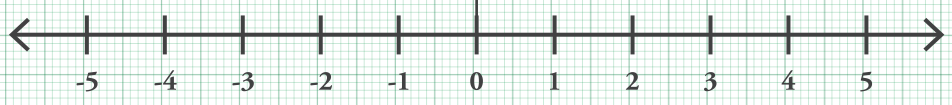
At Right Angles

A Resource for School Mathematics

PERCENTAGES

PADMAPRIYA SHIRALI



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INTRODUCTION

The word ***percentage*** has become part of our everyday language. 'I will come for the party, 100 percent.' 'There is no way I will get selected for the team, 0 percent chance.' 'I don't go to the gym regularly, maybe 50 percent of the time.' We use the word ***percent*** in a loose manner and often only as an approximation. However, through its usage we manage to communicate our intentions.

Many of our daily activities involve calculations involving percentages or comparisons based on percentages. 'How much will a shirt cost after the discount?' 'What is the highest percentage of marks in a subject?' 'What percentage of the school attends football coaching?' And so on.

Percentage is not a new concept. Percent is a form of fraction – a special fraction where the denominator is 100. Technically, percent means 'out of 100' and refers to a hundredth in decimal numbers. It is a way of expressing a number 'out of 100' using the symbol %.

What is the purpose of learning percentages? What does its usage achieve?

It builds the relational thinking of the students; a percentage is used to express how large or small one quantity is relative to another quantity.

Percentage is a topic closely connected with fractions and decimals. Students often encounter this topic after exposure to fractions and decimals. The teacher makes use of the prior knowledge of the students in the areas of fractions and decimal place value while introducing percentages. While solving percentage problems these earlier concepts are revisited and reinforced. Hence it is important to establish the connections between fractions, decimal place value and percentages in multiple ways at the start, so that students understand the linkages thoroughly.

The teaching of percentages incorporates estimation skills and building mental agility and dexterity in converting simple percentages to fractions and vice versa. Students should see the usage of percentage in varied contexts, as visuals and pie charts and should be able to model percentage problems.

I have long used the approach presented here and have found it to work well in developing students' mental arithmetic skills. As the approach proceeds in a gradual step-by-step manner, it focuses on one skill at a time and proceeds to build the connections.

Procedures and usage of notation are taken up at the second stage. Converting complex fractions and decimals to percentage and vice versa is taken up at the third stage.

Before starting on the topic, the teacher can focus on the word and its meaning.

The word 'percent' can be broken up into two parts: per and cent. Cent reminds us of words like century, centimetre, etc. It is common knowledge that a Century stands for 100 years and that 100 centimetres make a metre. 'Cent' stands for 100. The word 'Per' means 'out of.' So percent means 'out of 100,' a quantity expressed out of 100.

Students may be familiar with the word 'centipede.' (However, even though it is called a centipede it does not really have 100 legs!)

ACTIVITY 1

Objective: Build the students' capacity to use 50% in different contexts

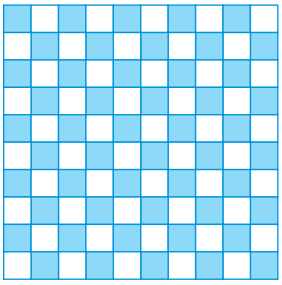
Materials: For demonstration: 100 square grid paper with 50% shaded, clock dial, transparent measuring jars, interlocking cubes, half drawn figures on square grid.

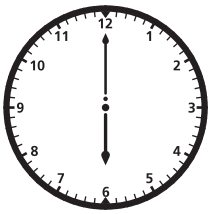
50%

For students: Square papers cut from newspapers, square grid paper sheets.

Show 50% in various contexts.

In a square grid of 100 where both 50 and 100, part and total are clearly visible.



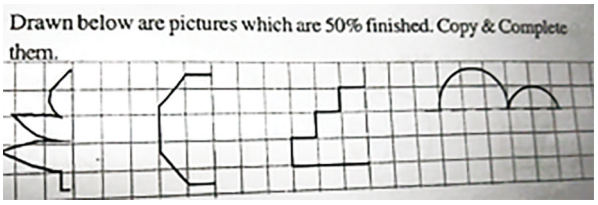


On a clock dial, show 50% of a circle.

