The Mystical Number 2997 (Kohli's Number)

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Introduction

This paper is about discovering and figuring out the working behind the magical number 2997.

Take any three-digit number, say xyz and multiply each digit by 111 and take the sum, i.e., x*111 + y*111 + z*111.

*[**Note:** Does not have to be a 3-digit number]*

For any number, repeating these steps of multiplying and adding will guarantee that the answer becomes 2997 in a maximum of 4 repetitions.

How does this happen?

It is easy to think that this is pure magic. I resolved to find out the mechanics behind this interesting result.

Consider the number 111, the number we use as a multiplier.

We see that 111 is divisible by 3, this can be confirmed by taking the sum of its digits and checking that it is divisible by 3. (1 + 1 + 1 = 3 which is the first multiple of 3.)

Consider any number (say xyz). Multiply each digit by 111 and take the sum. The sum is x*111 + y*111 + z*111, which can also be written as $111(x + y + z)$. We thus obtain a number that is divisible by 3.

For example, take the case of 999. (Incidentally, this is the only 3-digit number that will end up at 2997 after just one step.)

Step 1: Multiply each digit by 111. The products are 999; 999; 999.

Step 2: Take the sum: 999 + 999 + 999 = **2997**.

For other numbers, the sum may not be 2997 after the first try.

Let the sum $111(x + y + z)$ from the first try be the new number N. As pointed out earlier, N is divisible by 3. Multiplying each digit of N by 111 and taking the sum again, we get a number that is divisible by 9.

Let us consider an example, 4761.

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Step 1: Multiply each digit by 111. The products will be 444; 777; 666; 111.

Step 2: Take the sum: 444 + 777+ 666 + 111 = 1998.

Repeat Step 1 and Step 2: 111 + 999 + 999 + 888 = 2997.

There you have it.

For numbers not divisible by 3, it is necessary to repeat the steps to add another factor of 3. Try 57931 as an example.

The trick is that each time steps 1 and 2 are repeated, the new number gets an additional factor of 3, and once it becomes 111*(27), the number 2997 is obtained.

And when the sum reaches 999 or another number whose digits add up to 27 (the third multiple of 9), the result of multiplying the digits by 111 and taking the sum will be 2997.

Doing this continuously would guarantee an end at 2997.

The beautiful part is when we multiply each digit of the number 2997 itself by 111 and add the resulting numbers, we get ***2997****.*

The number does not have a significance in higher mathematics, but it serves as a recreational and entertaining fact, which shows that even high school mathematics can surprise us with patterns just waiting to be discovered.

Some more examples

1) 29
   222 + 999 = 1221
   111 + 222 + 222 + 111 = 666
   666 + 666 + 666 = 1998
   111 + 999 + 999 + 888 = 2997.

2) 739
   777 + 333 + 999 = 2109
   222 + 111 + 0 + 999 = 1332
   111 + 333 + 333 + 222 = 999
   999 + 999 + 999 = 2997.

3) 6307
   666 + 333 + 0 + 777 = 1776
   111 + 777 + 777 + 666 = 2331
   222 + 333 + 333 + 111 = 999
   999 + 999 + 999 = 2997.

4) 8
   888
   888 + 888 + 888 = 2664
   222 + 666 + 666 + 444 = 1998
   111 + 999 + 999 + 888 = 2997.

General Formulation, to Arbitrary Number of Digits

Let x (an arbitrary positive integer) be the number with say n (another arbitrary variable) digits.

Step 1:

Multiply all digits by 111. We get $111*a_1 + 111*a_2 + \dots + 111*a_n$

Step 2:

Take the sum $111(a_1 + a_2 + \dots + a_n)$.

If $(a_1 + a_2 + \dots + a_n)$ is divisible by 27

Sum: 2997.

Else:

Repeat the two steps. Each time we do this, the quantity acquires an extra factor of 3.

$3*37(a_1 + a_2 + \dots + a_n)$, i.e., $37*(3a_1 + 3a_2 + \dots + 3a_n)$

Continue until the sum becomes divisible by 27.

Sum: 2997.

Editor's Note: A formal proof of this result will be published in the next issue, meanwhile we encourage readers to send in their proofs, if they arrive at one.

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