in the classroom

Teacher's Diary on Classroom Assessment - III

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In my last entry on CCE, I had included a project as part of the formative assessment. In order to allow for both individual and collaborative work, I had ambitiously planned to include a single project with both these components. It was now time to get real on my plans. In my search for suitable projects which encompassed a wide spectrum of arithmetic, geometric and algebraic components with a focus on mensuration, I naturally turned to tangrams. This topic is a favourite for both teachers and project designers. I wondered if I could get off the beaten path while taking advantage of the opportunities this material offered. That was when a colleague showed me how to make a tangram from a single A4 sheet using paper folding. This is the project I designed based on her input.

My rationale for developing this project was:

1. My overarching goal of enabling students to move from "Concrete to Abstract". As students worked with paper cutting and then paper folding, they were able to see the dimensions change. For those comfortable with measuring the dimensions of their project at different stages, discussion with class mates would allow them to generalize and use algebraic terms instead of arithmetic quantities. I would of course aid this process with class discussions.

Keywords: *CCE, collaborative, project, paper folding, origami, tangram, quadrilateral, triangle, area, mensuration,concrete, abstract*

2. Algebraic simplification is never easy for students new to it. Very often, they simply don't see the point of it. Being able to calculate areas by algebraic simplification, and actually to verify the calculations, would help students tremendously.
3. The properties of quadrilaterals are often merely memorized. By asking students to create quadrilaterals, I hoped to make them understand and appreciate – by doing, not observing.
4. Working with paper and then generalizing in 2-dimensions would help students improve their spatial abilities.

Since the class was new to paper folding, I took time to explain the *valley fold* (inwards) and the *mountain fold* (outwards). As a group, we discussed the symmetries involved in folding. I was aware that this was not an easy project and that asking students to do independent work would result in them seeking external help. Throughout, I encouraged students to work with discussion, and I also had periodic whole class discussion sessions so that students could share difficulties. As each group had students working on identical individual projects they could always share notes and help each other along. I also asked students to document their progress, explaining that this would give them more credit than the 'right answer'.

Group Component: Each group of 7 students was given a single A4 sheet and asked to divide it into 2 quarters, 3 eighths and 2 sixteenths, as shown Figure 1. Each student takes a piece.

The group was asked to

[i] Show the calculation for the sum of all the pieces being equal to 1.

[ii] Find the ratio of the dimensions of the length l and the breadth b of the sheet of A4 paper.

[iii] Discuss and arrange the following in ascending order: $\frac{l}{2}, \frac{b}{2}, \frac{l}{4}, \frac{b}{4}, \frac{l}{8}, \frac{b}{8}$.

Individual Component: Next, each student in the group was given the following pieces with the accompanying instructions

[i] **Students 1 & 2:** Make a triangle out of the one- quarter A4 sheet. (Procedure: Triangle, Fig. 3)

[ii] **Student 3:** Make a triangle out of the one-eighth A4 sheet. (Procedure: Triangle, Fig. 3)

[iii] **Students 5 & 6:** Make triangles out of the one-sixteenth A4 sheet. (Procedure: Triangle, Fig. 3)

[iv] **Student 4:** Make a square out of the one-eighth A4 sheet. (Procedure: Square, Fig. 4

[v] **Student 7:** Make a parallelogram out of the one-eighth A4 sheet. (Procedure: Parallelogram, Fig. 5)

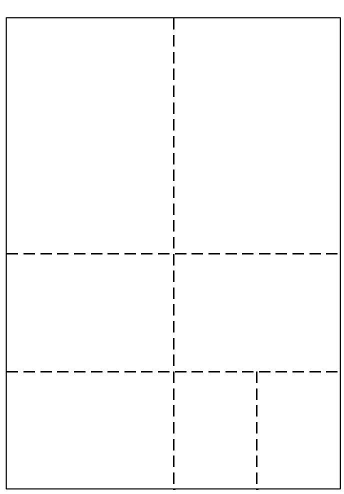

Figure 1. Cut the A4 sheet along the dotted lines to 7 parts - 2 equal large, 3 equal medium and 2 small equal rectangles

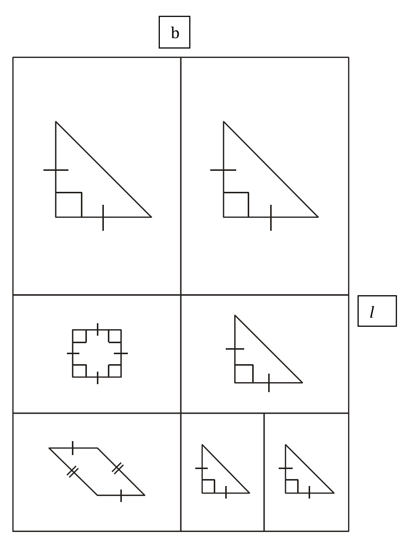

Figure 2.

