GEOMETRIC SOLUTION TO THE 120 DEGREE PROBLEM

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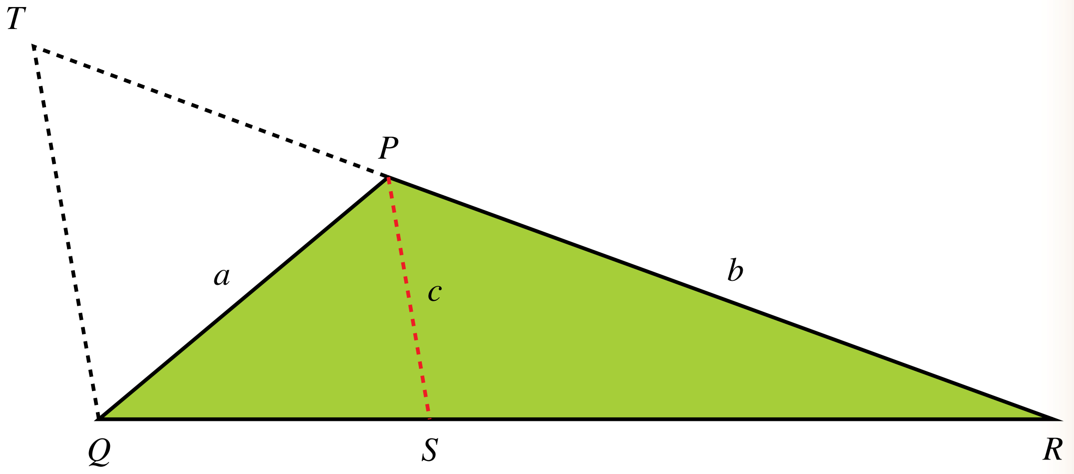
In the March 2013 issue of *AtRiA,* the following result had been stated in the article on 'Harmonic Triples': Let ΔPQR have $\measuredangle P = 120^\circ$. Let PS be the bisector of $\measuredangle QPR$, and let $PQ = a$, $PR = b$, $PS = c$; then $1 / a + 1 / b = 1 / c$. It had been proved using trigonometry, and the question was asked: Is there a proof using 'pure geometry'? We give just such a proof here.



Extend RP to T so that $PT = PQ$. Since $\measuredangle QPT = 60^\circ$ it follows that ΔPQT is equilateral, and hence (since $\measuredangle TQP = 60^\circ = \measuredangle QPS$) that $TQ \parallel PS$. Hence we have:

$$\frac{TQ}{PS} = \frac{TR}{PR} = \frac{TP + PR}{PR}, \quad \therefore \frac{a}{c} = \frac{a+b}{b}, \quad \therefore \frac{1}{c} = \frac{a+b}{ab} = \frac{1}{a} + \frac{1}{b}.$$

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