in the classroom

Towards mathematical disposition

Notes from a small school

Supportive learning spaces

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If there is one thing mathematicians or math educators are agreed upon, it is that the state of math education the world over is very unsatisfactory. Many schools have poor infrastructure and often an absent teacher. Even in schools with good infrastructure and teachers present, the curriculum is often dry and unimaginative and the textbooks more so. Significant numbers of teachers have a poor understanding of their subject, and are often burdened with large or mixed classes, and administrative duties. Not surprisingly, they seem unmotivated and uninterested in creative solutions. Vast numbers of students dread mathematics. The fear associated with learning mathematics often persists into adulthood. Moreover, many students do not achieve minimum learning standards.

What is more disappointing is that the motivation behind most attempts to reform math education is to create mathematically competent humans who will become part of a 'knowledge society' whose goal is to compete economically with other knowledgeable societies! This approach has not solved anything.

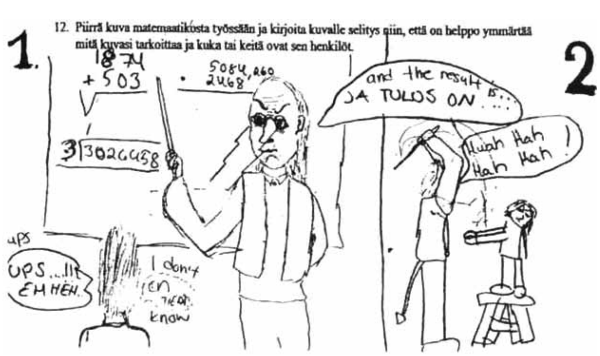
Though the global picture is depressing, at a local scale the situation can be completely different! In this article, I would like to share with you our experience at Centre For Learning in creating an environment where children enjoy learning mathematics.

Centre For Learning (www.cfl.in) is a small school in Bangalore started in 1990 by a group of educators interested in the nature of true learning in all its aspects. Based on my experience as a teacher of mathematics for the last 26 years, 17 of them at CFL, I can say with some certainty that it is possible to create a learning environment where children develop a love for mathematics, and a conceptual understanding that goes beyond textbook problems. Don't misunderstand me – we are not churning out mathematicians by the dozen! My point is the following: I firmly believe that if we are going to make mathematics a core subject in primary education, then we owe it to our students that they find their experience of learning both meaningful and enjoyable.

What is an enabling environment for the learning of mathematics?

At CFL we believe that for meaningful education it is important that we create a space where learning is not motivated by fear, competition, reward and punishment.

Unfortunately the word 'fear' has become indelibly linked with math learning, and the term 'math phobia' has become part of common parlance. A study published in 2000 by Susan Picker and John Berry entitled "Investigating Pupils' Images of Mathematicians" brings home the urgency of the problem. The researchers asked 12- to 13-year old children from the US, UK, Finland, Sweden and Romania to draw a picture of "a mathematician at work". The images are graphic. It comes across clearly that the children have no clue what mathematicians do for a living, but their stereotypical images of mathematics teachers are more damning! They experience themselves as helpless, their mathematics teachers as authoritarian and intimidating, and the learning process as highly coercive.



Source: [1]. Reproduced by kind permission from Dr Susan Picker and Dr John Berry

I am sure we can find countless stories from our contexts where children experience learning mathematics as traumatic.

Given that fear and learning do not and must not go together, we feel that there has to be an environment of learning where the relationship between teacher and student is based on mutual trust and affection. It is important that the child's self-worth is not linked to intellectual ability. These conditions are absolutely necessary because when the opposite prevails—when fear and competition are the main tools of motivation—they do great harm to children and create uncaring and dysfunctional societies.

Source: [1]. Reproduced by kind permission from Dr Susan Picker and Dr John Berry.

