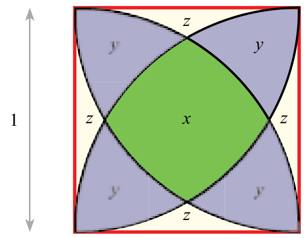
A Flower with Four Petals

Shown below is a square with side 1 unit, with four circular arcs drawn within it, each with radius 1 unit and centred at the four vertices of the square. The four arcs demarcate a region in the centre of the square, shown coloured green. The problem we pose is to find the area x of this region.



To find x we use a method that may remind you of Venn diagram computations. We start by assigning symbols to the areas of the other regions. The four regions marked y are clearly congruent to each other, as are the four regions marked z. Let their areas be denoted by the same symbols (y and z). Then the following relations are immediate:

$$y + 2z = 1 - \frac{\pi}{4}, \tag{1}$$

$$x + 2y = 1 - 2\left(1 - \frac{\pi}{4}\right) = \frac{\pi}{2} - 1, \tag{2}$$

$$x + 2y + z = \frac{\pi}{6} + \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}. \tag{3}$$

Keywords: *Unit circle, unit square, vertices, arcs, area, sector, segment, triangle, angle*

The three equations are readily solved for z, y, x (in that order). From (2) and (3) we get, by subtraction:

$$z = 1 - \frac{\sqrt{3}}{4} - \frac{\pi}{6}. \qquad (4)$$

Next from (1) and (4) we get:

$$y = 1 - \frac{\pi}{4} - 2 + \frac{\sqrt{3}}{2} + \frac{\pi}{3} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1. \qquad (5)$$

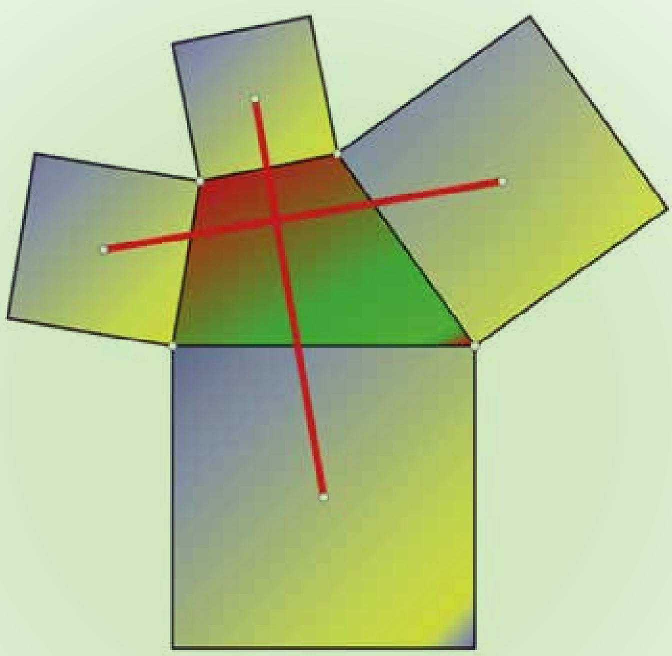
Finally from (2) we get:

$$x = \frac{\pi}{3} - \sqrt{3} + 1. \qquad (6)$$

— Thanks to Shri Bharat Karmarkar for suggesting the problem.

The **COMMUNITY MATHEMATICS CENTRE** (CoMaC) is an outreach arm of Rishi Valley Education Centre (AP) and Sahyadri School (KFI). It holds workshops in the teaching of mathematics and undertakes preparation of teaching materials for State Governments and NGOs. CoMaC may be contacted at shailesh.shirali@gmail.com.

THEOREM CONCERNING A QUADRILATERAL



The picture above demonstrates a beautiful result which is true of every quadrilateral. It should be self-explanatory: On each side of the quadrilateral, we draw a square facing outwards. Next, we join the centres of opposite pairs of squares (see the thick red lines). Then the two line segments thus drawn have equal length, and they are perpendicular to each other. Isn't that a beautiful result? Try finding a proof of it on your own.