

Review: Arrow Cards

By Math Space

Arrow cards are a simple manipulative to grasp place value or more generally the base-ten number-writing system that we use. Dr. Maria Montessori invented the static cards shown in Figure 1. These cards are used along with proportional material like static beads (unit = single bead, ten = 10 beads strung together forming a line, hundred = 10 tens strung together to form a square and thousand = 10 hundreds strung together to form a cube) to gain a sense of numbers – the quantities they indicate and the numerals that represent them and how they are linked. When these cards are superimposed, they form the multi-digit number 1232 as shown in Figure 2.

1	1 0	1 0 0
2	2 0	2 0 0
3	3 0	3 0 0
4	4 0	4 0 0
5	5 0	5 0 0
6	6 0	6 0 0
7	7 0	7 0 0
8	8 0	8 0 0
9	9 0	9 0 0
1	0 0 0	

Figure 1: Static cards – Montessori

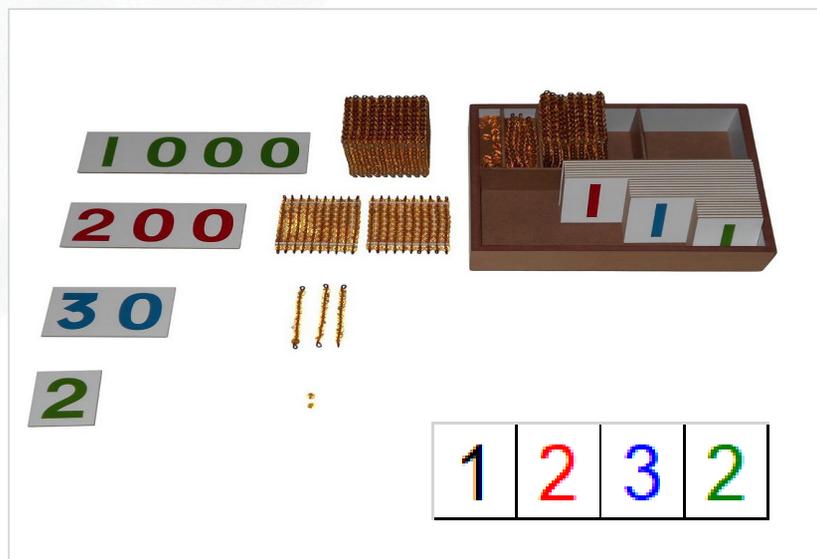


Figure 2: Combined static cards representing a multi-digit number

Keywords: manipulatives, arrow cards, numbers, place value

When these transitioned to regular schools, an arrow got added so that the cards can be held up for an entire class to see. The cards are supposed to be held only by one hand holding the arrows together. This ensures that a number like 327 can't be made with 300, 2 and 7. Even if a child tries to do that and succeeds thanks to friction, one flick of the hand would send the 2 flying out! So, the only way to make 327 would be to use the cards 300, 20 and 7 which is essentially a self-corrective feature, common to many Montessori materials. Figure 3 illustrates the arrow cards. Ideally, these should be introduced with some proportional material like (i) ganitmala, (ii) bundle-sticks, or (ii) 2D base 10-blocks, known as flats-longs-units (FLU) and ideally after introduction of zero.

1	1 0	1 0 0	1 0 0 0
2	2 0	2 0 0	2 0 0 0
3	3 0	3 0 0	3 0 0 0
4	4 0	4 0 0	4 0 0 0
5	5 0	5 0 0	5 0 0 0
6	6 0	6 0 0	6 0 0 0
7	7 0	7 0 0	7 0 0 0
8	8 0	8 0 0	8 0 0 0
9	9 0	9 0 0	9 0 0 0

Figure 3

There are two versions commonly available: (i) the colour-coded version as shown and (ii) the colour-less one where black is the only font colour. The utility of the second version is demonstrated in the following: Scatter all the cards in front of the children and ask them to pick say, forty – they have to distinguish 40 from 4, 40, 400, 4000, etc. However, the colour-coded version has many uses.