Creating a space where fear is not a motivating factor does not automatically eliminate fear in the child. Far from ignoring it, we tackle fear head-on. Students are encouraged to be aware of their fears, to express them and to observe how they may be impacting their learning. In looking for the roots of this insecurity, one may find that it arises because the student has linked his self-worth with the ability to perform. Often it arises because the student isn't confident about his understanding. This can be addressed by the teacher putting more energy and imagination into explaining, and also the student himself working at it.

Doing mathematics can make us acutely conscious of ourselves. It gives us constant feedback about how 'intelligent' we are. This is heightened in a society where ability to calculate quickly is equated with intelligence! Therefore, we need to dialogue not only about fear but also one's images of oneself as a learner. In a supportive environment, a student can recognise the reactions and emotions that block her learning, while at the same time coming to terms with her own strengths and weaknesses. The emphasis thus shifts from performance and self-worth to learning and self-understanding. This allows children to acquire meta-cognitive and self-regulatory skills, two important ingredients in an educational programme, which I will return to later in the article.

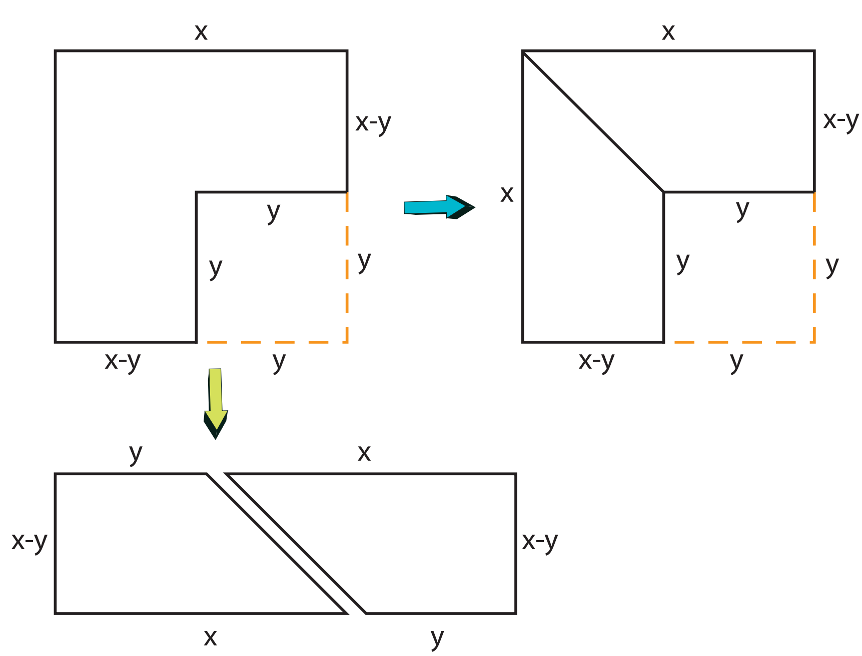
Creating the right environment for learning is necessary, but not sufficient, in meeting the challenges that the learning of mathematics throws up. We have to understand the underlying beliefs and attitudes that teachers and children have about mathematics, and what it takes to become mathematically competent. Let us look at beliefs to begin with.

Epistemological beliefs

Since the 1980s (see [2]) many researchers have studied the link between beliefs and competence in mathematics, so much so that positive beliefs are listed as a criterion for mathematical competence. It would be a very interesting exercise for teachers to jot down their own beliefs about the nature of the subject and why they are teaching it. Let me state the overarching ideas at CFL about the nature and teaching of mathematics.

Mathematics is deep and beautiful, and children should get a taste of this and experience the joy of understanding concepts and the pleasure of making connections. Mathematics can be viewed in many ways: as an art form, as the language of nature, as a tool to model our environment, as a tool for bookkeeping in the world of commerce. Children should be exposed to these different aspects, and no one view should dominate. While problem solving is an important part, there is more to it than that. Children should be exposed to theory building alongside problem solving. Mathematics is not just a body of knowledge but a lively activity consisting of recognizing patterns, making conjectures, and proving the conjectures. Children should learn how to play with ideas and patterns and learn to represent their recognition of these patterns using mathematical notation.

