Book Review

A Focus on Fractions – Bringing Research to the Classroom

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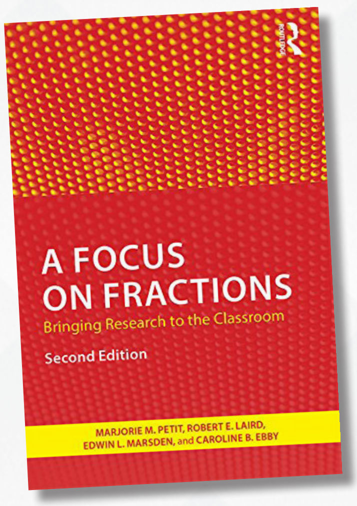

The reviewer works with the Azim Premji Foundation and in the course of his work on content development, he had to review a module made on *Operations on Fractions*. This book - he had earlier read its first chapter - was recommended as support for the task. A complete read led to his understanding of the usefulness of such books and this review is intended to share his experience. All illustrations used have been reproduced from the book.

The relevance of this book to my work is due to the fact that it is a terrific blend of conceptual understanding of fractions and the challenges faced in classrooms by the teacher during transaction of this concept. It made me re-visit my experiences in the field of education. This book has dual insights- data from cited research studies as well as from classroom work and is a valuable resource both for teachers as well as for teacher educators. Threads have been tied beautifully from start to finish of each chapter.

Chapter One (Modelling and Developing Understanding of Fractions) is a must-read for everyone who has earlier struggled during their own school days or is now struggling as a teacher/ teacher educator while dealing with the concept of fraction. Mathematical models or in the authors' words "mental maps," are used as tools to understand the concept and generalize the concerned mathematical idea. For fractions, the three most commonly used models are the Set Model (collection of discrete objects), the Linear Model (or Number Line) and the Area Model. I was able to see how well all three models complemented each other for the first time – as a student, these appeared to me as different, unrelated topics. I needed such a detailed and precise understanding for my daily work with government school teachers. For example, if one is drafting a worksheet for participants with regards to the meaning of fraction, one could easily do a diagnostic test with questions based on all three models

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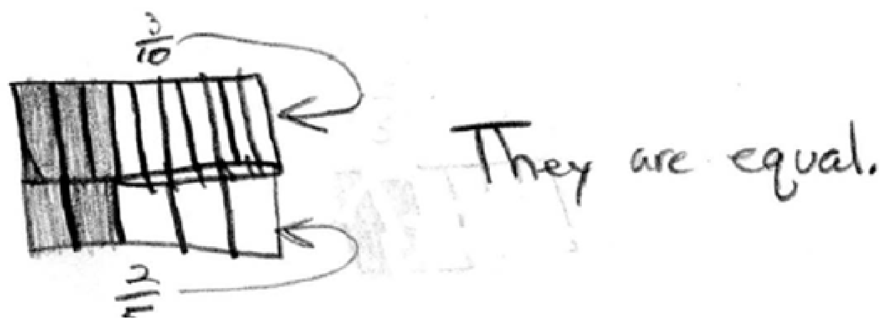
and according to the responses, make out how to proceed with his/her session. In many classrooms around the country, fraction is restricted to only one model (generally, the area model and specifically, area of a circle and/or rectangle). One may ask why all three models are needed - when students know all of them, they learn to select and use the model most applicable to the problem that they are solving.

In a few instances, where the fine motor skills of younger students are yet to be fully developed or when older students are dealing with fractions which are not significantly different from each other, models fail to make their expected impact and that's where manipulatives come into the picture. For example, when a student was asked to compare 3/10 to 2/5, he drew this



and wrote that they are equal. For such instances, manipulatives – such as Cuisenaire rods, paper folding, pattern blocks – and usage, have been brought into the picture.

