Low Floor
High Ceiling Tasks

Circle Time

Solid Learning!

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In the November 2014 issue, we began a new series which was a compilation of 'Low Floor High Ceiling' activities. A brief recap: an activity is chosen which starts by assigning simple age-appropriate tasks which can be attempted by all the students in the classroom. The complexity of the tasks builds up as the activity proceeds so that each student is pushed to his or her maximum as they attempt their work. There is enough work for all but as the level gets higher, fewer students are able to complete the tasks. The point, however, is that all students are engaged and all of them are able to accomplish at least a part of the whole task.

This time we focus on regular polygons inscribed in circles. As with all such hands-on activities, there is ample scope for skill development. But careful facilitation is key to this aspect of activities. Students should be given time for reflection, for discussion and, most importantly, they should not be afraid to make mistakes. While emphasis is given to rigour, this discipline is best imposed with gentle questioning – students must be encouraged to ask each other the question 'Why?' as often as possible.

The tasks require that each student has a large supply of paper, a compass, ruler, sharp pencil and scissors. Each student must use circles of the same radius throughout the activity; however, the radius can vary from student to student.

In the spirit of eco-friendliness, we recommend that you use paper which is used only on one side for this task. Folds to create the regular polygons should be made in toward the blank side.

These activities can be attempted by students in classes from 8 to 10 though some of the questions may need to be toned down for the younger students.

TASK 1: Construction to create a Square in a Circle

1.1   Draw a circle of reasonably large radius and any one diameter of it. Mark its end points B and C.

Construct the perpendicular bisector of this diameter and extend it to meet the circle at D and E. Mark the centre A of the circle and cut it out.
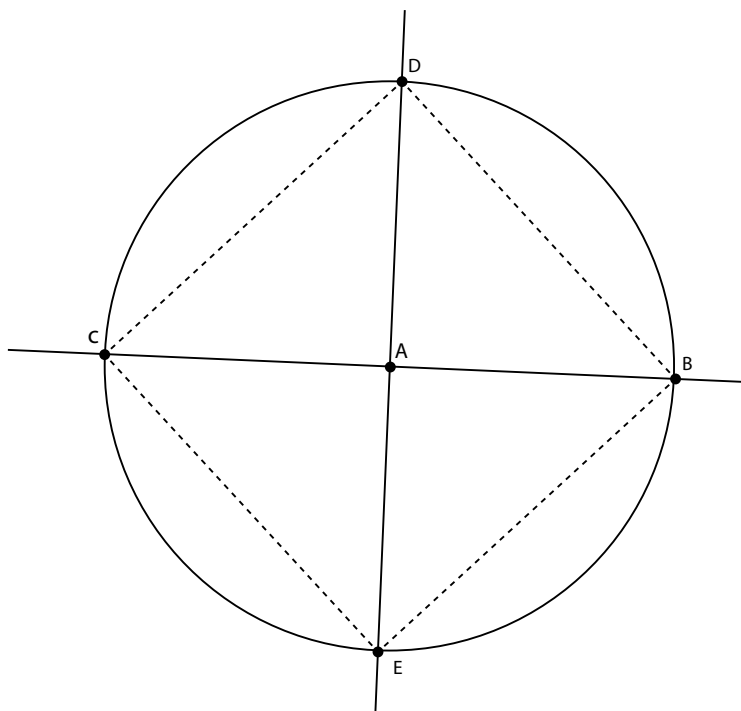

Figure 1: Fold along DC, CE, EB and BD, and verify that BDCE is a square. Explain how you verified this.

1.2   Prove that BDCE is a square.

1.3   If the side of the square is 1 unit, find the radius of the circle.

1.4   If the radius of the circle is 1 unit, find the length of the sides of the square.

1.5   How can you fold a regular octagon inscribed in another circle of the same radius?

1.6   Find the area of each sector of the circle which has been created in the figure, taking the radius to be one unit.

TASK 2: Construction to create a Triangle in a Circle

2.1   **Use another circle with the same radius and draw any diameter CB. Mark the centre A of the circle.**

