Squaring the Circle

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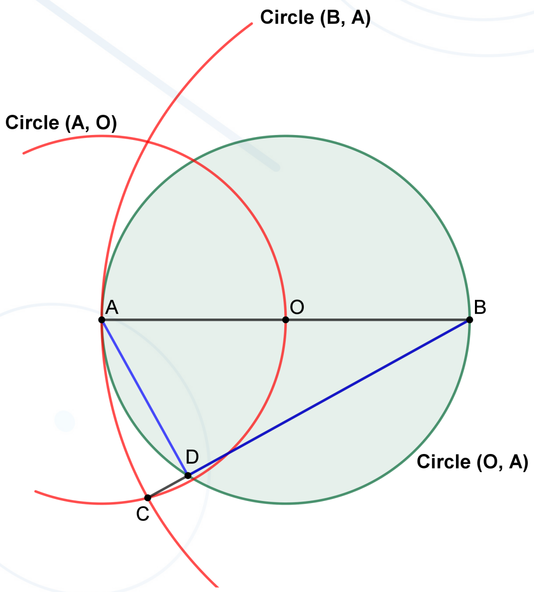
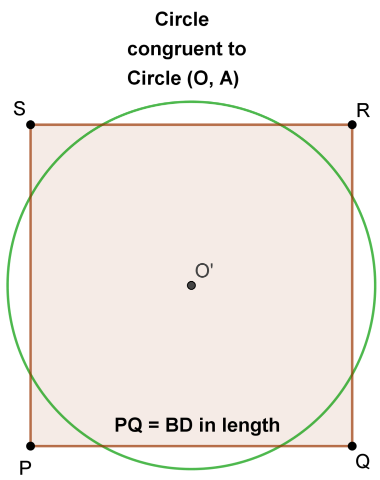
Construction steps

1. Draw Circle (*O*,*A*), with diameter *AB* and centre *O*.
2. Draw Circle (*B*,*A*), followed by Circle (*A*,*O*).
3. Mark *C*, one of the points of intersection of Circle (*B*,*A*) and Circle (*A*,*O*).
4. Join *BC*, and let it intersect Circle (*O*,*A*) at *D*.
5. Claim: BD^2 is almost equal to the area of Circle (*O*,*A*). [See the square at the right.]





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Let $AO = 1$ unit, $AB = 2$ units; then $BC = 2$ units, and $AC = 1$ unit. Hence, triangle BAC is isosceles, with equal sides 2 units and base 1 unit. (In the figure, AC has not been joined.)

By construction, $\angle ADB$ is a right angle ("angle in a semicircle").

The length of the perpendicular from B to AC is $\sqrt{2^2 - \frac{1}{2^2}} = \frac{\sqrt{15}}{2}$. Therefore, the area of triangle BAC is $\frac{\sqrt{15}}{4}$ square units.

Hence, $\frac{1}{2} \cdot AD \cdot BC = \frac{\sqrt{15}}{4}$, implying that $AD = \frac{\sqrt{15}}{4}$.

Using Pythagoras's theorem, $BD^2 + AD^2 = AB^2 = 4$, so $BD^2 = 4 - \frac{15}{16} = 3\frac{1}{16}$.

Hence $BD = \frac{7}{4}$, i.e., BD is $\frac{7}{4}$ times the radius of the circle.

So the area of the square on side BD is $3\frac{1}{16} = 3.0625$, and the area of the circle on AB as diameter is $\pi \approx 3.1416$.

So the error percentage is approximately -2.51% . (The error is on the negative side.)

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