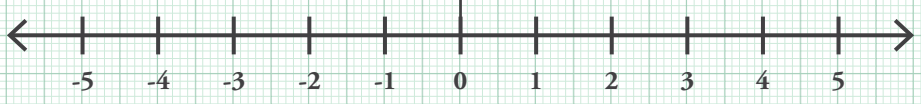
At Right Angles

A Resource for School Mathematics

ANGLES

PADMAPRIYA SHIRALI



Azim Premji University

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ANGLES

Measurement of line segments and the concept of length arise naturally in students. However, measurement of a turn or an angle formed between two rays is more complex to understand. Comprehending the concept of a degree as a measure of a turn and developing the skill of using a protractor takes time.

It is a gradual process and we need to guide students through this process by exposing them to varied situations. It entails the teacher having to pay individual attention to skill development in students. This can be a challenge in a large class.

Fortunately, we find angles everywhere, in the corners of the blackboard, between the fan blades, between the two hands of a clock, in a pair of scissors, a seesaw, a slide in the park, etc. Angle is seen in two forms, one as a fixed form, say between two fan blades or between the length and breadth of a blackboard and secondly in a dynamic form, i.e., as a varying turn, as in the case of the hands of a clock or between two scissor blades.

To focus on the skill of measurement before students have developed a proper conceptual understanding of angle will lead to the common mistakes that students make. Wrong usage of the protractor in measuring angles, lack of estimation, incorrect placement of the protractor resulting in non-alignment of the base line with the arm of the angle and other errors can be reduced if sufficient focus is given to the concept in the initial stages.

Using a kinaesthetic approach by bringing in yoga to demonstrate different angles and playing with map routes involving turns can build readiness in students.

Playing around with angles and matching angles, using estimation prior to measurement can all aid in the process of developing a robust understanding of this topic.

Protractor as a tool of measurement can be a challenge to most students and one can ease the process of understanding it by helping students to use simpler tools. Students can create all these tools with the help of the teacher, so that they understand the concept embedded in a protractor.

Further explorations can be done using dot paper, geoboards and varied geometric shapes to notice and arrive at properties of a straight angle, sum of angles at a point, and so on.

Artwork using angles can be easily incorporated to build their skills and make it enjoyable at the same time.

OBJECTIVES OF THE UNIT

Understand that an angle represents an amount of a turn.

Measure the size of an angle to the nearest degree using a protractor.

Draw any angle to the nearest degree using a protractor.

Estimate the size of an angle.

Use appropriate language.

ACTIVITY 1

Objective: Observing angles in *Yoga Asanas*

Materials: Mats for students, Printed sheet of simple yoga postures

Vocabulary: Angle, sharp angle, blunt angle, straight angle, square angle

Teacher can get a student to raise his arm gradually and get others to observe the angle that forms between the arm and the torso.

We see several angles in each pose. The teacher can direct the attention of the students to a particular angle and point out the arms of the angle in each case.

Students can do bending exercises from the waist to show different angles.

They can lie down on the mats and raise their legs to demonstrate different angles.

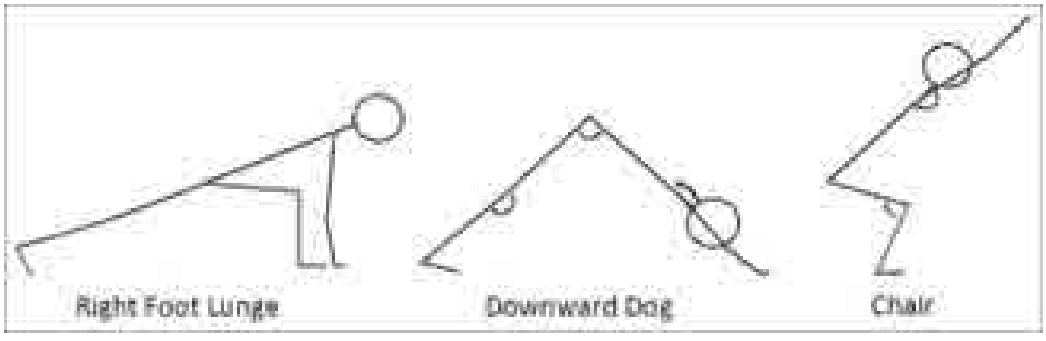
They can begin to describe these angles as sharp angles, blunt angles, square angles (instead of right angles) and straight angles.

