 – 1 = ___?

b. So, is 4 + (-1) = 4 – 1?

c. If Tulu had to spend ₹900, then what would have happened?

d. If Tintin made a loss of ₹900 and Tulu had to spend ₹900, who is poorer?

7. Sums of integers

Consider the following pairs of sums: 5 + (-1) and (-1) + 5 (Figure 8) as well as 2 + (-5) and (-5) + 2 (Figures 9). What do you observe?

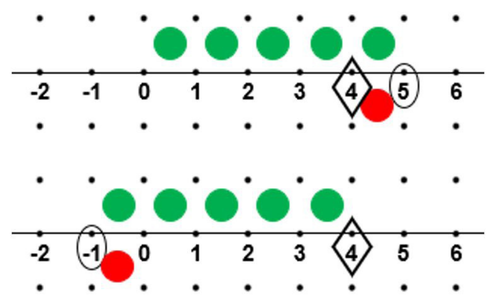


Figure 8

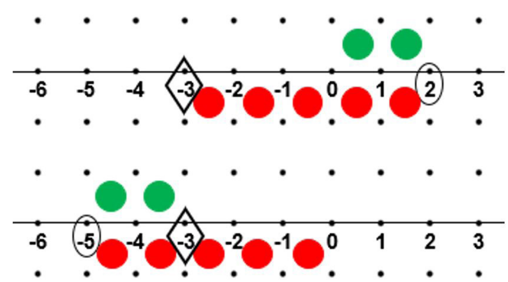


Figure 9

(a) How are the two pictures in Figure 8 related? Can you reflect the top one along some vertical line to get the bottom one?

(b) Where is this vertical line or the mirror for Figure 8?[2]

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1 Adding $-n$ is equivalent to subtracting n, its additive opposite

2 Figure 8: mirror is $x = 2 = (5 + (-1))/2$ … this can be observed by noting the common portion for both diagrams and therefore the common interval, and then finding the midpoint of that interval

(c) Observe the number this mirror is passing through. How can you get this number from 5 and -1?

(d) Can you find a similar mirror for Figure 9? Which number does it pass through? How can you get this number from 2 and -5?[3]

(e) Do observe similarly for (-2) + (-4) and (-4) + (-2). What about 3 + 4 and 4 + 3?

Try this with different pairs of numbers. What do you conclude?[4]

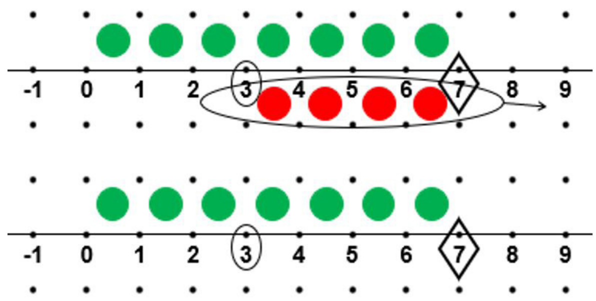

Figure 10

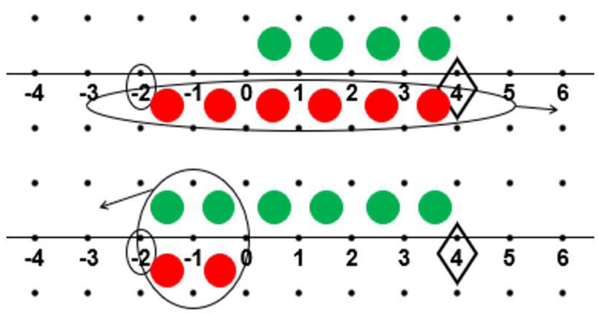

Figure 11

8. Subtracting negative numbers

When we subtract negative numbers, we take not one, but several steps to the _____ (right/left) and you remove _________ (red/green) counters.

[Now you should be knowing what to do when you run out of counters.]