General questions to be attempted by each student in the group after receiving his/her piece of paper. [The answers are in the accompanying teacher notes.]

1. If the original A4 sheet had length *l* and breadth *b,* what are the dimensions of the piece of paper assigned to you? Give your answer in terms of *l* and *b* and indicate which of them is smaller and which one is larger.
2. What is the area of the piece of paper you got?
3. What is the ratio of the area of your piece of paper to the area of the original A4 sheet?
4. Instructions for folding most of the shapes require you to start by folding off a square. What are the dimensions of the largest possible square for your piece of paper in terms of *l* and *b*?
5. Creating this square requires you to fold off a small rectangular extension – what are the dimensions of this rectangle in terms of *l* and *b*?

Procedure Triangle:

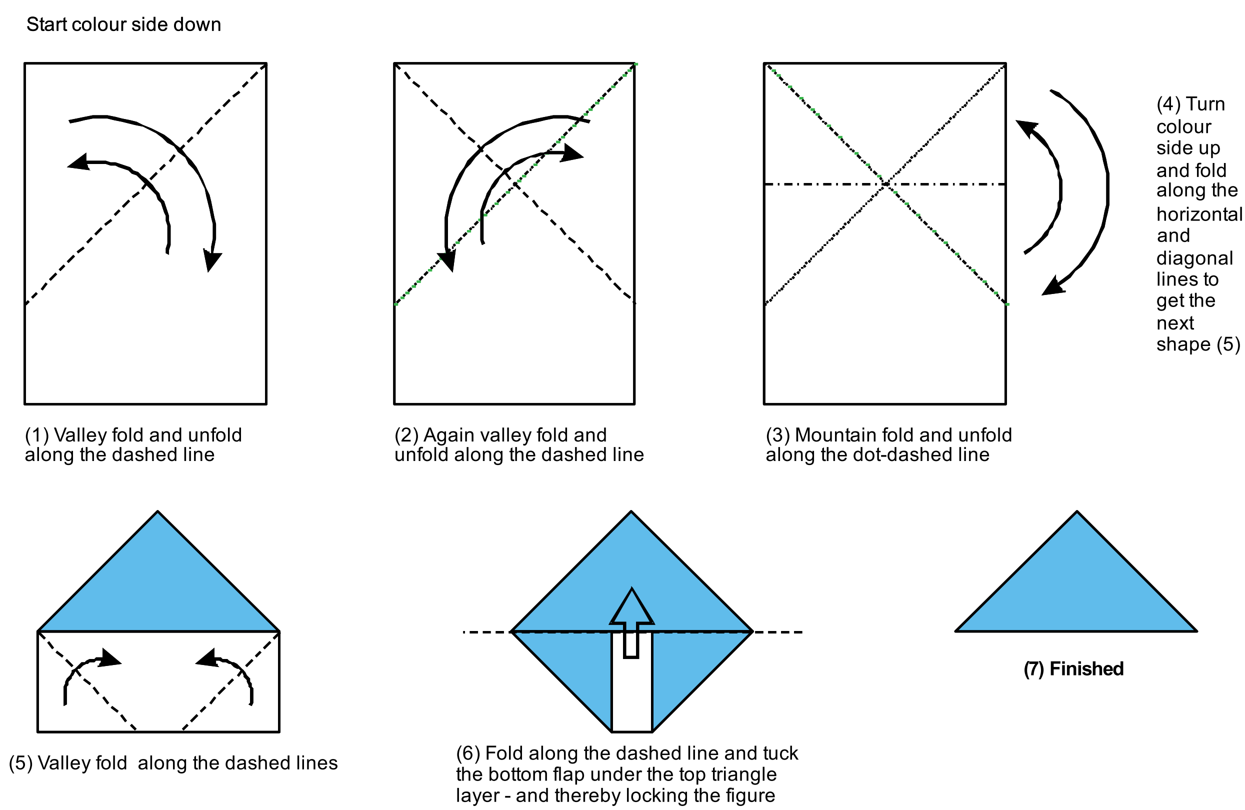

Figure 3

Specific questions to be attempted by the students making the triangle; the answers are in the accompanying teacher notes.

