A Utilitarian Math World: The Tail that Wags the Dog

ClassRoom

KUMAR GANDHARV MISHRA

One of the priorities of mathematics educators and teachers has been to improve mathematics learning in the classroom: to experiment with different teaching methods. The objective is to engage children with the mathematics they study by allowing them to explore and enabling them to think about the problem in multiple ways rather than memorising procedures and formulas. But, what about the world outside this classroom? What is happening in this world and how is it going to influence and affect classroom efforts in the future? This article takes the readers through a short journey of such a world and tries to highlight what the world often offers for mathematics in general, and mathematics education, in particular. Roaming around streets and markets in India, you would often come across posters, pamphlets and hoardings of coaching centres which claim to have ***Remedies for all your*** **Math Problems**. It is exciting to see these 'Mathematics' hoardings which stand tall along with the hoardings of cars and apartments for sale, near highways, streets, roads or in markets. Thousands pass by and see them daily. They would have absorbed some impression about mathematics. What is the (perhaps half-baked and partial) impression that people who have passed by these hoardings must have absorbed about mathematics?

A look at all the advertisements during a walk through Mukherjee Nagar, Delhi (a famous coaching hub in Delhi) made me wonder: **'Is Mathematics really frightening?', 'Does Mathematics need to be taught from the beginning?', 'Do students who pass out from college and school need to learn mathematics from the very beginning even at this stage?', 'Is mathematics that subject which needs to be taught with a guarantee stamp?'** or **'Are 10 ways sufficient to learn Mathematics?'** I am sure you have come across similar advertisements. Perhaps in some way, these statements

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reflect the status of mathematics in society and what mathematics means to society. If you happen to visit bookstores at a railway station or a bus station, you would often find books on mathematics tricks and formulas. Most of the coaching centres or coaching platforms mainly focus on 'competitive mathematics,' also known as quantitative aptitude which is an integral part of almost all competitive examinations leading to employment opportunities in India. During the last 4-5 years, the use of smartphones with affordable data services (internet) has significantly increased among students and graduates in India. For example: you would find YouTube flooded with videos on tricks. Many of these channels have grown with millions of subscribers (users or learners) as well as a million views. With an aim to gain maximum views and subscribers (monetary benefit), most of these videos appear with catchy thumbnails/captions to solve mathematics problems within *'seconds'* - not *'minutes.'* Many of these coaches 'guarantee' the selection of every subscriber (user) for various competitive examinations. (Though the number of seats is pre-known and applicants are in lakhs, they guarantee selection of every subscriber with their tricks.) A significant leap in revenue, number of subscribers and number of registered users in these coaching platforms and other Edtech companies during the times of the Covid-19 pandemic has also made headlines. The phrases used across various platforms (online and offline) confirm certain societal notions about mathematics:

i. Mathematics is important and an integral part of any competitive examination. (Mathematics as a gatekeeper in various employment opportunities.)
ii. Success in mathematics is measured by the speed in which the correct response is given.

These also popularise some notions (fallacies) about mathematics:

i. Mathematics is frightening (Math Phobia).
ii. People who are good in mathematics give the correct answer quickly.
iii. You become good at mathematics if you learn 'shortcuts,' 'magic formulas' and 'tricks.'

These examples also reflect how the widespread 'fear of mathematics' is shrewdly (mis)used by these platforms and it also shows how mathematics is being made synonymous with formulas, shortcuts and tricks in the public domain.

A short glimpse of some of the popular tricks

The syllabus for most of the competitive examinations is almost the same. Generally, the syllabus comprises of these topics: number system and arithmetic operations, divisibility, surds, exponents, GCD and LCM, elementary algebra, ratio and proportion, rates, commercial arithmetic, alligation/mixture problem, etc. Some examinations also include probability, data handling, mensuration, trigonometry, etc.

The terms 'tricks' and 'shortcuts' are often used without any distinction in day to day conversations. In colloquial language, a *shortcut* suggests a route which takes less time to reach the destination. In the context of a mathematics problem, any approach which helps find the answer quickly compared to another approach (conventional) is a shortcut. One approach may be a shortcut for one problem but not for another. For example: factorisation, formula or trial and error are all methods to solve quadratic equations, but the shortest method would depend on the quadratic itself.

Example 1: A popular trick[1] on multiplication of 2 two-digit numbers close to 100 is discussed here. Illustration and presentation of these tricks varies from coach to coach. Some present it as ***'multiplication of any 2 two-digit numbers'*** while some present it as ***'multiplication of 2 two-digit numbers close to 100'***.

Problem 1: Multiply 88 and 92.

Multiplication through conventional method

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  88
× 92
 176
7920
8096
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Figure 1

Multiplication through a trick

Step 1: Subtract each number from 100 and note down the corresponding difference. Place these below the two numbers.

88	**92**
12	**08**

Figure 2

Step 2: Multiply the differences. Denote this result by A.