At a later point they can be taught acute angle (acute means sharp), obtuse angle (obtuse means blunt), right angle and straight angle.

What pose can the students use to show a square angle? Can they show it using legs and the torso? Can they show it using the arms and the torso? Can they show it using two arms?

Is there a pose using both the arms that can show a straight angle?

Will it make a difference to the angle if someone has longer arms?



ACTIVITY 2

Objective: Angle hunt

Take students for a walk around the school and observe various objects all around to spot angles in nature, in man-made objects, etc.




ACTIVITY 3

Objective: Creating angles, naming and comparing angles

Materials: Straws of different sizes and staple pins/paper clips

Vocabulary: Angle, Acute angle, Obtuse angle, Straight angle, Right angle

Let students use two straws and staple them to create different angles and describe them using proper vocabulary. If one tooth of the staple pin goes through each straw, it allows for a full turn.

Can they compare two angles? How can they make out which is bigger?

Do the students think of aligning one arm of the two angles to compare?

Do they think that angles made with longer straws are bigger than the ones made with shorter straws? Discuss this aspect so that they clearly see that an angle is a measure of a turn. They can also cut the straws to see if the angle changes.

What can they do if they want to compare three angles?

ACTIVITY 4

Objective: Matching angles with slots and building estimation skills

Materials: Set of angle pieces and angle slots

Students can use a cardboard piece and make some angular cuts on the edges, as shown in the picture.

They can now use the entire set to guess and match the card with the right slot.

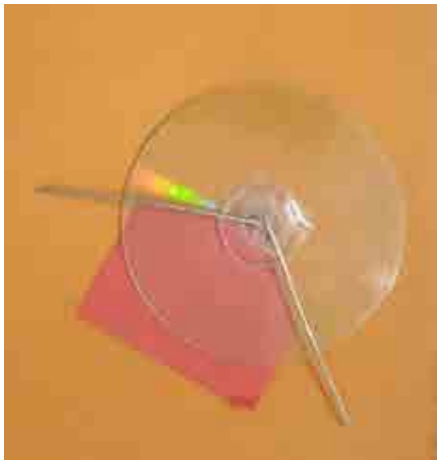
ACTIVITY 5

Objective: Spot angle pairs with a rotagram

Materials: Rotagram, Sheet with several pairs of matching angles

Preparation of a Rotagram: Materials: Circular plastic sheet (Old CD covers or waste CDs), two metal wires

Teacher can fabricate a rotagram using a circular plastic sheet. Pierce the centre and fix two stiff metal wire loops that pass through the centre as shown in the picture. One should be able to rotate them around the circular disc easily.



Students can now place the rotagram on an angle in the worksheet and align the wires with the arms of the angle. They have to spot the matching pair of the angles by fitting the rotagram over other angles.

ACTIVITY 6

Objective: Classify angles as right angles, acute angles, obtuse angles, and straight angles.
Materials: Right angle formed from a paper without straight edges, worksheet with different angles.

Construction of a right angle

Students should be shown how to create a right angle by folding any paper twice, once horizontally and once vertically as shown in the picture.

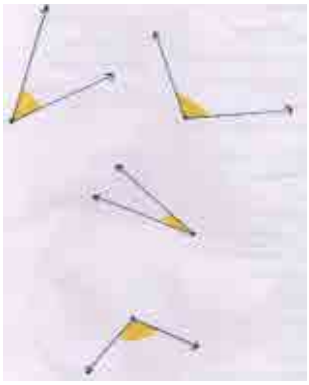
While making the second fold, ensure that the first crease folds on top of itself.

This forms a right angle. Open the paper to see that the two lines are perpendicular to each other and form four right angles.

Why does it work?

Hint: Examine what happens to the straight line that has formed when we fold it over a second time.

Students can use this paper to identify various angles as acute, obtuse, etc.



Are they aligning the two bases?

Do they see that if both lines of the paper align with the arms of the angle, then the angle is a right angle; if one arm is visible in the gap, then the angle is an obtuse angle; and if one arm is hidden from view, then the angle is an acute angle?

GAME: ANGLE CHALLENGES

Challenge 1

How can you place two sticks to make just one right angle?

Now make just two right angles using the two sticks.

Can you make three right angles? Four right angles?