Construct the perpendicular bisector of AC and extend it to cut the circle at E and F. Mark the intersection D of AC and EF. Cut out the circle.
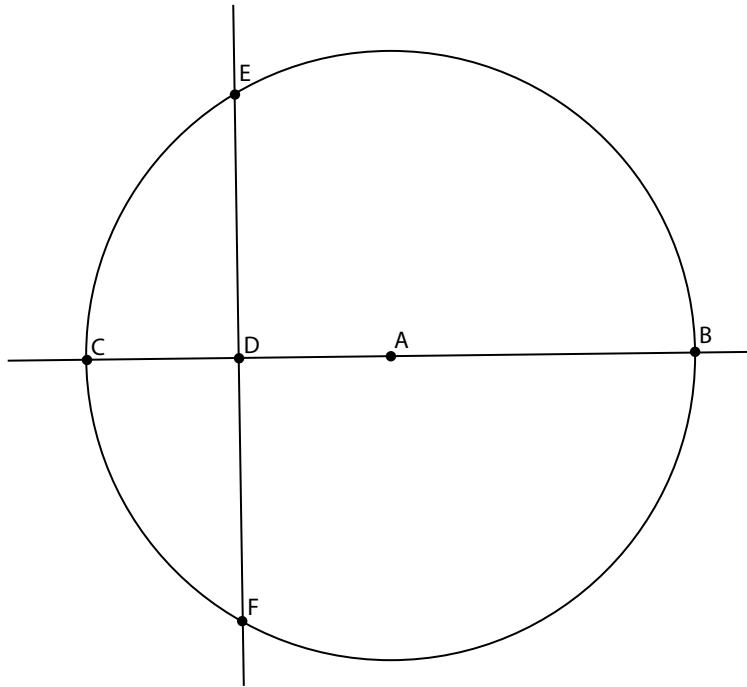

Figure 2

2.2   **Verify that the points E, F and B trisect the circle. Explain how you verified this.**

2.3   **Fold the lines joining E, B & F to get a triangle. What kind of triangle is this? Justify.**

2.4   **If the side of the triangle is 1 unit, find the radius of the circle.**

2.5   **If the radius of the circle is 1 unit, find the lengths of the sides of the triangle.**

2.6   **How can you fold the above paper to get a regular hexagon inscribed in the given circle?**

2.7   **Join BE and BF and find the area of each sector of the circle in the figure.**

Teacher's Note:

Tasks 1 and 2 are simple activities which are a good opening exercise that helps students practise the skills they have learnt in their geometry classes. It would be particularly interesting for them to attempt to do the same activity with paper folding instead of constructions and see the similarities between both the activities.

Notice the Low Floor activity 'Verify' which students can easily do with a protractor and ruler, and which helps them to revise their understanding of regular polygons.

In the subsequent step up to the High Ceiling, students are asked to 'Prove'- a task that they could do by using the concepts of angles in a semi-circle, chord properties, congruent triangles and by applying Pythagoras' theorem or simple trigonometric ratios. The last question in each task helps them to use mensuration formulae and symmetry, and their findings will be collated in a subsequent task. Some amount of familiarity with operations on irrational numbers is required.

TASK 3: Construction to create a Pentagon in a Circle.

3.1   **Draw a circle of the same radius used previously. Mark the centre A and any point B on the circle and draw the radius AB. Construct the diameter perpendicular to AB and mark any one of its ends. Call this point C and locate the mid-point D of AC. Calculate BD in terms of 'r', the radius of the circle.**

3.2   **With D as centre and radius BD, draw an arc. Mark the point of intersection E of this arc with the diameter AC. (E is inside the first circle). Calculate BE in terms of 'r'.**

3.3   **With B as centre and radius BE, draw another arc. Mark the point of intersection F of this arc and the radius AC of the first circle. Also mark the points of intersection G and H of this arc with the first circle.**

3.4   **Now using the same radius BE, draw 2 arcs centred at G and H. Mark the points of intersection J and L of these arcs with the first circle. Draw the polygon BHLJG. What are the sides of this polygon in terms of 'r'?**