(a) Subtract -3 from -5 (b) Subtract -4 from 3
(c) Subtract -6 from -2

(a) (-5) – (-3): Take ___ steps to the ______ (left/right) of -5, removing 1 counter at each step.

(b) 3 – (-4): Take ___ steps to the ______ (left/right) of 3, removing 1 counter at each step.

If we add zero-pairs, that is equivalent to adding which numbers? (See Figure 10)

Can you complete the steps? What comes after 3 in the last step?

3 – (-4) = 3 – (-4) + 0 = 3 – (-4) + ___ + (-___) = 3 ______________

Did you get the same answer both times?

(c) (-2) – (-6): Take ___ steps to the ______ (left/right) of -2, removing 1 counter at each step. What happens if we add zero-pairs instead? How many zero-pairs do we need? Can you complete the steps? [Hint: see Figure 11]
(-2) – (-6) = (-2) – (-6) + 0 = (-2) – (-6) + ___ + (-___) = (-2) ________________
Did you get the same answer?

Therefore, we can add ____________ (positive/negative) counters as we take these steps.

So, **subtracting a negative number** is equivalent to:

(i) _______________ (adding/removing) negative counters

(ii) _______________ (adding/removing) positive counters

Is this similar to anything we have already done?

So, **subtracting a negative number** is equivalent to ______________ (adding/subtracting) **a positive number**.

How are the two numbers related?[5]

3 Figure 9: mirror is x = -1.5 = ((-5) + 2)/2

4 Commutative property of addition for integers

5 Subtracting *-n* is equivalent to adding *n*, its additive opposite

Try these:

XII. 6 – (-2)

a. Do you reach/cross 0 on the number line for this?

b. How many zero pairs do you need to add?

c. Try this one step at a time. How many positive counters do you add?

d. To which number do you add these positive counters?

e. So, adding ___ positive counters to ___ is ___ + ___. How is this related to 6 – (-2)?

f. How are the number of zero pairs and the number of positive counters related?

XIII. (-3) – (-4)

a. Do you reach/cross 0 on the number line for this?

b. How many (minimum) zero-pairs do you need?

c. Try this one step at a time. How many positive counters do you add?

d. To which number do you add these positive counters?

e. So, adding ___ positive counters to ____ is ____ + ___. How is this related to (-3) – (-4)?

f. How are the number of positive counters and the minimum number of zero pairs related?

Think:

I. On a chilly winter night, Srinagar's temperature was -6°C while Leh's was -17°C and Gangtok was 8°C

a. Which city was colder: Srinagar or Leh?

b. _________ was colder than _______ by (____ – ____) = ____ °C.

c. Gangtok was warmer than Srinagar by (____ – ____) = ____°C.

J. Titir and Tinku started the day with ₹100 each. Titir made a profit of ₹300 while Tinku made a loss of ₹200.

a. How much did each one have at the end of the day?

b. Who was richer and by how much?

c. Titir's profit is ___ and Tinku's loss is ___.

d. ______ (Titir/Tinku) is richer and by ____ – ____ = ____ .

K. Next week, they again started with ₹100 each, but both made losses – Titir ₹500 and Tinku ₹300.

a. How much did each one have at the end of the day?

b. Titir ended with ___ while Tinku with ___.

c. ______ (Titir/Tinku) is richer and by ____ – ____ = ____ .

The idea of this article was triggered by an exploration by Jauhar K M and Nagendra Singh, MA Education students at Azim Premji University, as part of their Curriculum Material Development – Mathematics course.

MATH SPACE is a mathematics laboratory at Azim Premji University that caters to schools, teachers, parents, children, NGOs working in school education and teacher educators. It explores various teaching-learning materials for mathematics [mat(h)erials] – their scope as well as the possibility of low-cost versions that can be made from waste. It tries to address both ends of the spectrum, those who fear or even hate mathematics as well as those who love engaging with it. It is a space where ideas generate and evolve thanks to interactions with many people. Math Space can be reached at mathspace@apu.edu.in