Challenge 2

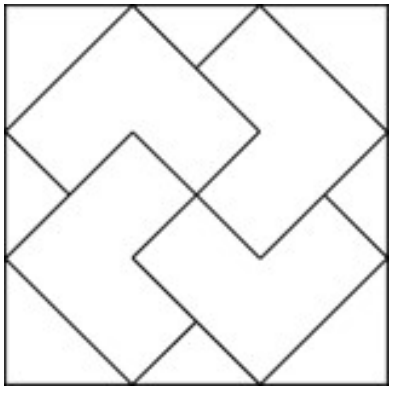
What happens when 2 right angles meet?

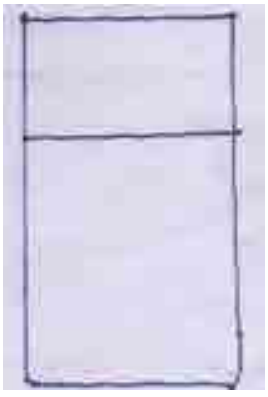
What happens when 3 right angles meet?

What happens when 4 right angles meet?

Challenge 3

How many right angles can you see in each of these drawings?

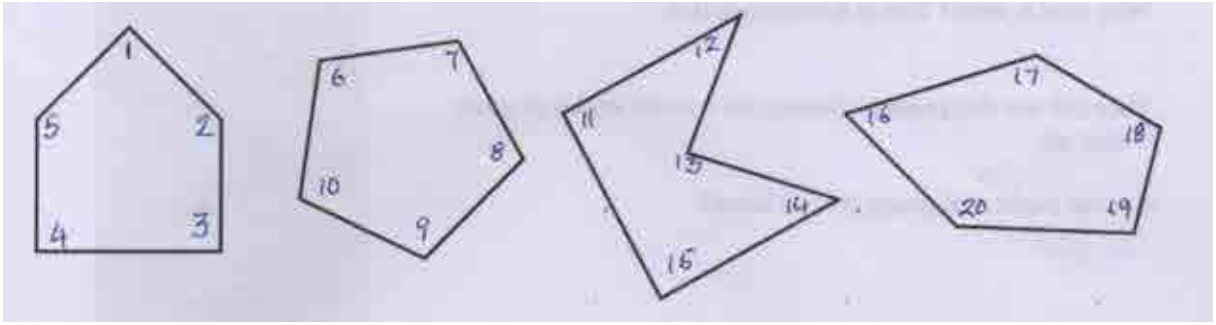




ACTIVITY 7

Objective: Classifying angles of polygons
Materials: Set of polygons with numbered angles

Students can classify the various angles under different categories.

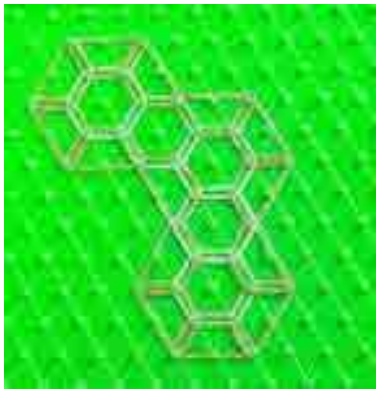


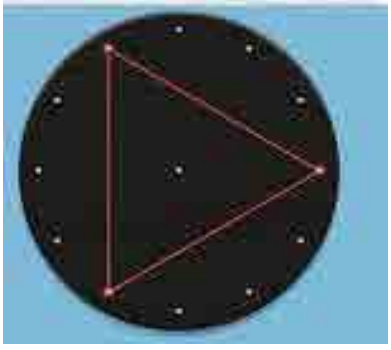
ACTIVITY 8

Objective: Identifying angles on a geoboard (square pin, triangular pin and circular)
Materials: Rubber bands

Students try to demonstrate various angles on the three types of geoboards.

Challenges: Can all angles be shown on each type of geoboard?





ACTIVITY 9

Objective: Identifying angles on a dot paper (square and isometric)

Materials: Dot paper

Students can draw various angles on dot paper, and mark the type of angles each time.

They can explore drawing triangles and see if they can make:

- A triangle with 3 acute angles;
- A triangle with 3 right angles;
- A triangle with 2 acute angles and 1 obtuse angle;
- A triangle with 2 obtuse angles and 1 acute angle, etc.

