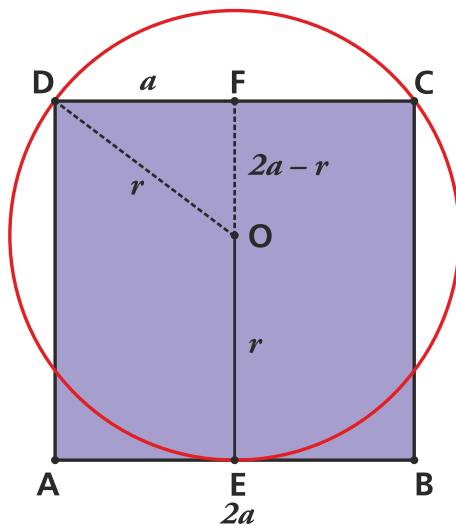


SOLUTION TO THE PROBLEM

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CoMaC

Square $ABCD$ has side $2a$. A circle of radius r touches side AB and passes through vertices C and D . The problem is to find r in terms of a . (In the actual problem, we had $2a=40$.)



Let E, F be the midpoints of sides AB, CD , and let O be the centre of the circle. Join OD . Then O lies on EF . Moreover, $OE=r$, $OF=2a-r$, $DF=a$, $OD=r$. From the right-angled triangle OFD , we obtain:

$$r^2 = a^2 + (2a - r)^2.$$

Hence $4ar=5a^2$, giving

$$r = \frac{5a}{4}$$

This is the required relationship. (In the given problem, we have $a=20$, so $r=25$.)

Comment: We also received a solution from Shri Tejash Patel. It uses coordinate geometry methods.

Construction of the figure

The answer suggests how we can construct such a figure, which otherwise is not at all obvious. For we obtain,

$$FO:OE=3:5.$$

So we must draw the midline of the square (i.e., the line segment EF joining the midpoints of a pair of opposite sides), and then locate a point O on EF such that $FO:OE=3:5$. The circle with centre O , passing through E , is then the required circle.