Show 50% of a cake.

Show a measuring jar which is half filled with sand or water.

- Ask the students to show 50% of a square paper through paper folding. How many ways can this be done?
- Give them some polygon shapes, ask them to trace the outline and colour 50% of the shape.
- Give them the half-drawn figure cards, ask them to copy and complete.



Do the students grasp the relationship between 50% and ½?

Pose some mental arithmetic questions.

What is 50% of 60?

50% of 30? 50% of 10?

50% of 48? 50% of 124?

You can gradually raise the challenge!
Ask what is 50% of 3? 50% of 17? 50% of 101?

Pose questions where the 50% is given and the students have to find the 100%.

What is 100% if 48 is 50%?

100% if 28 is 50%? 100% if 9 is 50%?

100% if 112 is 50%? 100% if 4008 is 50%?

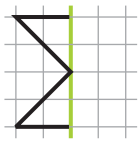
Raise the challenge gradually.

What is 100%, If 50% is 10 ½?

100% if 50% is 2.5? 100% if 50% is 99.5?

Pose some questions based on visuals.

If this shape is 50%, what is 100%?



Use some word problems as oral questions.

Example: There are 12 children left in a classroom. The other 50% are in the library. What is the strength of the class?

ACTIVITY 2

Objective: Build the students' capacity to use 25% in different contexts.

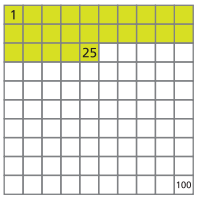
Materials: For teacher demonstration: 100 square grid paper with 25% shaded, clock, transparent measuring jars, interlocking cubes, quarter figures on square grid.

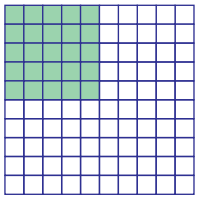
25%

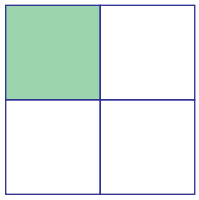
For students:
Square papers cut from newspapers, square grid paper sheets.

Show 25% in various contexts.

In a square grid of 100 where both 25 and 100, part and total are clearly visible.







On a clock face, show 25% of a circle.

Show a measuring jar which is quarter filled with sand or water.

- Ask the students to show 25% of a square paper through paper folding. How many ways can this be done?
- Give them some polygon shapes, ask them to trace the outline and colour 25% of the shape.
- Give them the quarter figure cards, ask them to copy and complete.

Do the students grasp the relationship between 25% and ¼?

Pose some mental arithmetic questions.

What is 25% of 20?
25% of 120? 25% of 96?
25% of 200? 25% of 1000?

What method did the students use to find the answer?

Let students share with the class how they obtained the answers.

Some would have directly calculated ¼ of the number in a single step.

Some would have calculated ½ first and then calculated ½ of the ½.

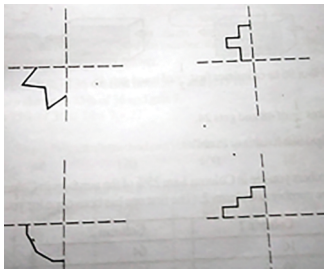
Example: ½ of 120 is 60 and 1/2 of 60 is 30.

Gradually raise the challenge.

Ask what is 25% of 2? 25% of 14? 25% of 25?

Pose some visual questions.

If this shape is 25%, what is 100%?



Pose numerical questions where the 25% is given and the students have to find the 100%.

What is 100% If 16 is 25%? If 30 is 25%?
If 19 is 25%? If 105 is 25%?

Raise the challenge gradually.

What is 100%, If 25% is 12 ½? If 25% is 1¼?
If 25% is 3.2?

If 25% of a number is 60, what is 50% of that number?

ACTIVITY 3

Objective: Build the students' capacity to use 75% in different contexts

Materials: Show 75% on a square grid and on a clock face.

75%

Pose the question What is 75% of 40?'

How do the students respond to it?

If they are able to give the right answer, ask them to explain the method they have used.