ACTIVITY 10

Objective: Studying clock angles

Materials: old analogue clock

When does the clock make a straight angle?

How many times during a span of twelve hours do the hands of a clock form a right angle?

If the hour hand points exactly to two what is the angle (clockwise) between the two hands?

If the hour hand points exactly to five what is the angle (clockwise) between the two hands?

GAME

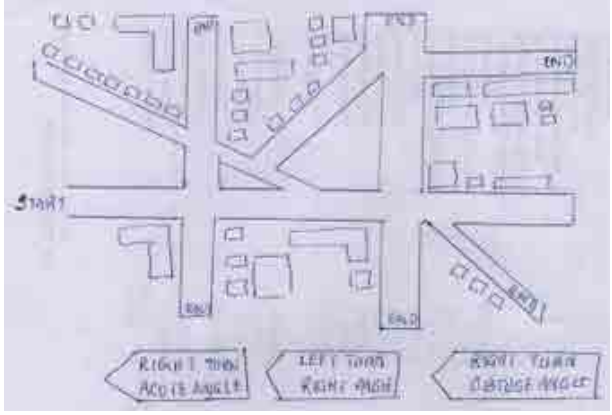
Objective: Angle walk

Materials: Any city map, set of instruction cards indicating the direction of turn and the angle, coloured counter

Each student is dealt a few instruction cards.

Can the student use the instructions on the cards to help move the counter from the starting point to all the endpoints?

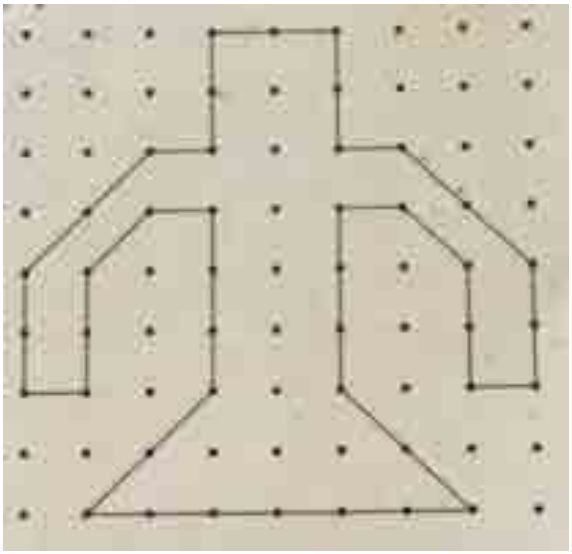
Examples of instruction cards are at the bottom of the map.

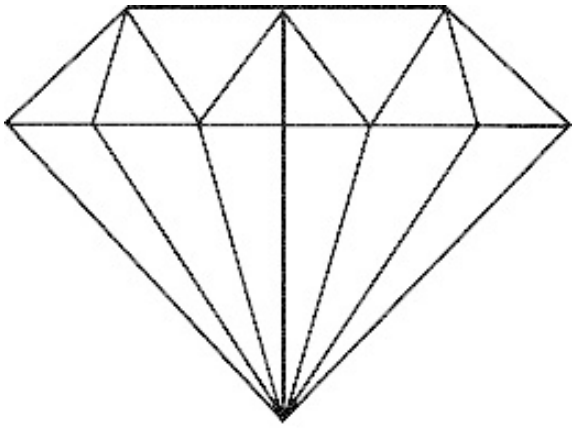


ACTIVITY 11

Objective: Spotting different angles
Materials: designs with lines

How many types of angles can you see in these drawings?





ACTIVITY 12

Objective: Teach the students that we measure a turn in degrees
Materials: Old Clock

Introduce students to the idea of measuring turns in degrees and that a full turn means 360 degrees.

Follow this up with a discussion on half turn (straight angle, 180 degrees), quarter turn (right angle, 90 degrees), three-quarter turn (270 degrees).

By discussing the divisions on a clock, further measures can be recorded.

They can also fix two hands at the centre and show various acute, obtuse angles.

They can fold a circular paper plate into equal divisions and calculate the angle of each portion.