1. Obtain the area of the square that you mark off in terms of *l* and b in two different ways. Show your calculations.
2. Mountain folds are made along two creases in step 3. Why do these folds result in a triangle (surmounting a rectangle)?
3. Once the extra paper is tucked in, what kind of triangle do you get?
4. What are the angles of this triangle?
5. What are the lengths of the sides of the triangle? (Hint: You will need to use Pythagoras' theorem for this)
6. Find the area of the triangle in two different ways.

Procedure Square:

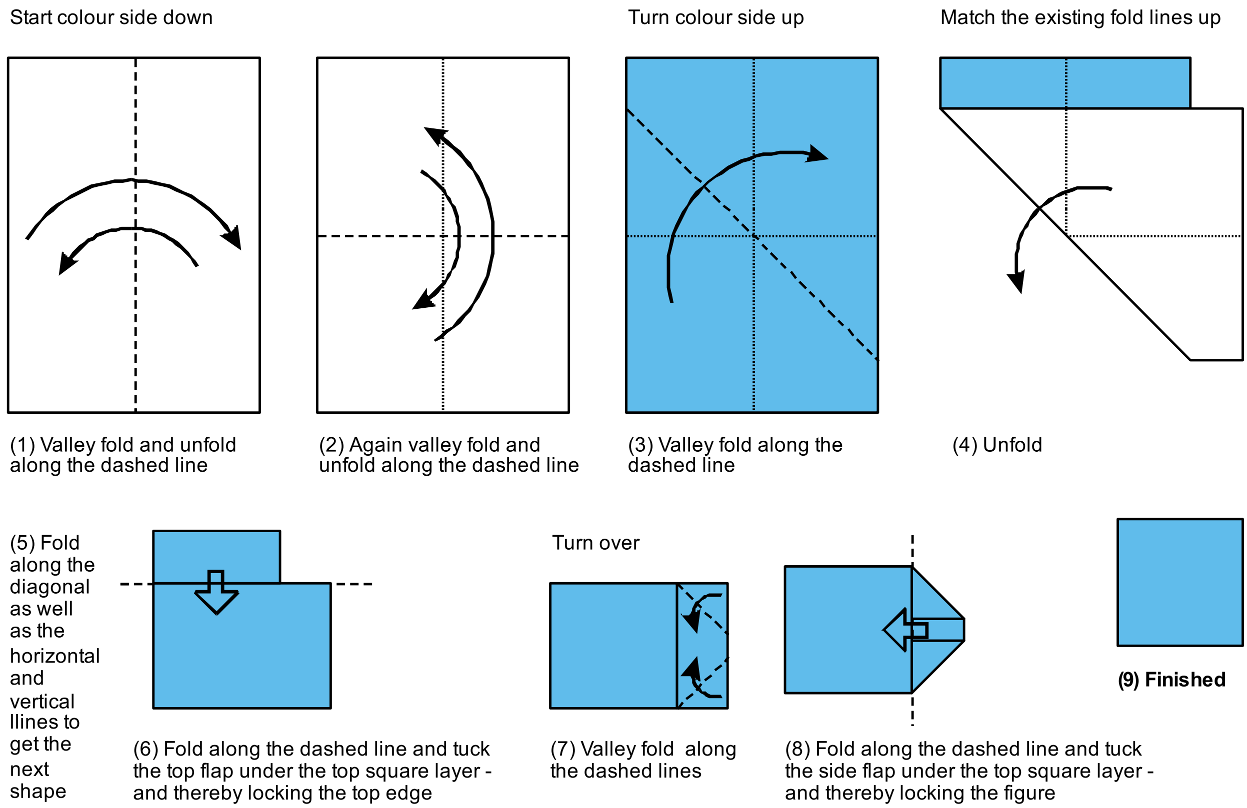

Figure 4

Specific questions to be attempted by the students making the square; the answers are in the accompanying teacher notes.

1. The three creases shown all pass through one point. What is this point?
2. The procedure for making the square involves marking off two rectangles on either side of a central quadrilateral. What are the dimensions of these two rectangles?
3. What angle does the valley fold in step 3 make with the vertical and the horizontal?
4. How do these angles enable the valley fold to create the fourth and fifth steps of Fig. 4?
5. The valley fold marks off a quadrilateral with two rectangles on either side in steps 5-9 of Fig. 4. What are the sides of this quadrilateral?
6. What type of quadrilateral is this? Give reasons for your answer.
7. What is its area?