12 × **08**

Figure 3

A = 96. Note: If the result is a one-digit number (say 2), write it as 02.

Step 3: Subtract one of the differences from the other number (diagonally opposite position). Denote this result as B.

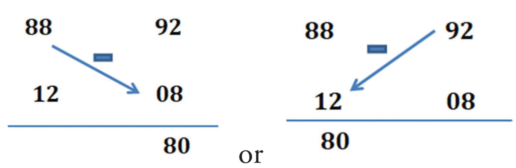

Figure 4

Here, B = 80.

Step 4: Place **B** before **A** i.e. *BA*, and this (8096) is the answer.

How does this trick work?

Let us understand the reasoning behind this trick through a general case. Let the 2 two-digit numbers 88 and 92 be P and Q respectively.

88	**92**
12	**08**

As per Step 1, the representation is:

P	**Q**
100-P	**100-Q**

Figure 5

Let the difference when P and Q are subtracted from 100 be x and y respectively i.e. $x = 100 - P$ and $y = 100 - Q$. In other words, $P = 100 - x$ and $Q = 100 - y$.

So, another equivalent representation (diagram) of Figure 5 is:

100-*x*	**100-*y***
x	***y***

Figure 6

Now, as per step 2 of the trick, Step 2: A (A used in trick) $= xy$ Step 3: $B = (100 - x) - y$ or $(100 - y) - x = 100 - (x + y)$. [From Figure 6, you can see why the difference of the two pairs of diagonally opposite numbers in the Figure 4 yields the same result.]

As per the trick, placing B before A i.e. BA yields the result. Let us see how the rule of 'placement' works in this case.

The product of P and Q can be written as

$PQ = (100 - x)(100 - y) = 10000 - 100x - 100y + xy = 10000 - 100(x + y) + xy = 100\{(100 - (x + y)\} + xy = 100B + A$

The product (100B) after multiplication of **B** by 100 will always have zero in its tens and units place and so when you add a **two-digit number** A to **100B**, the result will be the same as placing **B** before A (left of A). That's how a visual representation BA becomes an answer without any explicit operation like addition. (This is why A is always written as a two-digit number.)

Conditions for A to be a two-digit number (to be represented by two digits): $0 \le xy \le 99$

If the product $xy > 99$, the 'placement' rule will not work. For example: if 88 changes to 87, the product of the differences of 87 and 92 from

100 (13 and 8 respectively) becomes 104 which is a three-digit number. A three-digit number can't be placed or fit into '00', so the trick doesn't work in this case. In fact, the two 2-digit numbers used for illustration are intentionally selected to suit this condition (that $xy \leq 99$). Cases where $xy > 99$ are conveniently omitted in spite of the trick being advertised (and the audience fooled) as a general ***multiplication trick for 2 two-digit numbers which finds the product within seconds.***

Example 2: Figure 7 depicts a famous trick used in the coaching world for *Alligation* (mixture) problems. Here, a mixture problem where the Cost Price of Product A < Mean Price of mixture of A&B < Cost Price of Product B is discussed:

Problem 2: How many kg of rice at ₹ 52 per kg should a shopkeeper mix with 25 kg of rice at ₹ 24 per kg so that on selling the mixture at ₹40 per kg, the shopkeeper can gain 25% on the outlay?

Solution using conventional method

Selling Price (SP) = 40; A gain of 25% has been made by selling the mixture at ₹ 40/kg.

Here, CP + 0.25CP = SP, (Cost Price = CP)

So, 1.25 CP = 40 or CP = 32

The cost price of the resulting mixture is ₹ 32/kg. Let x kg of rice at ₹ 52/kg be mixed with 25 kg of rice at ₹ 24/kg.

As per the conditions:

$$52x + 25 \times 24 = 32(x + 25)$$

Solving this equation, we get $x = 10$. So, the required amount of rice should be 10 kg.

Solution using Trick

Step 1: $CP = \frac{100\ SP}{100+P}$ (A readymade formula for calculating CP, here P is the profit percent). Using this formula, $CP = \frac{100 \times 40}{100+25} = 32$ (which is the mean cost price of the mixture of two varieties of rice).

Step 2: ***Alligation Diagram***

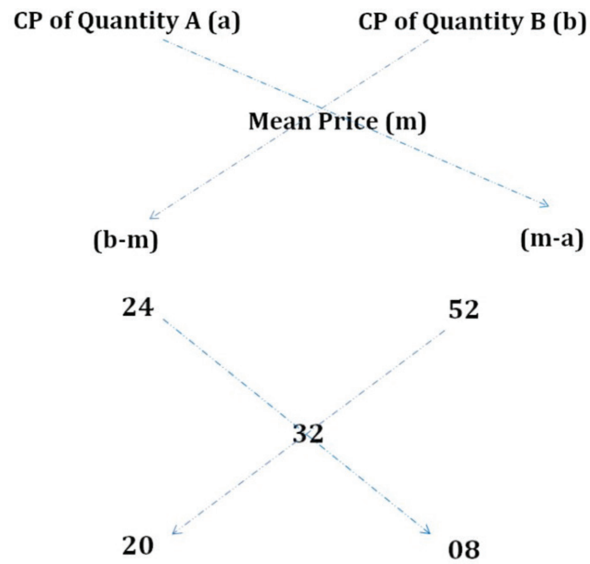

Figure 7
Quantity of A: Quantity of B = (b – m): (m – a)

Quantity of rice A (₹ 24/kg) : Quantity of rice B (₹ 52/kg) = 20:8 which is 5:2.