ACTIVITY 13

Objective: Measuring angles with an angle measurer

Materials: Cleaned CD or CD Cover

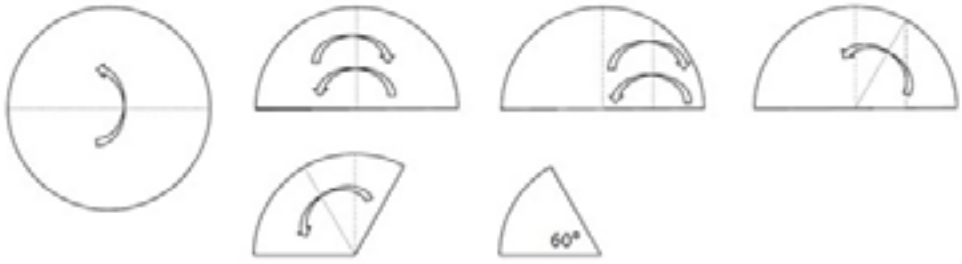
Construction of angle measurer: A transparent CD sheet can be used to create an angle measurer. The centre should be marked and using a right angle paper a mark can be made to denote the positions for 90 degrees, 180 degrees, 270 degrees.

A right angle paper can be folded twice to mark 45 degrees, 135 degrees, etc.

Students can be shown how to create a 60 degree angle which can be folded to make 30 degrees, 15 degrees, etc.

Now we have our own angle measurer with many degrees marked, and we can use it to measure various angles in approximate degrees.

Ensure that students align the wires of the angle measurer with the arms of the angle.



ACTIVITY 14

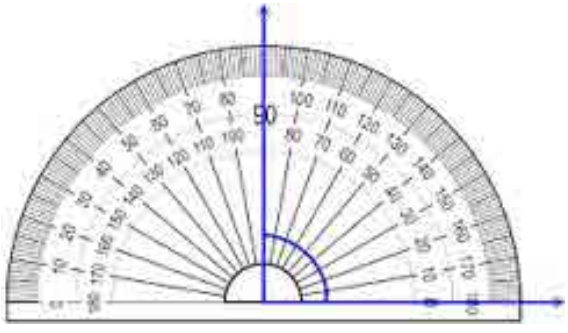
Objective: Teach the use of a protractor to measure angles.

Materials: Protractor

Point out the angles and the degrees that students are already familiar with and show them how a degree is the basic measure of a turn.

However, the fact that a protractor is marked both ways in degrees causes some confusion to students in the initial stages. One way of addressing this problem is to get the students to use only the outer ring for some time by turning the angle drawing if necessary. They need to observe the angle as a turn from 0 degrees while reading the measure.

After gaining some practice with the outer ring, they can gain practice in using the inner ring of measures.



Their prior experiences with a rotagram and angle measurer will help them to understand the idea of a protractor more easily.

Paper protractor by Arvind Gupta

'Paper folding is essentially a geometry laboratory. With a square piece of paper, you can create a protractor with a dozen different angles. This is very empowering because you can fold it anywhere. You can easily see angles of 45, 60, 90, 75, 120, and 150-degrees in your paper protractor.'

http://www.arvindguptatoys.com/toys/protractor.html

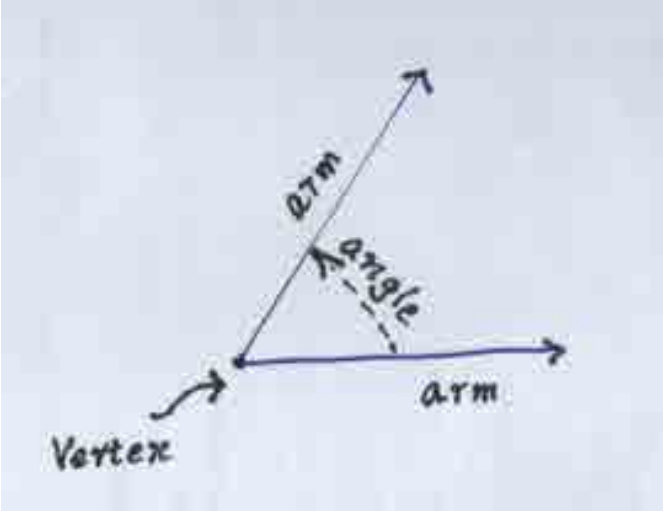
ACTIVITY 15

Objective: Teach the vocabulary

The corner point of an angle is the *Vertex.*

The two straight lines are the *arms.*

The angle is the amount of turn between the arms. It can be either clockwise or anti clockwise.



