The Rascal Triangle Revisited

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In the *At Right Angles* October 2021 webinar, Dr. Shashidhar Jagadeeshan and Dr. Shailesh Shirali discussed The Rascal Triangle, referencing a 2016 article of the same title by Ishaan Magon, Maya Reddy, Rishabh Suresh and Shashidhar Jagadeeshan.

The speakers elaborated on methods that students had discovered for finding a particular entry in the triangle and stressed the value of allowing students to explore and enjoy mathematics. While the webinar also explored many other interesting problems, I would like to share a few additional ideas about the fascinating Rascal Triangle. Please visit [1] to view the recording, and see the articles at [2] and [3].

Using the example from the webinar, suppose we want to know the 3rd term in row 5, as shown in Figure 1, with the diamond showing $a = 3$, $b = 5$, $c = 4$ and x.

Note: In this article, the rows and elements are numbered as in the original article, see [2]. Rows start at 0 and each element in the rows also starts at 0.

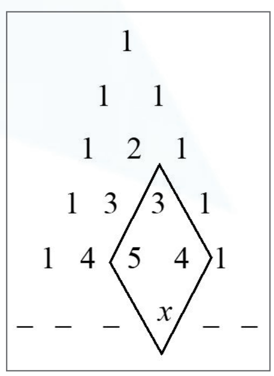

Figure 1

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The value of x can be found using the expression $a + n - 1$, so we use a and n instead of b and c. With $a = 3$ and $n = 5$, $x = 3 + 5 - 1 = 7$. For another example, if we want to find the 2nd element in row 8, $a = 6$ by inspection and $n = 8$, so $x = 6 + 8 - 1 = 13$.

Figure 2 shows a modified diamond diagram from page 47 of [2], with $x = \text{Entry}\ (n, k) = k(n - k) + 1$ as established in the article.

$$a = \text{Entry}\ (n - 2, k - 1)$$
$$= k(n - k) - n + 2$$
$$b = \text{Entry}\ (n - 1, k - 1) \qquad c = \text{Entry}\ (n - 1, k)$$
$$x = \text{Entry}\ (n, k) = k(n - k) + 1$$
$$= [k(n - k) - n + 2] + n - 1 = a + n - 1$$

Figure 2

Thus, the value of x is $a + n - 1$.

We can also determine the 3rd number in the 5th row in a way similar to what is used with Pascal's Triangle (Figure 3).

We need to consider the triangle (not the diamond) as we do with Pascal, but we must also use n. We use the formula $x = (b + c + n)/2$, so $x = (5 + 4 + 5)/2 = 7$. It is easy to show this is valid since $x = a + n - 1$ and $b + c = a + x - 1$. Replace a in the second equation with $x - n + 1$ and the result follows easily. For another example, the 5th number in the 8th row is $(13 + 11 + 8)/2 = 16$. When the row number is even, then the sum of b and c is even (both b and c are odd) and when the row number is odd, then the sum of b and c is odd (one is even and the other is odd), echoing the webinar exploration of the sequence of triangular numbers.

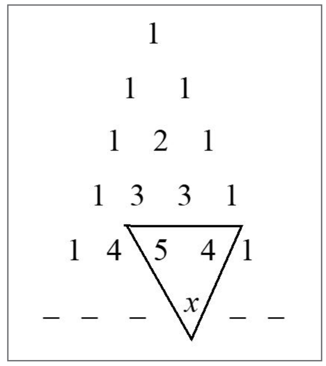

Figure 3

Another way to look at the two different formulas given in the webinar is to observe that $ax = bc + 1$ and also $a + x = b + c + 1$, and solving for x in each gives the two formulas. What a delightful set of 4 numbers! No doubt, both the American students and the CFL students might have discovered their respective formulas from this observation.

In Pascal's triangle, the sum of the numbers in a given row is a power of 2. In the Rascal Triangle, the sum of the numbers in row n is given by $(n^3 + 5n + 6)/6$, a nice result that students can get using successive differences.

References

[1] *At Right Angles* Webinar: Rascal Triangle, https://www.youtube.com/watch?v=fryhomqA0dI

[2] The Rascal Triangle, https://bit.ly/3FF65en

[3] The Rascal Triangle, a Rascal full of Surprises, https://bit.ly/3BxNuhM

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