3.5   **What kind of polygon is BHLJG? What are the angles subtended by each side of this polygon at the centre A of the circle? Cut out the circle and fold along the sides of the polygon.**
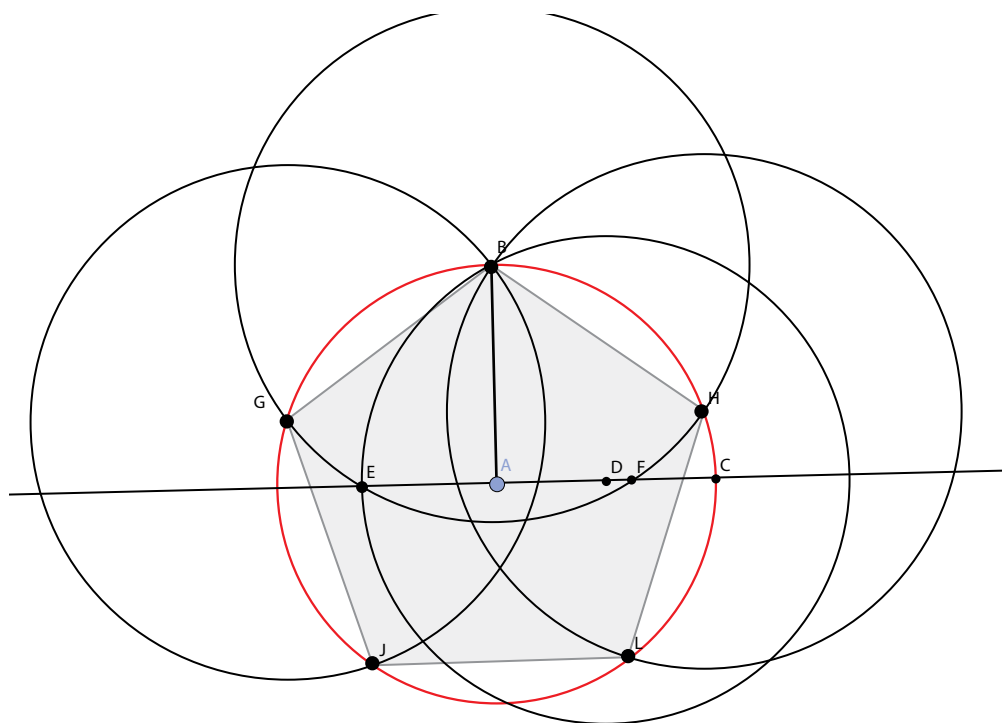

Figure 3

This is a slightly more complex construction and students may get confused with the number of circles that have to be drawn. It would be a good idea to draw the original circle in a different colour. The calculation of the side of the pentagon in terms of 'r' may be difficult for students who are not comfortable with symbolic manipulation; in this case, we suggest that the student is asked to assume that the radius of the circle is 1 unit.

Refer to the Low Floor High Ceiling article "The Midas Touch" in the November 2015 issue of At Right Angles. Explain how the construction is related to the Golden Ratio.

An alternative way of getting a pentagon by a ruler and compass construction is found at http://www. mathopenref.com/constinpentagon.html and it would be useful for students to find similarities between the two methods. Using paper folding to get an inscribed pentagon is slightly complex; you could find clear instructions at https://www.youtube.com/watch?v=1X-UAjSGCpU. It's rich with mathematical connections and a great 'pause and question why' exercise for a teacher to do with a class.

TASK 4: From 2D to 3D

4.1   Fill in the following table

Regular Polygon	Interior angle α	Angle subtended by a side at the centre of the circle	Length of side in terms of 'r'	Multiples of α which are less than 360
Quadrilateral				
Triangle				
Pentagon				
Hexagon				
Octagon				

In this part of the activity, you will be making platonic solids. A platonic solid is a polyhedron in which all the polygonal faces are regular and congruent, and which has the same number of polygons at each vertex with all angles at the vertices equal. Make multiple cutouts of circles of the same radius which have equilateral triangles, squares, hexagons and pentagons inscribed in them. Take several congruent circles with the same regular polygon inscribed in each and join the segments of different circles to form solids.

4.2   Can you make a solid which has 3 equilateral triangles at each vertex? If so, how many faces does this solid have?

4.3   Can you make a solid which has square faces on all sides? How many squares meet at each vertex?

4.4   Can there be more than one solid which has square faces on all sides? If so, how many squares meet at each vertex in that case? How many faces does this solid have? How many such solids exist?

4.5   Can there be more than one solid which has equilateral triangles on all sides? If so, how many triangles meet at each vertex? How many faces does this solid have? How many such solids exist?