ACTIVITY 16

Objective: Angles on a straight line add up to 180 degrees
Materials: Two straws

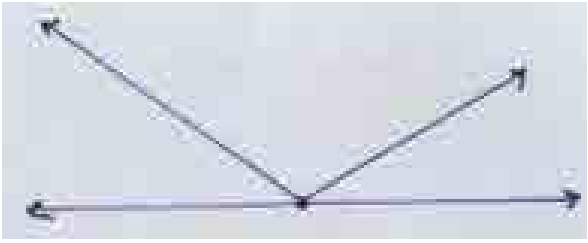
Students can staple the second straw to the centre of the first straw and study the angles formed.

Examine what happens when we rotate the second straw.

Let students draw some straight lines with one or two lines radiating from the centre as shown in the picture.

They can measure the angles and verify the sum.

How does this link with a full turn angle?



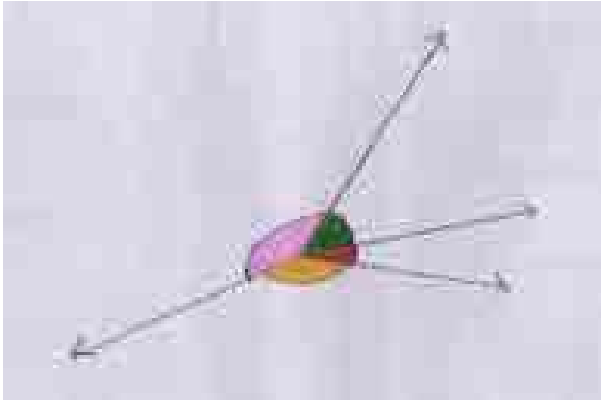
ACTIVITY 17

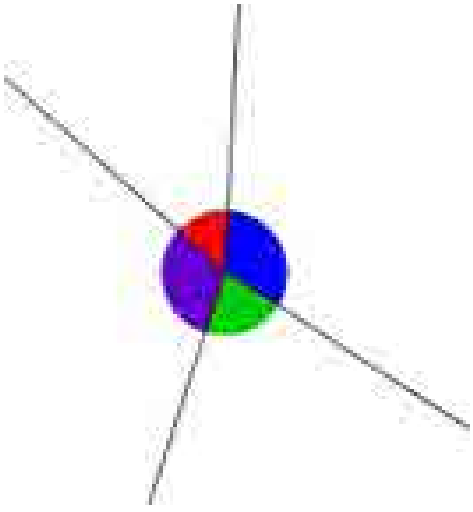
Objective: Angles at a point add up to 360 degrees

Let students draw some straight lines radiating from a point as shown in the picture.

They can measure the angles and verify the sum.

How does this link with a full turn angle?



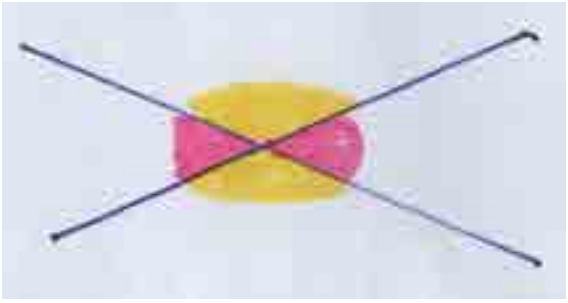


ACTIVITY 18

Objective: Angles of intersecting lines

Let students draw several intersecting pairs of lines, measure the angles, and then discover the relationship between vertically opposite angles.

Why should vertically opposite angles be of the same size?



ACTIVITY 19

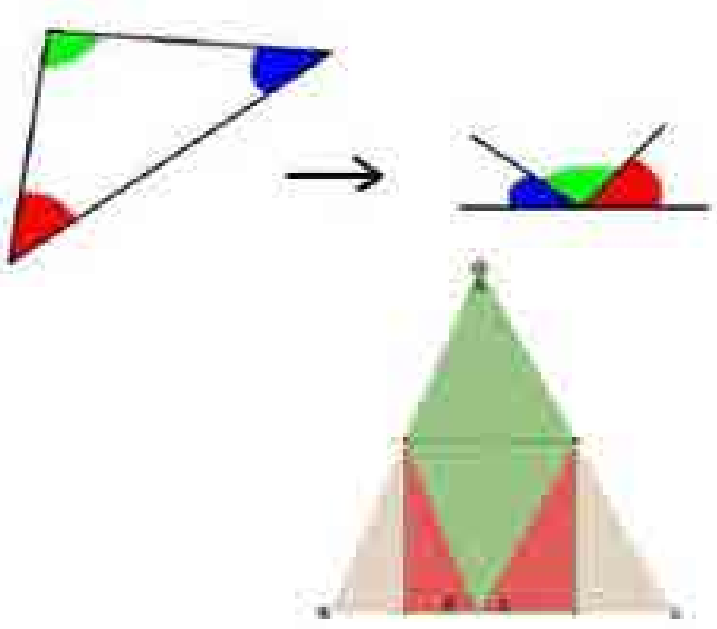
Objective: Angles of a triangle add up to 180 degrees

