Age Digit Reversals

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Razvi was born in 1974 and his father was born in 1947, so the tens and units digits of their birth years are reverses of each other. In 2021 Razvi will be 47 years old and his father will be 74 years old, again reverses of each other and also reverses of the tens and units digits of their birth years.

We can prove the more general case for a pair of years in the same century, C.

- Let x = the tens digit of the year of birth of the father, which will be the units digit of the year of birth of his son.
- Let y = the units digit of the year of birth of the father, which will be the tens digit of the year of birth of his son.
- Year of birth of the father: $C + 10x + y$.
- Year of birth of his son: $C + 10y + x$.

In year $C + 10x + y + 10y + x$, the age of the father will be $10y + x$ and the age of the son will be $10x + y$, so their ages are reversed. Of course, $10x + y$ must be less than $10y + x$.

In the 20th century, besides 1947 and 1974, what other pairs of years have this reversibility feature? If we make a list, starting with the pair 1901 and 1910, then 1902 and 1920, we soon see that the age difference is a multiple of 9, in fact 9 times the absolute value of the difference of the tens and units digits. We will assume that the age difference must be at least 18 and at most 63. In the first decade, there are 6 practical pairs: 1902 and 1920, with an age difference of 18, 1903 and 1930, with an age difference of 27, continuing until 1907 and 1970, with an age difference of 63. In the second decade there are also 6 pairs, starting with 1913 and 1931, and ending with 1918 and 1981. Likewise, the third decade

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has 6 pairs. There are only 5 in the next decade, between 1935 and 1939, then 4 between 1946 and 1949, continuing with one less pair each decade until there is only one pair, 1979 and 1997, for a total of 33 practical pairs in a century.

The Magic of 11

Returning to Razvi and his father, because of the magic of 11, their ages are also reverses of each other every 11 years before and after 2021. Since we know the ages are reverses in the year $C + 10x + y + 10y + x = C + 11x + 11y = C + 11(x + y)$, we know that years of the form $C + 11n$ will have the age reversal property, as shown.

When the father is born in 1947 and the son in 1974: difference of ages is 27 years		
Year	**Age of Father**	**Age of Son**
1977	30	03
1988	41	14
1999	52	25
2010	63	36
2021	74	47
2032	85	58
2043	96	69

Let us observe another case:

When the father is born in 1926 and the son in 1962: difference of ages is 36 years		
Year	**Age of Father**	**Age of Son**
1966	40	04
1977	51	15
1988	62	26
1999	73	37
2010	84	48
2021	95	59

In general, if the difference in the ages is $9k$, with $k = 2, 3, 4..., 7$, there are $10 - k$ age reversals.

This investigation began with a simple observation that opened the door to some beautiful mathematics. Indeed, mathematics resides everywhere.

Reference

[1] NCTM The Math Forum Ask Dr. Math, "Reversal of Age Digits Every Eleven Years", http://mathforum.org/library/drmath/view/71681.html

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