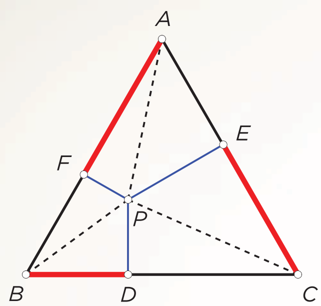
LETTER

Cousin to Viviani's Theorem = Clough's Conjecture = Clough's Theorem

In Issue-I-2 of *At Right Angles* we had presented an account of Viviani's theorem, proved it using vector algebra, and found that the proof gave rise to a corollary in an unexpected and yet very natural way. We called it a 'cousin' to Viviani's theorem:



Let ΔABC be equilateral with side a, and let P be a point in its interior. Let perpendiculars PD, PE, PF be dropped to sides BC, CA, AB respectively. Then $BD+CE+AF = 3a/2$ for all positions of P.

It turns out that this result has been known for a decade, and has a curious history behind it. In the literature it is known as *Clough's Conjecture*. We came to know this through a letter received from Professor Michael de Villiers of the Department of Math Education, University of KwaZulu-Natal, South Africa. He refers us to a paper of his, "An example of the explanatory and discovery function of proof". It was presented at ICME 12 and has now been published in the online journal 'Pythagoras' at: http://www.pythagoras.org.za/index.php/pythagoras/article/view/193.

Readers are urged to download this very readable paper and learn how the result was discovered *empirically* by Duncan Clough, a Cape Town grade 11 student, during a dynamic geometry session in which the students were exploring Viviani's theorem and attempting to prove it; he reported it to his teacher Marcus Bizony, who wrote to de Villiers; and that's how it got the name "Clough's Conjecture" (but it is now a theorem, proved by de Villiers himself). In the paper, the author notes that the incident provides an illustration of the fact that the search for proof sometimes uncovers new results. This is the central thesis of the paper, and it is a matter worth dwelling on as it has important pedagogic implications. He also provides a few proofs of the theorem, shows that it follows from the main Viviani theorem, and deduces some extensions, e.g., to a rhombus and to an equi-angular pentagon.

Many thanks to Prof de Villiers for this communication.

— *The Editors*