Let me illustrate this with an example.
When teaching the identity $x^2 - y^2 = (x-y)(x+y)$, we can conduct the following simple investigation. Start by asking them to compute the difference of squares $x^2 - y^2$, with $x - y = 1$. Under these conditions, students will soon see that $x^2 - y^2 = x + y$. Do not forget to ask them to state the conditions on x and y! Then ask them to compute the difference of squares $x^2 - y^2$, with $x - y = 2$. Students will soon see that $x^2 - y^2$ in this case is $2(x + y)$. Again, ask them to state the conditions on x and y. Continue to compute difference of squares by changing the value of $x - y$. They will then discover in general that $x^2 - y^2 = (x - y)(x + y)$. One can further reinforce this concept with the following geometric 'proof' (students can be asked to come up their own geometric 'proof').



There are many myths about mathematics that need to be constantly dispelled: the teacher knows everything, mathematics is all about 'calculating', there is only one way to do a problem, if I am not good at mathematics then I must be stupid, and mathematics does not afford experimentation and exploration.

Mathematical Disposition

Once we are clear about an enabling environment and epistemological beliefs, we must be clear what it means when we want our students to be mathematically competent. It is best to look at what math education research has to say in this regard. I have taken the liberty to edit an excerpt from [2] for brevity.

There is currently a consensus among scholars in the field of mathematics education that becoming competent in mathematics can be conceived of as acquiring a ***mathematical disposition****. Building up and mastering such a disposition requires the acquisition of five abilities:*

1. *A well-organized knowledge base involving the facts, symbols, algorithms, concepts, and rules of mathematics*
2. *Heuristic methods, i.e., search strategies for problem solving, which increase the probability of finding the correct solution: for instance, decomposing a problem into sub goals.*
3. *Meta-knowledge, about one's cognitive functioning on the one hand, and about one's motivation and emotions on the other hand (e.g., becoming aware of one's fear of failure when confronted with a complex mathematical task or problem).*
4. *Positive mathematics-related beliefs, about mathematics education, about the self as a learner of mathematics, and about the social context of the mathematics classroom.*
5. *Self-regulatory skills, i.e., one's cognitive processes (planning and monitoring one's problem-solving processes) on the one hand, and skills for regulating one's volitional processes/activities on the other hand (keeping up one's attention and motivation to solve a given problem).*

Categories 1 and 2 have to do with curricular content and delivery; the rest have more to do with attitude and a culture of learning. Experience tells us that some students will achieve much of this mathematical disposition *in spite of* their learning environment! However, our goal, as I mentioned in the introduction, is to help *all* children enjoy the process of acquiring competence in mathematics, and to impart an education concerned with more than acquisition of skills. For this to happen, we consider it vitally important that we create the right learning environment, understand our belief systems, have a coherent curriculum, choose appropriate teaching materials, and pay attention to the process involved in acquiring mathematical competence.

Acquiring Mathematical Disposition

Some of you may say, "All this theory is fine; tell me what happens in the classroom". In our classroom practice, every attempt is made to demonstrate that mathematics is a human endeavour. This is done by talking about the history of mathematics and stories of mathematicians, trying to discover why humans might have needed/developed the mathematics being taught. The classroom environment is kept light, yet rigour is not sacrificed for informality. The students' collective attention has to be steadily focused on what is being learned, the teacher keeping track of each child as the lesson progresses. As and when possible, the teacher will try and connect what appear to be different parts of mathematics, so that the child's learning is not compartmentalised. Teachers spend a significant amount of class time explaining concepts, and children are often called upon to articulate what they have learnt, as far as possible in precise language.