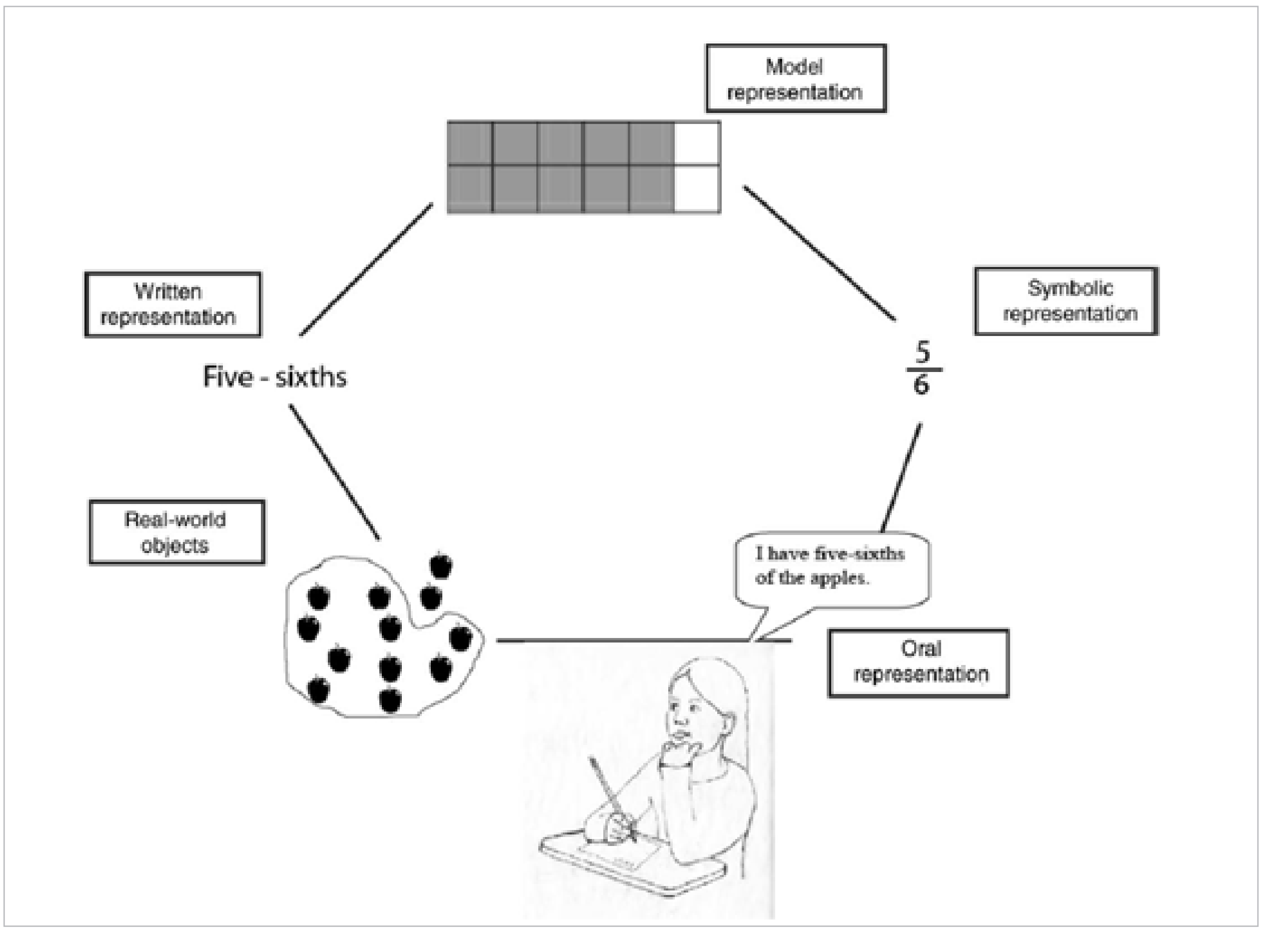
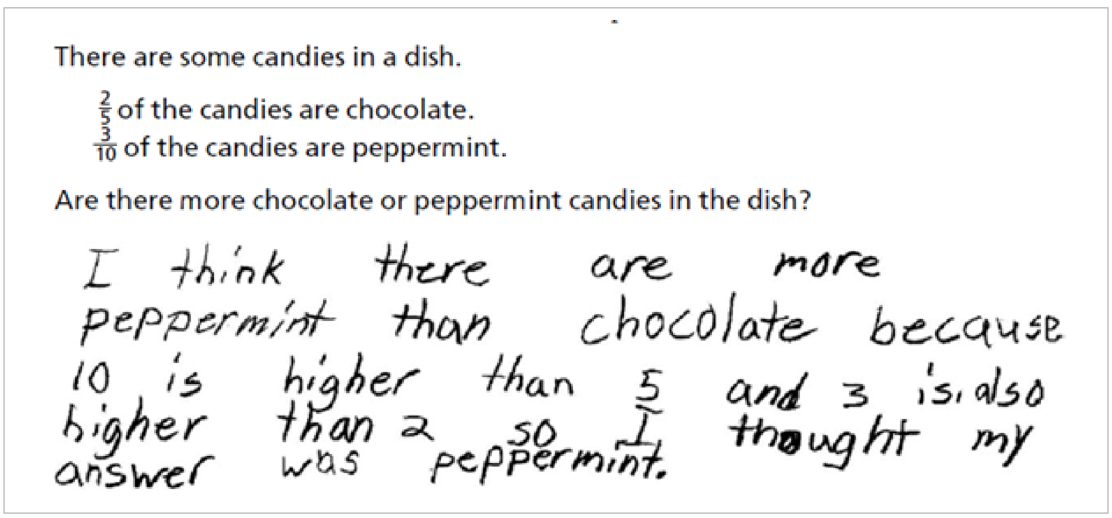


