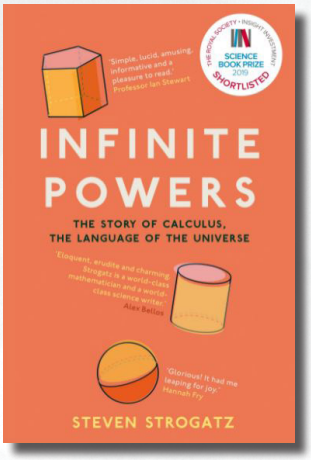
The Language God Talks? A Review

Infinite Powers: The Story of Calculus - the Language of the Universe

Reviewed by Shashidhar Jagadeeshan



Author: Steven Strogatz

I. The infinity principle and the language God talks

The book by Steven Strogatz,[1] the highly accomplished applied mathematician, teacher and communicator of mathematics, begins with a bang:

> "Without calculus, we wouldn't have cell phones, computers, or microwave ovens. We wouldn't have radio. Or television. Or ultrasound for expectant mothers, or GPS for lost travellers. We wouldn't have split the atom, unravelled the human genome, or put astronauts on the moon. We might not even have had the Declaration of Independence.
>
> It's a curiosity of history that the world was changed forever by an arcane branch of mathematics."

The unusual lens that Strogatz uses to view calculus is

> "**The Infinity Principle:** To shed light on any continuous shape, object, motion, process or phenomenon -no matter how wild and complicated it may appear -reimagine it as an infinite series of simpler parts, analyse those, and then add the results back together to make sense of the original whole."

A classic illustration of this principle is finding the volume of a sphere. The sphere is sandwiched between inscribed and circumscribed cylindrical slices. The radius of these discs vary, but their thickness is identical, and can be made smaller and smaller. One can easily find the volumes of these cylinders and by summing up an infinite series of volumes and taking limits, one derives the formula for the volume of a sphere.

1 Regular readers of At Right Angles will be familiar with Prof Steven Strogatz who was introduced in the November 2018 issue (Volume 7 | Issue 3).

This allows him to take what he calls a 'big tent' view with which to question the conventional narrative of calculus. The narrative holds that calculus began by great insights from Newton and Leibniz, created a 'gold-rush' of results, but lacked proper rigour, which led to serious questions about the foundations of calculus. Mathematicians in the nineteenth century 'expunged infinity and infinitesimals' to clean up calculus.

This view is too 'blinkered' for Strogatz, who prefers the lens of the Infinity Principle, according to which calculus has its origins way back in the history of mathematics. This allows him to include many descendants and spin-offs of calculus, and to feel that calculus is very much alive today with a vibrant future.

Along with re-examining the very idea of what calculus is, the book has two other aims, both illustrated in a story. In a conversation Richard Feynman asked the novelist Herman Wouk if he knew calculus, and when Wouk said he did not, Feynman told him *"You had better learn it. It's the language God talks".*

Strogatz wants to convince the reader of the truth of this statement (page viii of the introduction):

> "By inadvertently discovering this strange language, first in a corner of geometry and later in the code of the universe, then by learning to speak it fluently and decipher its idioms and nuances, and finally by harnessing its forecasting powers, humans have used calculus to remake the world."

That is the central argument of this book.

His other aim is slightly more modest! Apparently when Wouk finally got around to learning calculus, he found no suitable material. Given that calculus is considered one of the greatest intellectual achievements of mankind, Strogatz has written this book for a person from a non-mathematical background to learn all about it, without having to go through the grind of doing calculus.

II. The story of calculus and the calculus debate

There is no doubt that Strogatz is a master storyteller. Using the Infinity Principle as his point of view, he builds up a riveting history of calculus over eleven chapters.

Through the understanding of the work of an array of scientists from Archimedes to Zeno, we learn how the Infinity Principle has been working its way through mathematics, reaching its high point in The Fundamental Theorem of Calculus at the hands of Newton and Leibniz, after which it explodes and enters almost all areas of mathematics both pure and applied. Along the way we learn about Galileo's work, Cartesian geometry, Descartes's and Fermat's idea of the slope of a curve, and the laws of Kepler and Newton.

Strogatz spends tremendous energy educating the reader in a variety of applications of calculus, ranging from how it is used in animation movies, GPS, PET scan and predicting the trajectory of a moving space craft. The chapter on HIV treatment is a personal favourite of mine. He also explains how calculus has played a role in modern physics - for example in the work of Maxwell, Einstein, Dirac and Feynman.

He has also taken great pains to include the work of many scientists, not all of whom are European men! Thus we learn about Al-Hassan Ibn al-Haytham, Katherine Johnson, Sophie Germain and Sophia Kovalevskaya, among many others.

This non-gendered, non-Eurocentric approach is refreshing. Yet I still have a bone to pick with Strogatz. It is to do with the calculus debate (see [2], pages 348-357) and will need some background material to explain.

