

Explorations in Teaching

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In this short note we describe some incidents in mathematics teaching— as they actually occurred in the classroom.

Finding the sine of 15 degrees

It is class IX. The teacher is teaching trigonometric ratios of 30° , 45° , 60° . Suddenly an idea pops up in his mind. He draws Figure 1 and asks the students, “Can anybody calculate $\sin 15^\circ$ from this figure?” This topic is not in the IX-th standard text book. But a few students accept this challenge, a few more silently appreciate their efforts to solve the problem, and the rest wait for the period to end! Two days later some students find a nice solution.

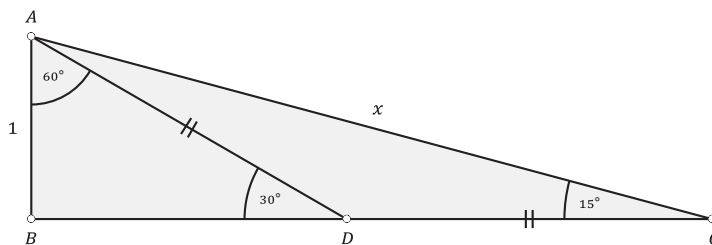


Figure 1.

They write: Let $AB = 1$; then $AD = 2$ (since $\sin 30^\circ = 1/2 = AB/AD$), and $CD = 2$ as well because $\angle DCA = \angle DAC$. Also, $BD = \sqrt{3}$, so $BC = 2 + \sqrt{3}$. Let $AC = x$. Using the Pythagorean theorem,

$$x^2 = AB^2 + BC^2 = 1^2 + (2 + \sqrt{3})^2 = 1 + (7 + 4\sqrt{3}) = 8 + 4\sqrt{3}.$$

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Hence:

$$\sin^2 15^\circ = \frac{1}{x^2} = \frac{1}{8 + 4\sqrt{3}}$$

In taking the square root, some simplification occurs, since

$$4 + 2\sqrt{3} = (1 + \sqrt{3})^2.$$

Hence:

$$\sin 15^\circ = \frac{1}{\sqrt{2(4 + 2\sqrt{3})}} = \frac{1}{\sqrt{2}(1 + \sqrt{3})}.$$

This can be written this in different ways, e.g.:

$$\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}.$$

Dividing a line segment in a given ratio

It is class X. The teacher is teaching the chapter on similarity and angle bisectors. A question is posed: "Draw a line segment AB of length 8.3 cm. Using compass and ruler, locate a point P on it such that $AP : PB = 5 : 2$." Most students use a method based on parallels, but Figure 2 shows an imaginative solution, based on a different idea; it is presented without any words. Can you explain why it works? Try it!

- $AB = 8.3$ cm (given)
- $AC = 10$ cm
- $BC = 4$ cm
- P is the required point

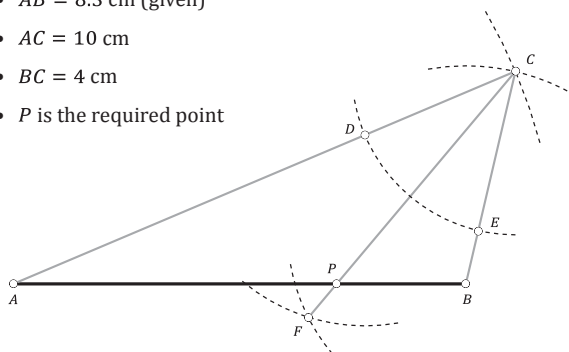


Figure 2.



BHARAT KARMAKAR is a freelance educator. He believes that learning any subject is simply a tool to learn better learning habits and a better aptitude; what a learner really carries forward after schooling is learning skills rather than content knowledge. His learning club, located in Pune, is based on this vision. He may be contacted at learningclubpune@gmail.com.