If not, pose some additional questions as a hint.

What is 50% of 40? What is 25% of 40?

Some students may have realised that 75% is the same as ¾ and calculated ¾ of 40 in a single step.

Some may have computed 50% first and then 25% and added the two results.

Let the students discuss this and realise that both methods give the same answer.

Pose some mental arithmetic questions.

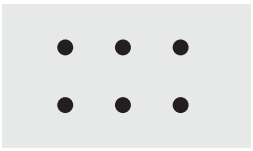
What is 75% of 48? 75% of 120?
75% of 400? 75% of 12?

You can gradually raise the challenge.

Ask what is 75% of 2? 75% of 18?

Pose some visual questions.

If this shape is 75%, what is 100%?



Pose questions where the 75% is given and the students have to find the 25% and 50%.

- If 75% of a number is 12, then what is 25% of that number? What is 50% of that number?
- What is the actual number?
- If 75% of a number is 1.5, then what is 25% of that number? What is 50% of that number?
- What is the actual number?

ACTIVITY 4

Objective: Build students' capacity to use 10% in different contexts

10%

Pose the question 'How will you find 10% of a number?'

By now the students see the relationship between percentage and fractions clearly to be able to state that 10% is the same as 1/10 of a whole.

Finding 10% is the same as dividing the number by 10.

Ask 'what happens to the digits of a number when you divide by 10?' Do the students see that when you divide a number by 10, each digit shifts by one place to the right? From a tens place, it shifts to the units place. From a units place, it shifts to the tenths place. And so on.

What is 10% of 60? 10% of 500?
10% of 45? 10% of 2?

If 5 is 10% of a number, what is the number?

What is 10% of 100?
10% of 200?
10% of 300?

Do the students see the pattern?

ACTIVITY 5

Objective: Build students' capacity to use 20% in different contexts

20%

Pose the question 'What is 20% of 80?' How do the students respond to it?

Some students may have realised that 20% is the same as ⅕ and calculated ⅕ of 80 in a single step.

Some may have computed 10% first and then doubled the result.

Has anyone tried any other method?

Let the students discuss this and realise that there are different ways of solving the problem.

Pose some mental arithmetic questions.

What is 20% of 40? 20% of 120?
20% of 300?

If 10% of a number is 5,
what is 100%? What is 50%?

If 20% of a number is 4,
what is 100%? What is 60%?

ACTIVITY 6

Objective: Students to develop methods to calculate 5%

5%

Ask the students if they can develop a method for computing 5% of a number. Discuss the methods they develop.

How will they compute 1% of a number?

What happens to the digits of a number when the number is divided by 100?

How will they compute ½ % of a number?

Introduction of Symbol: Teacher can now introduce the symbol % for percentage and show that a whole stands for 100%.

ACTIVITY 7

Objective: To reinforce that a whole represents 100%

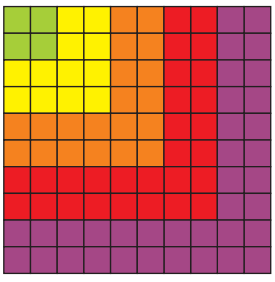
Materials: 100 square grid, Circle divisions.

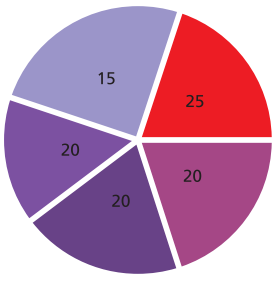
100%

To reinforce that parts of a whole add up to 100%

Students to make a coloured design in a 100 square grid and write the percentage of each colour.

Let them total the percentages of all the colours and notice that the percentage figures of the different colours add up to 100%.





ACTIVITY 8

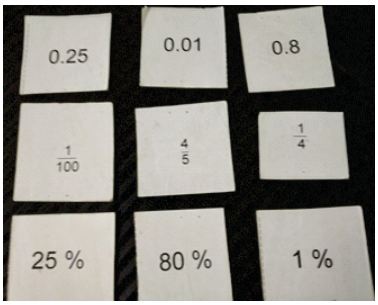
Objective: To create sets of matching cards

Materials: Percentage kit consisting of equivalent percentage, fraction and decimal cards

The kit can be used for multiple activities. Show the percentage card and have students pick up the matching fraction and decimal card.