As early as 1832, through the work of a Britisher, Charles Whish, and later on from the 1940's, modern historians of mathematics have been aware of the work of the so-called Nila school of mathematics and astronomy from Kerala (see [1],[2],[3]). The founder of the school is believed to be Madhava (c. 1360-c.1430) of Sangamagrama, and the school was active from the

fourteenth to sixteenth century of the common era. The Nila school had discovered the power series expansion for trigonometric functions, the surface area and volume of the sphere, basic ideas of differentials and integrals, the discrete fundamental theorem of calculus, integration by parts, multiple integrals and much more!

Several scholars (such as Diwakaran and Mumford) strongly feel that the birth and the early steps of calculus took place on the banks of the river Nila and its inventor was Madhava, who too "stood on the shoulders of giants" like Aryabhata and Bhaskara.

On the other hand, historian Victor J Katz in a paper entitled *Ideas of Calculus in Islam and India* (see [4], page 173) says, "There is no danger, therefore, that we have to rewrite the history texts to remove the statement that Newton and Leibniz invented the calculus."

Strogatz does not enter into this messy debate at all, and I can understand why. After all, issues of priority can get nasty, just look at what happened between Newton and Leibniz! Moreover, as we have seen earlier, Strogatz's take is very different when it comes to the history of calculus. To give him due credit, he does acknowledge that the Nila school had discovered the infinite series expansion for trigonometric functions 250 years before Newton. My bone however, is that while he has taken great care to apply the Infinity Principle to study the work of many of the masters and explained how calculus emerged from their work, he has not done this for Madhava. To quote Mumford:([3], page 389) "It seems fair to me to compare him with Newton and Leibniz."

It would also have been really fruitful to see an analysis of the so-called Indian method of mathematics. In Mumford's words ([3], page 389) "Simply put, these are recursion, induction, and careful passage to the limit."

On a separate note, while the birth of calculus might have happened in Kerala, it did not spread like wildfire, nor did it have a similar impact on science and technology as in Europe. Traditionally, this has been attributed to the genius of Newton and Leibniz in their formulation of mathematics and its application to the natural world, the beginning of the scientific revolution and support from the ruling hierarchy. The author of Sapiens, Harari, alerts us that this view may be a bit naive, and one should not ignore the twin forces of 'imperialism and capitalism.' To quote him "The feedback loop between science, empire and capital has arguably been history's chief engine for the past 500 years." Kerala in the fourteenth to sixteenth centuries was hardly at the centre of the empire!

III. The language of the universe

As mentioned in the first section, Strogatz is convinced of and wants to sell the idea that calculus is the best language to understand the universe, whose mysteries are written in "sentences called differential equations".

How much of the modern world has really been shaped by calculus? I thought it best to ask several scientists this question (see the box for the survey sent). I deliberately chose scientists who are not pure mathematicians. They work on a wide spectrum of problems: how bees smell flowers, properties of polymers (with applications from plastics to DNA), behaviour of nano-materials, the application of group theory in physics, cellular and tissue biophysics and supermassive black holes. Interestingly, all of them do use calculus to solve their problems. Madan says, "Newton's math and its descendants pervade every aspect of my work and every physicist's work . . . " However, the jury is still out as far as Strogatz's strong claims!

For example, Ravi feels: "It is an exaggeration to say that calculus is at the heart of all scientific and technological progress, and to modern civilisation as we know it. It is definitely an integral part, and it is central to many of the achievements of mankind, but how does one separate one element of mathematics and give it the most prominent role?"

Shobhana wonders: "I do not know what to make of the claim that the Declaration of Independence would not have been made were it not for calculus! I am not sure what Strogatz had in mind. Maybe if people hadn't been able to calculate the trajectory of a cannonball using calculus, the outcome of some battle would have been different, thus changing the course of history?"

Prajval says: "Exceptionalizing calculus in technological developments unnecessarily undermines the importance of tinkering and trial-and-error in engineering..."

Knowledge creation is a process of building upon and constantly testing, interrogating and reaffirming (or not) past knowledge, and is perforce a collective, co-operative human endeavour. Therefore to create straw-persons that privilege some particular branch or toolkit that has contributed to the current positive aspects of the human condition becomes merely polemics."

Mukunda points out: "However, calculus by itself cannot lead us to the laws of nature, like Newton's equation of motion, or Maxwell's equations, or wave mechanics, or relativity. These are independent inputs from physics, to express which calculus is the most convenient language. So, the basic physical laws are not determined by calculus, but when discovered are expressed in the language of calculus. This distinction is very important."

Due to constraints of space we are not able to publish their complete, fascinating responses in this print issue. Please visit http://publications.azimpremjifoundation.org/3350/ to find their responses.

