in the classroom

Thoughts on the Division Operation

Notes from a Reader

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Among the four basic arithmetical operations, division is easily the most complex, and it is the one with which children have the most difficulty. In this regard, I would like to share the following thoughts with readers on how division can be taught. They may be regarded as alternatives to the way division was presented in the pullout of an earlier issue. In particular, the following questions have been addressed:

1. Why do we proceed from left to right in division whereas all the other operations proceed from right to left?
2. What is the role of place value in the standard division algorithm?
3. How do we estimate the quotient when the divisor has two or more digits?

Points 1 and 2 are connected. The argument I use with students is that it is just a matter of efficiency. In the case of addition and subtraction, a left to right algorithm will not be efficient, as illustrated below.

Left to right				
Step 1		T	U	We start by adding the tens.
		5	7	
	+	3	9	
		8		
Step 2		T	U	Next we add the units and write the unit digit of their sum and the tens digit of the sum separately.
		1		
		5	7	
	+	3	9	
		8	6	
Step 3		T	U	So a 3rd step requires **changing** the already written digit in the tens place.
		1		
		5	7	
	+	3	9	
		9	6	

Right to left				
Step 1		T	U	We start by adding the units and write the units digit of the sum and the tens digit of the sum separately.
		1		
		5	7	
	+	3	9	
			6	
Step 2		T	U	In the 2nd step we add up all the tens.
		1		
		5	7	
	+	3	9	
		9	6	
Step 3				Here we **didn't have to change** any digit in the answer.

A similar situation arises in subtraction as well. But because of the variety of cases which arise, I have doubts on whether to teach or even demonstrate this to children.

For division a right to left algorithm is not the most efficient one, as shown below.

Right to left					
Step 1			T	U	Let's start with the units, i.e. we divide 6 by 2 
				3	
	2	)	3	6	
				6	
Step 2			T	U	 Now we try to divide the 3 tens We are left with 1 ten
			1	3	
	2	)	3	6	
				6	
			3		
			2		
			1		

Left to right					
Step 1			T	U	 We start with the tens, i.e. we divide 3 by 2 and we are left with 1 ten
			1		
	2	)	3	6	
			2		
			1		
Step 2			T	U	 Now we exchange this ten to 10 units and add the 6 units from the dividend to get 16 units
			1		
	2	)	3	6	
			2		
			1	6	

Right to left — Step 3

		T	U
		1	8
2	)	3	6
			6
		3	
		2	
		1	0
		1	0

So we have to exchange this remaining ten into 10 units

Then we distribute these 10 units

∴ we have to increase the unit's digit by 5 i.e. **change** it from 3 to 3 + 5 = 8

Left to right — Step 3

		T	U
		1	8
2	)	3	6
		2	
		1	6
		1	6

Now we divide these 16 units by 2. Here again we **didn't have to change** any digit in the answer (or quotient).

Note: The exchange of units is not obvious in the algorithm and students benefit if the teacher demonstrates the conversion from tens to units by using material manipulatives such as flats and longs or bundles of ten.

So if we show these to students or, even better, get them to work out both ways – left to right and right to left – and then reflect on the efficiency of each method, they will have a much better understanding of why the standard algorithms are the way they are.

I have tried to document how place value is invoked in the standard division algorithm with a detailed step by step demo of both long and short methods simultaneously showing 8643 ÷ 7 at http://teachersofindia.org/en/video/division-pay-attention.

When the division algorithm is taught, we generally use the shorter version of the division algorithm that violates the subtraction rules that a child has learnt.

Longer version — Step 1

		T	U
		1	0
2	)	3	6
		2	0
		1	6

Writing the complete 20 doesn't violate the subtraction rule and we are actually subtracting 2 tens or 20*

Shorter version — Step 1

		T	U
		1	
2	)	3	6
		2	
		1	

Writing 20 as just 2 and then subtracting it from 36 is confusing since children may not realize they are subtracting 2 tens from 3 tens

Longer version — Step 2

		T	U
		1	8
2	)	3	6
		2	0
		1	6
		1	6
			0

Here we need to **change** the unit from 0 to 8. However, if this is shown with arrow cards, then this problem can be resolved.

Shorter version — Step 2

		T	U
		1	8
2	)	3	6
		2	
		1	6
		1	6
			0

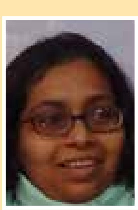
Here we **don't need to change** the unit. This is why we don't write the unit in step 1.

Point 3 is important since division is the only place where the standard algorithm requires estimation. For addition, subtraction and multiplication, no matter how large the numbers are, there is no need for estimation.

The strategy is multi-pronged as rounding can go either way. So we need to start with 2 digit divisors and round them to the nearest tens in order to estimate. Naturally, it is a good idea to give children enough practice with divisors that are multiples of 10, e.g. 40, 70 etc., before going into general 2 digit divisors. Next, if the divisor is 62, we can round it to 60 and look for the highest multiples less than the dividend in each step. Similarly, if the divisor is 37, it should be rounded to 40. Of course, numbers ending in 5 like 85 are always tricky – you can use either 80 or 90. It is also important to highlight here that the difference at each step should be less than the divisor. The estimation steps are as follows:

Estimation steps	Example: 256 ÷ 36	Example: 256 ÷ 33
1. Round off divisor to the nearest multiple of 10	36 rounded off to 40	33 rounded off to 30
2. Estimate quotient (or quotient digit) at that step using the estimate	256 ÷ 40 (or 25 ÷ 4) ≈ 6	256 ÷ 30 (or 25 ÷ 3) ≈ 8
3. Calculate quotient digit × actual divisor	6 × 36 = 216	8 × 33 = 264
4. Check a. if quotient digit × divisor > dividend: decrease quotient digit by 1 and repeat step 3 b. if not, proceed to check dividend – quotient digit × divisor > divisor: increase quotient digit by 1 and repeat step 3 c. If dividend – quotient digit × divisor ≤ divisor proceed to step 5	Check: a. 216 < 256 b. 256 – 216 = 40 > 36 ⇒ quotient = 6 + 1 = 7 Go back to step 3 using 7 as the quotient digit	Check: a. 264 > 256 ⇒ quotient digit= 8 – 1 = 7 Go back to step 3 using 7 as the quotient digit
5. Complete division step with the (modified) quotient	7 × 36 = 252, i.e. 256 ÷ 36 = 7 remainder 4	7 × 33 = 231, i.e. 256 ÷ 33 = 7 remainder 25

I welcome suggestions from readers.



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