Figure 1 - Five ways to represent the concept of Fraction

Not that modelling or manipulatives are self-sufficient - a big point that came out of this chapter was that modelling is only a means to understanding the concept of fraction and is, very certainly, not the end.

> ***"Learning is facilitated when students interact with multiple models that differ in perceptual features causing students to continuously rethink and ultimately generalize the concept." - (Dienes, cited in Post & Reys, 1979)***

Chapter Two deals with a common misconception (or rather a preconception) that most of the students beginning the study of fractions develop. This is applying the analogy of the concept of whole number. In Chhattisgarh where I work, a student is first introduced to fractions in the third standard, right after s/he has spent close to two and a half years forming concepts around whole numbers. So as soon as a fraction is presented to such a student, and if it hasn't been handled well, they start seeing fraction as two whole numbers rather than as a single quantity. When a student is presented with the number ½, s/he considers it to be two numbers – 1 and 2. It's all downhill from there and they start lagging behind as there seems to be no cohesion between concepts already formed in their minds. In the next year, where they begin operations on fractions, students start to shy away from the subject of Mathematics altogether. The book cities an example from their research-



I really do empathise with such students. To quote the position paper on Teaching of Mathematics (2006), by NCERT -

> ***"There is some evidence that the introduction of operations on fractions coincides with the beginnings of fear of mathematics."***

Which brings us to the all-important question that is also the title of Chapter Three (What is the Whole?) Could you define the whole, for me? Is it the Earth or the Milky Way galaxy or could it be just one chapatti? Behr and Post said - *"The concept of the whole underlies the concept of a fraction." (Behr & Post, 1992, p. 13)*. A fraction is nothing if not for the whole. Finding/identifying the whole when there is more than one part is often a challenge for students, as research has found. Not being able to visualize the whole becomes a major hurdle for students and when they start losing "sight" of the whole then they also possibly stop thinking about the meaning of the fraction in a given context. For example, students are taught that half is less than a whole. But half of a bag of 20 chocolates is greater than a whole bag of 8 chocolates!

Many students find similarities between fractions and the operation of division for several reasons, the symbolic representation predominant among them. Part of it is theoretically linked via Chapter Four of this book – Partitioning. Equal division of the whole is an important concept to be developed by students whatever the model used may be. This gets a notch higher when they're solving problems with several operations or solving problems through "partitioning in their heads," a catchy phrase that they have used.

Comparing and Ordering of Fractions come next. According to the authors, researchers have found that students use fives types of reasoning when they're able to correctly compare and order fractions –

- *Reasoning with unit fractions*
- *Extension of this reasoning to non-unit fractions*
- *Reasoning based on models*
- *Reasoning through the use of a common reference fraction such as ½*
- *Reasoning involving equivalence.*

The Unit Fraction is defined as a fraction having 1 as the numerator and any natural number as denominator. So it becomes easier to compare and order between 1/10, 1/5 and 1/3 as a tenth is a smaller part than a fifth which is smaller than a third. While comparing 7/8 (1- 1/8) and 4/5 (1-1/5), students use an extended version of the same unit fraction reasoning. They see how far away 7/8 is from the whole and similarly, how far away 4/5 is from the whole. Now, using unit fraction reasoning, it is comparatively easier to decipher that 1/8 (the distance of 7/8 from the whole) is smaller than 1/5 (the distance of 4/5 from the whole), hence we can conclude that 7/8 is greater than 4/5. Another way is to use benchmarks/ milestones like 0, ½ and 1. Comparing 13/24 to 24/50 becomes easier if a student compares them first individually with ½ and then as they lie on either side of the half, they can say that 13/24 > 24/50.