Materials: Cut triangles

Students can tear the corners and place all the angles on a ruler as shown. The three angles fit on a straight line. Hence, they add up to 180 degrees.

A second way of showing the same property:

The vertex at the top could come down first by a fold parallel to the base; then the other vertices can be folded in snugly to form a rectangle.

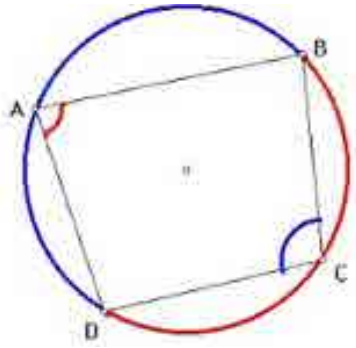


ACTIVITY 20

Objective: Angles of a four-sided figure add up to 360 degrees

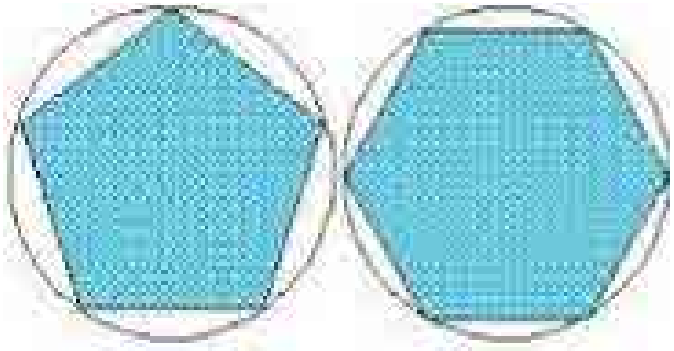
Materials: Circles

Students can draw several circles, mark four points on each of them, and label them as shown in cyclic order. (The use of a circle is to avoid the possibility of the quadrilateral becoming non-convex.) They can then join the points to make four sided shapes. Let them measure the angles and sum them.



What do they find?

Get them to explore further by marking 5 points and 6 points on a circle and measuring the sum of the angles formed.



ACTIVITY 21

Objective: Practice with protractor to make angles

Materials: Protractor, colour pencils

Protractor practice exercises can become very enjoyable if they lead to artwork.

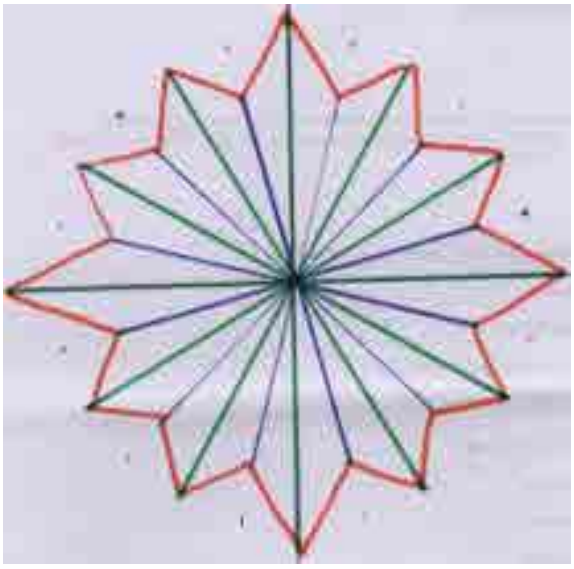
In addition, they have an in built check for accuracy, which makes the teachers' task of correcting easier.

