The Gregorian Calendar

An absorbing story that brings together history, religion, natural science and mathematics

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The calendar is a common device that regulates our day to day (even week to week and month to month) activities. The system of tracking days and assigning dates to them that is widely used now is called the Gregorian calendar, after Pope Gregory XIII, who held office from 1572 to 1585.

A calendar basically aims to integrate the cycle of earth's rotation around itself with the cycle of its revolution around the sun. The former phenomenon gives rise to the 'day' and the latter to the 'year.' Now a year is not an integral number of times as long as a day. The whole history of calendar making is the story of attempts to reconcile these two. (A calendar also tries to integrate a third phenomenon – the movement of the moon around the earth, which defines the month. This makes things a lot more complicated and so we shall not bring this in now.)

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A year is 365.2422 days long, correct to 4 decimal places. The immediate response to this situation would be to approximate this figure to 365.25 days in a year. As it would be awkward to have a quarter day added to 365 days to complete a year we could just have an extra day once in 4 years – a leap year. Though the earth's revolution around the sun and our reckoning of the year would get a bit out of phase every year, the situation would get more or less restored every 4 years. This is the basis of the *Julian calendar* which served much of Europe from 325 AD to the mid-sixteenth century.

The length of the year is actually a bit less than 365.25 days. Though the difference is small, over the centuries it accumulated to a noticeable extent. By the sixteenth century the actual revolutionary movement of the earth and the reckoning of the Julian year had fallen out of phase by 10 days. At this stage one may ask, 'So what?' To answer this question we consider the observed effects of the earth's revolution. This (combined with the tilt of the earth's rotation axis) gives rise to the seasons and associated phenomena such as temperature variation, variation in length of daylight, angle of sun's rays at a place, the equinoxes and solstices. These affect natural events such as rainfall patterns, germination and flowering in plants, breeding and migration in animals, etc. Besides, some religious and cultural events are tied to these phenomena. If our calendar year is not in phase with the earth's revolution, then there will be no correlation between a natural event and its date in the calendar. For instance, the date of the spring equinox will not be a constant. In fact this observation is what precipitated action on the part of Pope Gregory.

When the Julian calendar was initiated, the spring equinox occurred on 21 March. But in the following twelve centuries it moved gradually to 11 March. (Convince yourself that when the calendar year is longer than necessary, a natural event like the spring equinox moves backward in the calendar.) The festival of Easter is linked to the spring equinox (in a rather complicated way; we shall not go into it) and if nothing was done, the day of Easter would shift widely with the passage of decades. Conversely, Christmas, celebrated on a fixed date, 25 December, would move away from its seasonal moorings (winter in the Northern hemisphere) and take place in other seasons. This situation was not pleasing to the Catholic Church; so, taking inputs from astronomers and mathematicians, Pope Gregory ordered a reform to the calendar.

Two tasks needed to be done at this stage to arrest the trend. Firstly, the average length of the year had to be adjusted to be closer to reality. Secondly, the accumulated phase difference of 10 days had to be erased.

Taking up the former issue, it is apparent that the average length of the year in the Julian calendar is longer than necessary. This could be rectified by converting some leap years to ordinary years. The suggestion that came up was that 3 leap years could be dropped in a period of four hundred years; that is, instead of 100 leap years in 400 years we would have only 97. The average length of the year would then be 365+ (97/400) = 365.2425 days. The difference between this and the actual year length (correct to 4 decimal places) of 365.2422 days is 0.0003 days which is an error of less than one part in a million.

A further suggestion was that the years that reverted from leap years to normal years would be the century years except those that were divisible by 400. So we now have a block of four hundred years as the repeating unit of the calendar. The condition for a leap year could be stated as "If it is divisible by 4 it is a leap year; if it is also divisible by 100 it isn't; but if it is also divisible by 400 then it is."

Surprisingly, this block of 400 years is an exact number of weeks. (Convince yourself that this is true.) So any two dates differing by exactly 400 years would fall on the same weekday. Indeed, 400 is the least value of k for which the following statement can be made: For any x, the calendar of year x is the same as that for the year $x + nk$ for any integer n. But as only one cycle of 400 years has transpired after the Gregorian calendar came into effect, we cannot use this formula retrospectively.

Let's move to the other issue, namely, erasing the 10 day phase difference that had built up by the middle of the sixteenth century. To restore the spring equinox to its original date of 21 March, 10 days had to be dropped from the calendar. It was decided to declare that the day following 4 October 1582 would be 15 October 1582, a really bold step to take in the face of much public apprehension. This was effected smoothly in Catholic countries but other countries initially resisted the move. Britain and the British Empire made the transition in 1752, having to drop 11 days, as the discrepancy had grown. Russia switched to the new system in 1918, having to drop 13 days. The last European nation to adopt the Gregorian calendar was Greece, in 1923. Eventually the Gregorian system was almost universally accepted. But for a time different European countries followed different calendars and this led to some confusion regarding historical dates.

Some countries decided to make the transition in a phased manner, dropping a leap year here and there, but this led to further confusion – in the interim period they were out of phase with both calendars.

When the jump from 4 October to 15 October was made in 1582, the weekly cycle was not disturbed. 4 October fell on Thursday, and 15 October was taken to be a Friday. An unintended consequence of this was that the first day of this millennium, 1 January 2001, was a Monday.

An alternative to the Gregorian system would be to drop 4 leap years in 5 centuries, which would give an average year length of 365 + (121/500) = 365.242 days. Though this would be slightly closer to the true value (it would be a negative error), a 500-year cycle does not seem to have as much appeal as a 400-year cycle. (The number four seems to be a recurring theme in calendar circles.) Also, a 500-year cycle would not be an exact number of weeks. So it looks as though the legacy of a medieval Pope will be with us for a long time to come.

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