Middle School Problems on the Weighted Average

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A bit of theory before we present the problems.

The average (or arithmetic mean) of two numbers a and b is given by $\frac{a+b}{2}$. The average of three numbers p, q and r is given by $\frac{p+q+r}{3}$. These expressions could be rewritten as

$$\frac{1}{2}a + \frac{1}{2}b \text{ and } \frac{1}{3}p + \frac{1}{3}q + \frac{1}{3}r,$$

showing that the various components contribute similarly to the average value. However, in some circumstances one component may have a greater bearing on the average value. For instance, if 30 litres of water at a temperature of 80°C and 20 litres of water at 20°C are mixed together (without loss or gain of heat), the temperature of the mixture is not 50°C (arithmetic mean of 80° and 20°), but is given by

$$80^\circ \times \frac{30}{50} + 20^\circ \times \frac{20}{50} = 56^\circ\text{C}.$$

Each temperature (the variable in this case) is multiplied by the volume fraction of that component and added to get the 'weighted average.' The volume fractions are the 'weights,' which always add up to unity. Use this idea to solve the following problems.

1. The average monthly income of the 28 men working in an office is Rs.80,000, while that of the 22 women working there is Rs.72,000. What is the overall average income of an employee?
2. A trader sells seven-tenths of his stock of grains at 60% profit, one fifth at 20% profit and the rest at 10% loss. What is his overall profit percentage?
3. A trader acquires a stock of electric heaters at a certain wholesale price. He sells four-fifths of his stock at 80% profit, but the rest are completely destroyed in a fire accident. What is the overall profit percentage achieved by him in the deal?
4. Nine litres of a 6% solution of a salt in water is mixed with 15 litres of another solution of the same salt in water. If the resulting mixture has a salt content of 8.5%, what was the percentage of salt in the latter? By 'percentage solution' we mean the number of grams of solute in 100 ml. of solution.
5. From stocks of a 9% solution and a 16% solution of pesticide, a farmer wants to obtain a 10.5% solution. In what ratio should he mix the two stock solutions?
6. How much pure water should be added to 10 litres of a 7% solution of a salt to obtain a 3% solution?
7. Masses of 5 units, 7 units and 8 units are placed at locations −8, 0 and +15, respectively, on the X-axis. Where would their centre of mass be located?

Solutions

1. The weighted average of the given incomes (which is the variable in this case) is given by

$$80,000\times\frac{28}{50}+72,000\times\frac{22}{50} = \text{Rs. } 76,480.$$

2. We can consider loss to be negative profit, and form the expression

$$60\% \times \frac{7}{10} + 20\% \times \frac{1}{5} - 10\% \times \frac{1}{10}$$

which equals 45% profit on the whole. It is not necessary in such situations to know the actual cost price, selling price or the quantity sold.

3. Destroyed goods indicate a 100% loss. So we can form the expression

$$80\% \times \frac{4}{5} - 100\% \times \frac{1}{5}$$

which leads to 64% – 20% = 44% profit.

4. Here, the solution strength is the variable. Let the unknown solution percentage be P%. Then $6\% \times \frac{9}{24} + \text{P}\% \times \frac{15}{24} = 8.5\%$. On simplification, this yields

$$15\text{P} = 8.5 \times 24 - 54 \text{ or } P\% = 10\%.$$

5. Let the required ratio be a:b. Then $9\% \times \frac{a}{a+b} + 16\% \times \frac{b}{a+b} = 10.5\%$. This equation leads to $9a + 16b = 10.5(a + b)$ or $1.5a = 5.5b$ or a:b = 11:3.
6. Pure water can be considered a 0% solution. If we take the volume of pure water required to be x litres, then we can form the equation

$$7\% \times \frac{10}{10+x} + 0\% \times \frac{x}{10+x} = 3\%.$$

This leads to $70 = 3(10 + x)$ or $x = \frac{40}{3}$ litres.

7. Here the position on the X-axis is the variable. So we form the expression

$$-8 \times \frac{5}{20} + 0 \times \frac{7}{20} + 15 \times \frac{8}{20}$$

which yields +4 as the location of the centre of mass.