Step 3: For every 5 kg of rice **A**, 2 kg of rice **B** is used. So, for 25 kg of rice **A**, 10 kg of rice **B** will be used.

This trick (Figure 7) is a modified version of the conventional method. The *Alligation Diagram* is nothing but a visual aid for solving linear equations (at least for those who haven't understood or applied the conventional approach). The *trick* is to be mugged up by those students who are not well aware of the reasoning process involved in this problem.

By using tricks repeatedly, children and even adults begin to experience the satisfaction of getting the right answer, but in the process, they begin to think that *'this is the way mathematics works and coaches/teachers who 'invent' or come up with these tricks are definitely champions or kings of mathematics'*. Coaches are described with adjectives like Magician, King and Wizard and called 'Mathematicians' and 'Educators' in the public domain. You may come across such discourse among social media users, common public and students in coaching classrooms about glorification of similar tricks as well as of their math coaches.

Some Consequences

In general, the audience which attends coaching classes or virtual platforms for competitive examinations are not school children. Wide circulation of billboards, posters, hoardings in public spaces and circulation of videos (both online and offline mode) with thumbnail-captions emphasising shortcuts helps create and strengthen a utilitarian and narrow impression of mathematics among parents, general public and college and school students (future parents, teachers and mathematicians). Some of the far reaching and long-lasting consequences on children, parents and society in general can be:

A. Parents: When parents absorb such impressions about mathematics, their expectation from their children of being good in mathematics would be reduced to being good with tricks and of giving the right answer quickly. A notion of competition is prevalent in society where parents try to compare their children's ability and success with that of other children. Sadly, a child who is dubbed a 'young mathematician' is often nothing more than a trained performer.

B. Children: Children who don't have the privilege to be taught through various pedagogical interventions, classroom engagement, teaching resources, games and activities are more vulnerable to rely on the power of tricks and formulas and absorb similar beliefs about mathematics which carry on into their adulthood. This culture has also started making its way to school students by targeting the syllabus of school mathematics. For example: Trick based or readymade solutions are being made available in the name of NCERT or CBSE mathematics problems. Though tricks are not emphasised in an ideal school classroom, these have the potential to divert students in getting readymade answers once they get exposed to these. It can help them create their own impression about mathematics: a narrow impression that mathematics means only numbers, tricks, mechanical procedures and correct answers. Based on these, a contrast between the teacher's and parent's expectations can create an uncomfortable situation for children.

Children have also joined YouTube and other platforms where you would find them publishing their own channel and videos on multiplication tricks, addition tricks, etc. with the use of phrases like that used by math coaches. They are also appreciated and glorified for the presentation of tricks. Peer pressure leads to such practices spreading. The prevalent culture and its consequences for children are also contrary to the vision envisaged in the position paper of the teaching of mathematics (NCF 2005) which explicitly states that equating mathematics with formulas and mechanical procedures does great harm. Unfortunately, such activities are taking place outside the classroom. However, it would be a good initiative if children are asked to explore the reasoning behind working of their tricks and formulas. They should also be asked to explore the conditions under which their tricks work or fail. Developing understanding behind the working of many of these techniques can also help them become problem solvers in the true sense of the word.

Concluding comments

No doubt, tricks and readymade formulas come to rescue in various competitive examinations and it is unfair to expect anyone to refrain from tricks and formulas in tests where speed matters. In such an environment, everyone has the freedom to teach and solve the way they want until they get correct answers, but it is unfortunate that tricks and formulas have become a ubiquitous part of mathematics. Many a time, tricks are glorified in the name of mathematics education which is contrary to the goals of mathematics education and voices of mathematics educators across the globe. One may say that it depends on discretion of viewers and learners to follow and use these tricks. But, irrespective of the usage of tricks, marketing and popularising tricks with misleading thumbnails and posters spreads a wrong and partial impression about mathematics. The unfair depiction of mathematics in public spaces through these advertisements can construct such a perception in society where **getting answers within the shortest possible time = being successful** may emerge as the new equation.

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References

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KUMAR GANDHARV MISHRA is an independent researcher in Mathematics Education. He completed his Masters in Mathematics Education – M.Sc. (Mathematics Education) from Cluster Innovation Centre, University of Delhi. He often writes on social and cultural aspects of mathematics, and mathematics education. He can be reached at mishrakumargandharv@gmail.com