4.6   Which other regular polygon can be used to make a platonic solid?

4.7   Which polygons cannot be used?

4.8   How many platonic solids are there?

This part of the activity is a shot at the ceiling but with concrete models to work with, students can definitely arrive at the conclusion that there are only 5 platonic solids. The table in the first part of the activity will help students direct their reasoning and it is good practice for them to draw conclusions by extrapolating from available data. Of course, no teacher can resist bringing up Euler's formula of $F + V - E = 2$ and this is a solid opportunity for students to verify this formula. Do encourage your students to use this formula to arrive at other proofs of the fact that there are only 5 platonic solids. If the teacher is so inclined, (s)he can take this a step further into Archimedean solids.

Here is a question which could trigger off such an investigation:

If a solid had the same combination of 3 faces (triangle, square, pentagon, hexagon or octagon- all of them regular) at each vertex, what are the possible solids that could be formed?

You may be surprised at what students will come up with as they venture into trying all possible combinations. Do encourage them to write down their discoveries and begin to reason what can work and what won't. You can even bring in a parity argument and help them see that it must be odd-even-even or even-even-even (and therefore 3-3-even, 5-5-even, 3-5-even and 3-3-5 and 3-5-5 are ruled out.)

These two would give them several of the Archimedean solids, though not all…

Conclusion

Going round in circles has never been so much fun! These Platonic Solids make for great decorations as I am sure our photos reveal. Students work so much with pen and paper that 3 dimensions often fazes them – ironic, since it has been prescribed that the study of mathematics should connect with their experience of the real world. Do share photos of your students' creations on AtRiUM – our FaceBook page.

The Making of the Tetrahedron

Figure. 4: Equilateral Triangles inscribed in congruent circles.
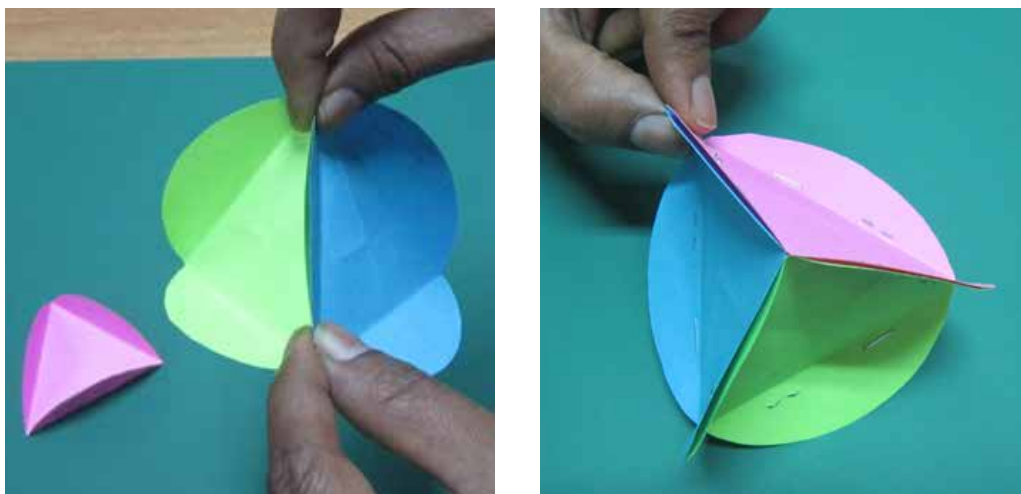Notice the one-sided paper and the folds toward the blank side.