Show the fraction card and have the students pick up the matching percentage card.

Give two percentage cards and have the students find a fraction or decimal card which lies between the two percentages.

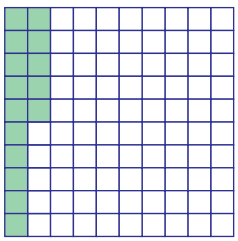


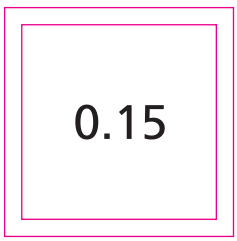
GAME 1: Four Set!

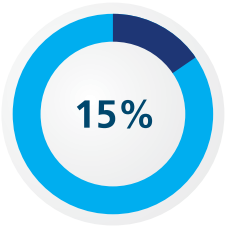
Objective: To make a full set of equivalent cards.

Materials: Percentage Kit: Several sets of equivalent percentage, fraction, decimal and visual cards

No. of players: 4







$$\frac{15}{100}$$

Four set is a card game in which you try to improve the hand that you have been originally dealt. You can do this whenever it is your turn to play, either by drawing cards from a pile (or stock) or by picking up the card thrown down by the opponent and then discarding a card from your hand.

It is quite similar to the game of Rummy (for those who are familiar with this game).

Distribute the cards to students so that each student gets 4 cards. Leave one card open and place the remaining cards in the centre, face down. Each student is allowed to pick up the topmost card from the pile or the card just thrown down by the opponent. Each time he picks up a card, he has to discard one card. The second player may pick up the discarded card or pick up a new card from the pile. This continues until one of the players manages to get a full equivalent set.

If all the cards in the closed pile finish, they can shuffle the cards left on the ground, pile them face down and repeat the process. And so on.

ACTIVITY 9

Objective: Converting fractions with denominators which are factors of 100 to percentages.

While using the percentage kit, students would have already come across how percentages are represented by fractions which are reduced to the lowest form. Example: 50% is 50/100, i.e., ½.

The teacher can reinforce this using several examples. She need not take up fractional or decimal percentages at this point. That can come later, once the percentage concept has been thoroughly understood.

The teacher begins with fractions where the denominator is a factor of 100.

Example: ⅖, ¾, 7/10, 11/20, etc.

Students are familiar with equivalent fractions and they will be able to write:

⅖ = 40/100 = 40%

¾ = 75/100 = 75%

7/10 = 70/100 = 70%

11/20 = 55/100 = 55%

ACTIVITY 10

Objective: Converting fractions to percentages. Converting decimals to percentages.

Discuss the method the students used for fractions such as 2/5 or 3/10 to generate the fractions 40/100, 30/100, etc.

Help them notice that they multiplied the numerator and denominator by the same number to get a fraction with denominator 100.

Point out that multiplying numerator and denominator by the same number does not change the value of the fraction.

Now ask the students to multiply 2/5 (the numerator and denominator) by 100.

- It will be ⅖ × 100/100. By 100 (/100) can be replaced by % symbol as the symbol stands for /100.
- It can now be written as ⅖ × 100%.
- Now they can do cancellations to simplify the answer.
- Teacher can now explain to the students that converting fractions to percentage can be done by multiplying by 100% (which is the same as 100/100).
- ¾ × 100/100 is the same as ¾ × 100% = 75%

Note: Most textbooks and teachers ask students to multiply the fraction by 100. This is incorrect as it alters the value of the fraction, whereas multiplying by 100% is the same as multiplying by 1 which does not alter the value of a fraction.

Students can now use the method of multiplying by 100% to convert fractions to percentages.

Note: The teacher can show that the same method of multiplying by 100% works for converting decimals to percentages.

ACTIVITY 11

Objective: Calculating given percentages of a quantity.

Students can prepare a set of matching cards for practice.

Teacher can create a grid to be filled in with answer cards. Students can note down the time they take to complete a grid.