The next two chapters deal with the number line and density of fractions (which could also be extended later to rational numbers). They describe a few advantages of using the linear model over the area and set models and how this helps students. This is followed by a description of some of the disadvantages and difficulties faced by them.

It is mentioned that students who think sequentially about the number line face more difficulties than the ones who think about it proportionally. Sequential thinking is where the numbers are just aligned in a sequential manner from 0 to 4, 5 and beyond and the distance between these points isn't given any weightage. Thinking proportionally means, that in addition, the same distance is maintained between consecutive numbers. Their argument that number lines help students understand important aspects of fractions has been observed to some extent in our classroom experiences, though there remains a sense of doubt about supplementary pedagogy. The density of fractions has been explained with a simple example of a TV remote's volume feature where, say, the sound level of 4 units is not audible in a room while that of 5 units is too loud as per the user's expectations. Obviously, there ought to be something in between which is louder than Level 4 but quieter than Level 5 but alas, there isn't any in whole number, whereas there are an infinite number of fraction values between any two fraction or whole numbers. I still feel that this remains a challenge to explain to a grade 5 student in practical scenarios. We try certain questions during our workshops with teachers, such as, name five fraction values between two fraction numbers and participants, of course, come up with various answers- often ending with a question on where there is a limit to fraction values. Could a similar line of questioning be used in a classroom with students? This is an area that hasn't been explored much in my experience. Authors also use the concept of average to elaborate on density as an average of any two numbers is always a number between them. Then, successive averages could be figured between numbers that are closer together, to find a midpoint between them. This successive use of averages generates an unending list of different fractions between two fraction numbers.

Roger Antonsen mentions in his Ted Talk (https://www.youtube.com/watch?v=ZQElzjCsl9o), *Math is the hidden secret to understanding the world,* that math could be of great use in understanding different perspectives of various people; similarly, the same analogy could be used for understanding equivalent fractions (Chapter – 8), which is nothing but (infinitely) different names (symbols) given to a fraction. Equivalence becomes a necessity when students are to add & subtract fractions. So having a procedural fluency in it is a pre-requisite although merely applying algorithm without understanding that 2/5 is the same as 4/10 (even though one has multiplied numerator and denominator by two) doesn't go too far.

Chapter 9 – Addition and Subtraction of Fractions talks about how procedural fluency and conceptual understanding work together to deepen students' understanding of fraction addition and subtraction. The following quotes from National Research Council [NRC], 2001, explain further:

> ***"Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently."***
>
> ***"Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful."***

The number line helps in dealing with the subtraction of mixed fractions. Estimation becomes an important skill while operating with fractions though some students tend to forget the meaning of fraction and do not apply them as soon as they start to solve problems that have operations in them. Being accustomed to different models and then being able to apply one when required forms the baseline of successful operations on fractions.

The last, but definitely not the least, chapter is, of course – probably the most difficult part for students, i.e., multiplication and division of fractions. The generalized concepts of whole numbers with regard to these two operations don't apply as we are now dealing with fractions. Ideas such as "multiplication makes numbers bigger while division makes numbers smaller" start conflicting with previously learnt concepts in students' minds. Visual representation of multiplication and division of fractions becomes harder and harder for many teachers and so does its abstract understanding for their students. Visually seeing that multiplying by ¼ is the same as dividing by 4 is necessary, be it using only the area model. Teachers should still make sure that students aren't beginning to generalise here as well. For example, dividing fractions will result in a bigger number or vice-versa for multiplication. This will fall short when mixed fractions are involved. Day-to-day life experience is also provided to throw some light where if one was preparing 2½ times the recipe, the calculation would be 2½ × ¾ cups of flour and the recipe would require more than ¾ cup of flour.

Each chapter of the book ends with a section, namely *Instructional Link—Your Turn*. These are prompts in table format which enable teachers to reflect on their lessons with questions such as 'Do you encourage students to use a variety of models in all aspects of understanding fractions?' or 'Are students provided with the opportunity to answer questions in which models are used?' Such prompts enable the reader to action their understanding in a very practical manner.

Just as Antonsen mentioned in his Ted Talk, the concept of fractions too, has, in my opinion, a hidden secret of understanding the world and this book comes as a good resource. For all educators and teachers out there, do try to get your hands on it.