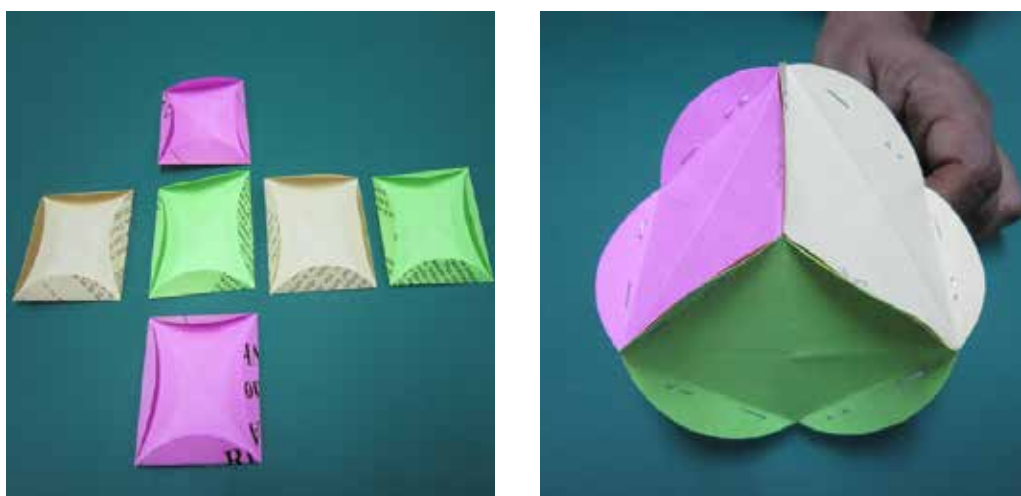Figure. 5: Joining the circles along the folds to get the tetrahedron

Figure. 6: The inscribed squares (notice the arrangement of the circles in the net of the solid) and a vertex of the cube formed when the six faces are joined.

Figure. 7: Preparations for the octahedron – and the finished product

Figure. 8: The inscribed pentagons join together to form the 12 sided dodecahedron
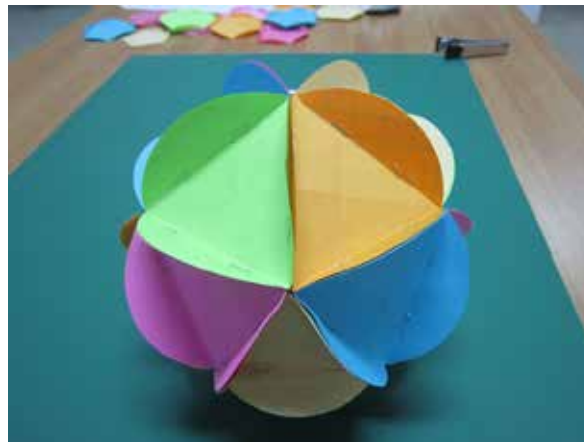

Figure. 9: And the grand finale: the assembling of 20 equilateral triangles to get the icosahedron. Notice the colour scheme; at least 5 colours are needed for no two circles at the same vertex to have the same colour.
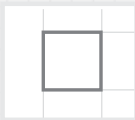
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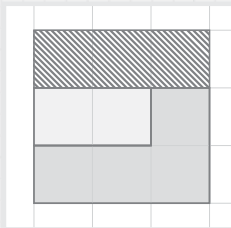
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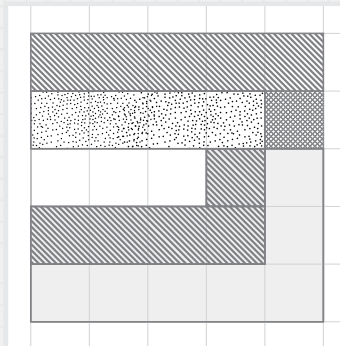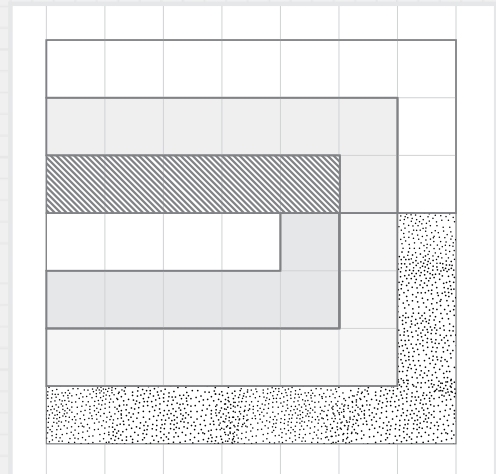
PICTURE SUMS FOR THE **ODD** SQUARES

1

$2 + 3 + 4 = 3^2$

$3 + 4 + 5 + 6 + 7 = 5^2$

$4 + 5 + 6 + 7 + 8 + 9 + 10 = 7^2$

REFERENCES
Preebesh Kumar, https://www.facebook. com/photo.php?fbid=1809354995952471& set=gm.1215164325181388&type=3&theater, AtRiUM, June 19, 2016