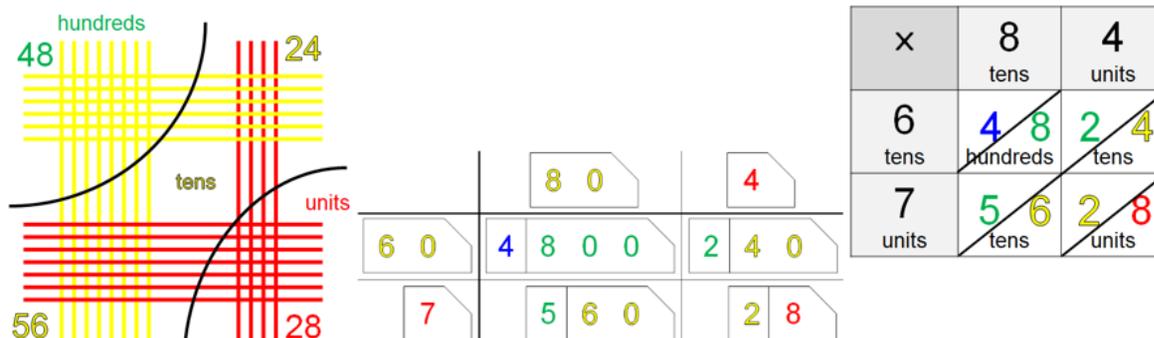


Figure 5: Stick multiplication with arrow cards

1. Expansion of a multi-digit number e.g. $327 = 300 + 20 + 7$ becomes automatic and effortless.

2. It can be linked to the absence of a particular 'bundle' in a number with 0 as an in-between digit. For example, in 307, the colour of ten (red) is missing. Observe that all 30 tens in this number are inside the 3 hundreds and there is no ten on its own outside a hundred. The absence of a ten (on its own) is associated with the absence of its corresponding colour in the arrow card representation of the number. Similarly, for 4050, all hundreds are inside the thousands, and no hundred is outside a thousand. Likewise, all units (or ones) are inside tens or thousands, and none is on its own. The absence of units and hundreds correspond to the absence of their colours in the arrow card representation. See Figure 4.

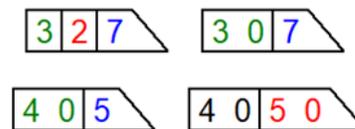


Figure 4

3. The colours provide a vital clue as to why the stick-multiplication or Napier board works (see Figure 5). We encourage the reader to check the following Power Point presentations for further details:

- <http://teachersofindia.org/en/presentation/initiating-multiplication>– initiating multiplication with FLU and arrow cards
- <http://teachersofindia.org/en/presentation/deciphering-stick-multiplication>– unpacking stick multiplication with colour-coded sticks and arrow cards

In addition, arrow cards play quite a powerful role to decipher any algorithm based on place value. It can help understand the division algorithm of finding square roots as well!

However, it should be noted that arrow cards just by themselves do not foster the understanding of 240 as 24 tens or 1500 as 15 hundreds, etc. That aspect is understood better with proportional material like FLU. Arrow cards help in understanding that a 32 is $30 + 2$ and not 3 and 2.

1	0	0	.	1	0	.	1	.	0	.	1	0	.	0	1	0	.	0	0	1
2	0	0	.	2	0	.	2	.	0	.	2	0	.	0	2	0	.	0	0	2
3	0	0	.	3	0	.	3	.	0	.	3	0	.	0	3	0	.	0	0	3
4	0	0	.	4	0	.	4	.	0	.	4	0	.	0	4	0	.	0	0	4
5	0	0	.	5	0	.	5	.	0	.	5	0	.	0	5	0	.	0	0	5
6	0	0	.	6	0	.	6	.	0	.	6	0	.	0	6	0	.	0	0	6
7	0	0	.	7	0	.	7	.	0	.	7	0	.	0	7	0	.	0	0	7
8	0	0	.	8	0	.	8	.	0	.	8	0	.	0	8	0	.	0	0	8
9	0	0	.	9	0	.	9	.	0	.	9	0	.	0	9	0	.	0	0	9

Figure 6

But more interestingly, note that the arrow cards can be expanded for larger numbers beyond 4-digits as much as one wants. Also, it can be



Figure 7

modified to include decimals (Figure 6). The arrow is replaced by a groove for the decimal point. So, the cards have to be held together at the decimal point (Figure 7). This facilitates the notion of lining up numbers so that the decimal points are aligned – a crucial step in column addition-subtraction with decimal numbers.

Introduction to decimal arrow cards should also be done with the decimal version of FLU.

Note that the decimal arrow cards can also be extended beyond hundreds on the left and beyond thousandths on the right, and thus can be used to represent any decimal number.

While both kinds of arrow cards can be bought from various resource organizations, they can be made easily with old A3 size (or larger) posters or chart papers. More details on how to make can be found at <http://teachersofindia.org/en/article/making-your-own-arrow-cards> with suggested size and possible layouts.

We hope to review FLU and many other manipulatives in future issues.

MATH SPACE is a mathematics laboratory at Azim Premji University that caters to schools, teachers, parents, children, NGOs working in school education and teacher educators. It explores various teaching-learning materials for mathematics [mat(h)erials] both in terms of uses and regarding possibility of low-cost versions that can be made from boxes etc. It tries to address both fear, hatred and dislike for mathematics as well as provide food for thought to those to like or love the subject. It is a space where ideas generate and evolve thanks to interactions with many people. Math Space can be reached at mathspace@apu.edu.in