	120	1080	1800	640
10%				
50%				
25%				
75%				
90%				

GAME 2: Percentage dominoes

Materials: Domino cards

Objective: Matching fractions and percentages

Prepare a set of domino cards as shown.

Let students arrange the matching cards to complete the domino train.

ACTIVITY 12

Objective: Given the percentage of a number to find the original quantity.

Questions can be posed in the following manner and the students figure out the original quantity.

40% of a number is 64. What is the number?

30% of a number is 27. What is the number?

ACTIVITY 13

Objective: Understanding of percentages that are more than 100.

Materials: Cubes or buttons.

Understand that a percentage can be zero

Modelling percentages

Call 6 students. Give 8 cubes to each student.

Tell the first student to increase the cubes by 50%. The student will take 4 more.

Tell the second student to increase the cubes by 25%. The student will take 2 more.

- Ask the third student to increase the cubes by 75%. The student will take 6 more.
- Ask the fourth student to increase the cubes by 100%. The student will take 8 more.
- Ask the fifth student to increase the cubes by 200%. What should the student do?
- Ask the sixth student to increase the cubes by 0%. What will the student do?

Discuss various percentages greater than 100 like 150%, 250%, 300% and so on.

Discuss some real life examples.

I planned to finish 10 problems today. I did 15 problems. This was 150% of my plan!

I expected the meal to cost ₹ 60. But it cost ₹ 120. This was 200% of the expected price.

ACTIVITY 14

Objective: Percentage increase/decrease through modelling

Materials: Shapes, cubes, problem cards

Pose problems that require application of percentage increase and decrease using modelling.

Example: Yagya ate 4 strawberries yesterday. Today he ate 75% more than yesterday. How many strawberries did he eat today?

Let students pair up, share their understanding of the problems and explain their thinking.

They can model the problem using materials.

They could also represent large numbers on a graph which will aid their visual appreciation.

- 450 people watched a film on the first day. On the second day the number of people who watched the film is 10% less than the number who watched on the first day. How many watched on the second day?
- If 150 decreased to 120, by what percentage has it decreased?
- The number of students in the school today is 744. That is 7% less than the number present in the school yesterday. How many were present yesterday?

ACTIVITY 15

Objective: Estimations

Materials: Problem cards

Estimations in word problems

Students should be encouraged to use estimation while solving problems. Again, the teacher can give some problem cards to students and ask them to work in pairs.

Have students explain in pairs how they would estimate the following.

9% of 140 (10% of 140 is 14, so 9% would be less than 14)

14% of 180 (10% is 18, 5% is 9, so 15% is 18 + 9 = 27, so 14 percent is around 25)

26% of 320 (25% is 80 so 26% will be around 84)

In a classroom, 7 of the 49 students like movies based on true stories. What percentage is this?

First ask students to estimate a percentage.

(7/49 = 1/7

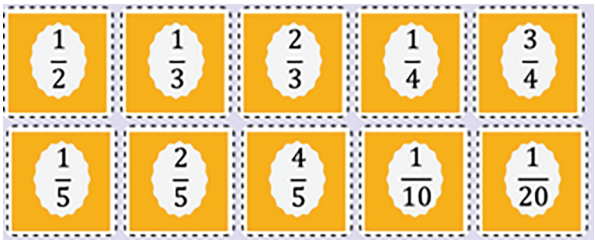
1/5 is the same as 20%, and 1/8 is the same as 12.5%

So 1/7 will be in-between 12.5% and 20% but closer to12.5%)

ACTIVITY 16

Objective: Practice in fraction, decimal and percentage conversions (including mixed fractions)

Materials: Matching kit with common percentage and fraction cards.



Let students get plenty of practice through matching activities.

Both the fractions and percentage cards given above can be modified to include mixed fractions and corresponding percentages. Ex. 1 ½, 150%, 2 1/4, 225%.

Provide some tables as shown for students to practice conversions.

Fraction	Decimal	Percentage
$\frac{1}{2}$		
	0.75	
		30%
$\frac{3}{100}$		
		$33\frac{1}{3}\%$
	0.625	

The teacher can collect clippings from newspapers of usage of percentages in real life and discuss them.

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com