There is of course the great wonder as to why the universe is comprehensible by a tiny species with a large brain; and why is mathematics the best language to express the laws of nature? This is a very deep philosophical debate and I will let the reader enjoy Strogatz's point of view.

IV. Read the book!

I would strongly urge anyone interested in studying or teaching mathematics to read the book, especially because Strogatz takes an unusual point of view and often makes dramatic and provocative statements! I am not sure if he has succeeded in conveying what calculus is to the lay person (one of his intentions as mentioned earlier), but he definitely tries very hard to lay out all the background in as non-technical a manner possible, and conveys the vast sweep of ideas using many metaphors, illustrations and examples.

If you are a teacher of calculus, then there is much to learn, both in terms of mathematical content and its exposition. I particularly enjoyed the way he explains the proof of The Fundamental Theorem of Calculus. You will also learn many anecdotes in the story of calculus which you can use in the classroom. And when your students ask you, as they inevitably do, "what is the use of learning all this stuff?", you can regale them with mind boggling applications from their DNA to their smart phone!

V. To end on a sombre note

The book is an unbridled celebration of the human ability to comprehend the universe, control it and produce marvels of technology that have helped some humans enjoy longer life, better health and vast wealth.

The genus *Homo* evolved on earth 6 million years ago, and for the last 13000 years the only species in this genus to survive is *Homo sapiens*. For most of this time, we have had very little impact on the earth. The last five hundred years, however, has seen the mass extinction of several thousand species, serious biodiversity loss, intolerable inequality among humans, the possibility of all life on earth being wiped out by nuclear bombs, and as a direct result of human activity, a change in our climate that has started and will continue to unleash huge suffering.

If calculus helped us "reshape civilization", should we also consider the role of human thought and knowledge, even specifically calculus, in creating our current crisis?

References

[1] Divakaran, P.P. *Calculus in India: The historical and mathematical context*, Current Science, Volume 99, No 3, 10 August 2010, p 293-299.

[2] Divakaran, P.P. *The Mathematics of India: Concepts, Methods, Connections*, Hindustan Book Agency, New Delhi 2018.

[3] Mumford, D. *Mathematics in India: Reviewed by David Mumford,* Notices of the AMS, Volume 57, Number 3, March 2010, p 385-390.

[4] Katz, V.J. *Ideas of Calculus in Islam and India,* Mathematics Magazine, Volume 68, Number 3, June 1995, p 163-174.

SHASHIDHAR JAGADEESHAN has been teaching mathematics for over 30 years. He is a firm believer that mathematics is a human endeavour, and his interest lies in conveying the beauty of mathematics to students and demonstrating that it is possible to create learning environments where children enjoy learning mathematics. He may be contacted at jshashidhar@gmail.com.

Did Calculus Change the World?

What do an astrophysicist, a biologist, a bio-physicist, a chemical engineer, a computational nano-scientist and a theoretical physicist have in common? They were all subject to the following survey:

(i) Could you briefly describe for a layperson the problem(s) you work on?

(ii) Can you explain how calculus plays a part in your work (again as non-technically as possible)?

(iii) Steven Strogatz begins his book, Infinite Powers, with the following claim (page vi of the Introduction)

> "Without calculus, we wouldn't have cell phones, computers, or microwave ovens. We wouldn't have radio. Or television. Or ultrasound for expectant mothers, or GPS for lost travellers. We wouldn't have split the atom, unravelled the human genome, or put astronauts on the moon. We might not even have had the Declaration of Independence.
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> It's a curiosity of history that the world was changed forever by an arcane branch of mathematics."

Later on he tempers the above statements by saying (page x of the introduction):

> "...This is a much broader view of calculus than usual. It encompasses the many cousins and spinoffs of calculus, both within mathematics and in adjacent disciplines. Since this big-tent view is unconventional, I want to make sure it doesn't cause any confusion. For example, when I said earlier that without calculus we wouldn't have computers and cell phones and so on, I certainly didn't mean to suggest that calculus produced all these wonders by itself. Far from it. Science and Technology were essential partners-and arguably the stars of the show. My point is merely that calculus has also played a crucial role, albeit often a supporting one, in giving us the world we know today."

(on page xi after describing Maxwell's work on electromagnetism)

> "Clearly, calculus could not have done this alone. But equally clearly, none of it would have happened without calculus. Or, perhaps more accurately, it might have happened, but only much later, if at all."

What is your opinion about this?

PROFESSORS RAVI PRAKASH JAGADEESHAN, N. MUKUNDA, SHOBHANA NARASIMHAN, MADAN RAO and **PRAJVAL SHASTRI** were kind enough to respond to questions (i) to (iii). Avehi Singh, a graduate student, has responded to questions (i) and (ii). Please visit http://publications.azimpremjifoundation.org/3350/ to read their responses to the survey.