"*... to achieve successful mathematical understanding, we must go beyond telling children how to solve mathematical problems; we must reach a point where children are not only successfully producing mathematical solutions but also understanding why the procedures work and when the procedures are and are not applicable. This point may be reached by providing children with, and requiring that they contribute, to adequate explanations in their mathematics classrooms*." - Michelle Perry [3]

Students speak as much as, if not more than, the teacher. They spontaneously explain to each other what they have learned, and answer each other's questions. Some comments may seem tangential or even irrelevant to a discussion, but, if followed up, often yield unexpected connections and ways of understanding. The student who finds math easiest is not the star of the math class! Everyone feels equally important in class, in terms of attention, appreciation and affection. Students often work in groups and learn cooperatively, making mathematics a social activity. They engage in thinking together in solving problems and help each other to build the solution without a sense of competition. Along with written work, students do projects, engage in 'thinking stories' and play mathematical games. Mathematics is also part of the overall consciousness of the school, with whole school presentations in mathematics by students and experts..

A teacher burdened with the pressure to complete a syllabus may wonder how this can be done. I think the key here is that, when the emphasis is on understanding material and helping children articulate their learning, though the process in the beginning may take time, once such a culture is established, teachers find that children master many concepts quite easily and so called 'lost' time can easily be made up. In fact, some topics (the more formula oriented or algorithmic ones) can be mastered by students on their own once they are confident about their learning.

Not everything that students do in the mathematics class is formally assessed. In the next section I discuss assessment at CFL in more detail.

Assessment at CFL

At CFL we have children from age 6 to about 18. The teacher to student ratio is 1: 8 at younger ages and in the senior school 1: 4. During the course of their school year they are not subject to exams, quizzes, surprise tests or terminal exams, except at the end of the 10th and 12th standards, when they appear for the IGCSE and A-levels conducted by Cambridge International Examinations.

So then, how does assessment happen at CFL?

First of all we save a tremendous amount of learning time because we don't spend it in preparing, administering and correcting exams. In a small class, teachers are aware of the level of understanding of each student as well as other markers of learning. What has she mastered well, what does she need to work on? What are her study habits, what does she resist? Teachers have also learned how to break down concepts into various components and to figure out difficulties that students have with them. As mentioned earlier, a lot of the teaching consists of discussion, so students receive feedback from each other too. Discussions are of great value, since a teacher's greatest challenge is to enter into and understand a student's world.

We do have written work in the form of assignments; these are corrected but not graded with marks. So students are more focussed on what they have and have not learnt. They are not bothered if their peers are doing better; it is difficult for them to make such a judgment! In correcting and giving detailed feedback, both teacher and student can take corrective action in real time during the course of teaching rather than waiting for the end of a term or a year. So-called mistakes are of tremendous value, because they help understand how a student is thinking, what are the mistaken assumptions, what are the gaps in basic skills and so on.

One area we are working on is to come up with a more quantitative way of recording our observations to pass on to parents, students and teachers. Currently these are communicated in detailed descriptive reports, and face-to-face conversations with parents. We see that we can improve on this. One reason for a lack of urgency in this regard is that we have been able to demonstrate that students can and do learn without comparative and summative assessment and actually do quite well in the school-leaving exams (for example, at the IGCSE so far 83% of the students have a B or higher and at the A-levels 45% get a B or higher)!

Our Challenges

Like all educational environments we face challenges. In fact, when external motivators such as fear, competition, reward and punishment are removed, educators confront the real issues of education. Despite all the thought put into our learning environment, we still encounter resistance to learning. This problem is a human predicament (all of us face it), and our question is how to address it without resorting to the usual tricks.

One question we often ask is: are we adequately challenging the student who is 'gifted' in mathematics? These students enjoy our basic mathematics programme and retain their love and sharp mind for mathematics in high school and beyond. They especially enjoy the projects where they feel stretched, since there is the scope to take on harder challenges. But there is no doubt that such students could have done more or gone much further.

Finally, the biggest challenge: the learner is not a blank disc on which all knowledge can be burned! The learner influences his own learning. Despite a conducive environment and the best of efforts, the learner's complexities (attitudinal, motivational, emotional) can limit his or her learning.

References

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