Examples

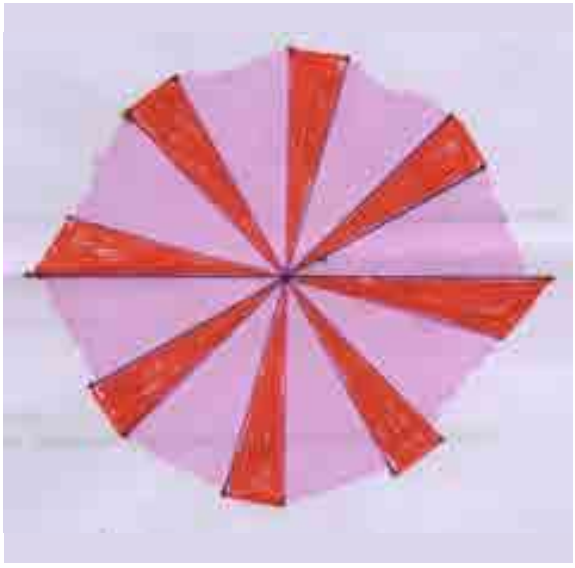
Ex 1: Draw a base line of length 12 cm. From the midpoint, draw an angle of 15 degrees. To give it an interesting effect the length could alternately vary. Repeat the 15 degrees angle several times to make a full turn. Students can join the ends or create some patterns.

Ex. 2: They can explore angle combinations that can add up to a full turn. The second example is a combination of 30 degrees and 15 degrees.

What other combinations can we try?



Example 1



Example 2

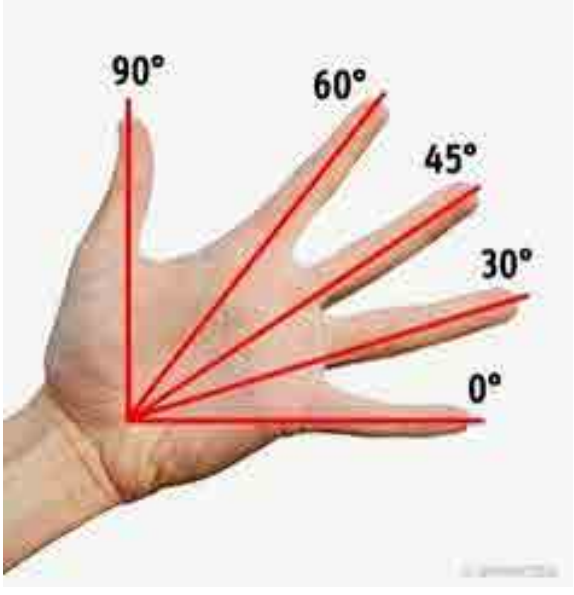
Do the students see that the combination sums have to be a factor of 360 degrees?

ACTIVITY 22

Objective: Angles on a palm

Is it so for your palm? Check.

Can it be used as an estimation measure?



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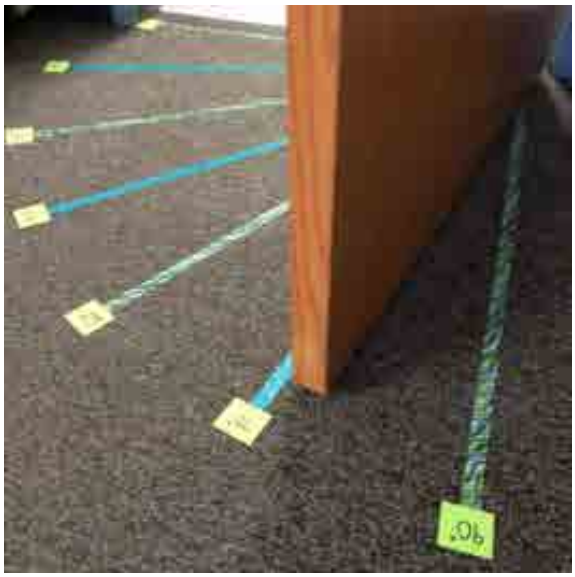
ACTIVITY 23

Objective: Class room door and floor

The floor can also become a bulletin board!



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Acknowledgements

- SMP (School mathematics project)

Links

- http://teachersofindia.org/en/presentation/construction-classroom
- http://www.teachersofindia.org/en/presentation/meet-angles
- http://www.teachersofindia.org/en/presentation/measuring-lengths-line-segments-and-angles

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box' Padmapriya may be contacted at padmapriya.shirali@gmail.com