Procedure Parallelogram:

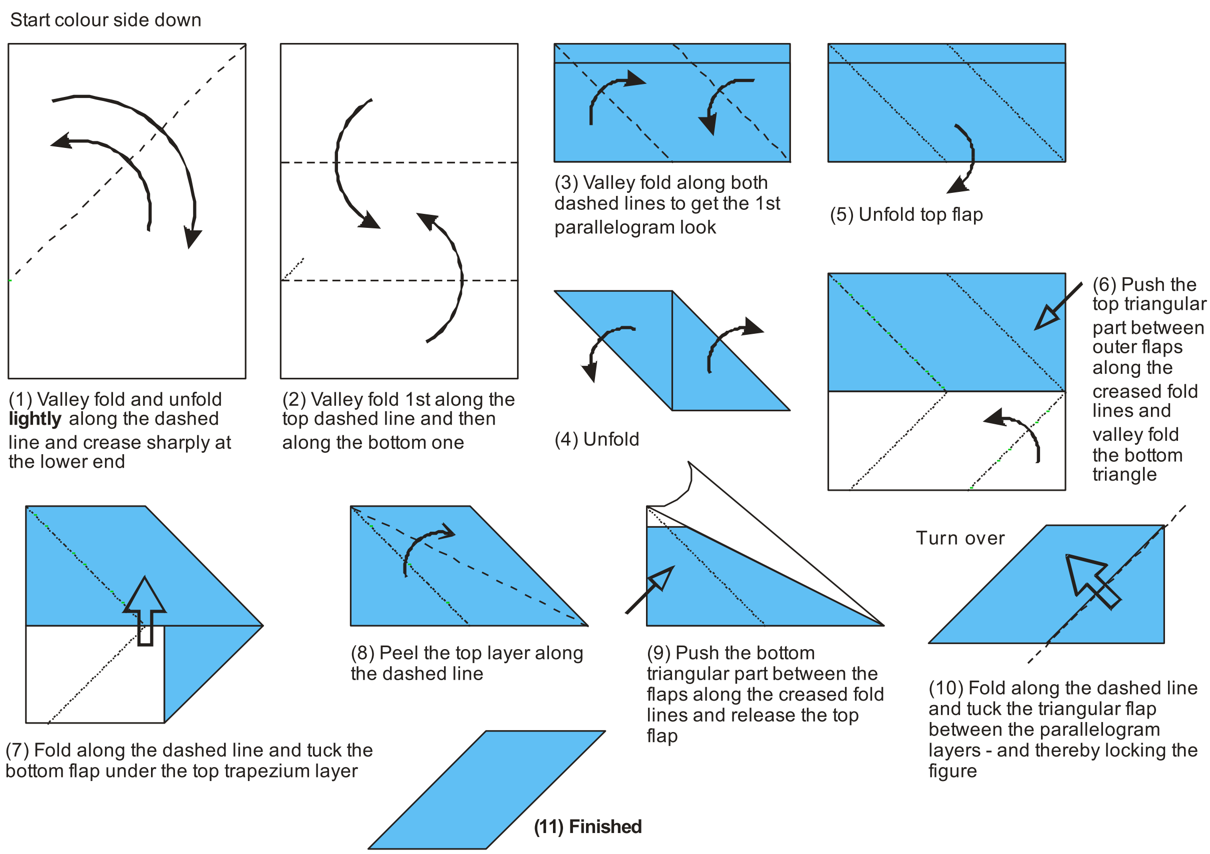

Figure 5

Specific questions to be attempted by the students making the parallelogram.

1. When the paper is folded into three, what are the dimensions of the larger rectangle that is obtained at the bottom (in step 3)?
2. Obtain the area of this rectangle in terms of *l* and *b* in two different ways. Show your calculations.
3. What is the ratio of the length to the breadth of this rectangle?
4. How does folding the two triangles give a figure whose opposite sides are equal and parallel?
5. What are the different kinds of polygons that emerge during this folding? Sketch or photograph them.

Assessment

When the students are being graded on their individual projects, I plan to focus on process rather than product. I will encourage them to put in as much detail as possible. I also plan to be open to their submitting assignments as Power Point presentations or other media where photographs or sketches are part of the work. The planned descriptors for the rubric are as follows:

1. Can the student follow the instructions given for the individual project?
2. Does the student seek help from peers or teachers to follow the instructions? (Note: This is not a negative marking, I encourage students to try on their own and to indicate in their submission how much they were able to manage on their own and what help they took from others.)
3. In answering the individual questions, has the student been able to sketch or describe the reasoning for the answer? Is the answer correct?
4. Has the student been able to obtain the answer by more than one method?

Answers to the general questions to be attempted by each student in the group after receiving his/her piece of paper.

1. **If the original A4 sheet had length l and breadth b, what are the dimensions of the piece of paper assigned to you? Give your answer in terms of l and b and indicate which of them is smaller and which one is larger.**

   Students 1 and 2:
   $l/2$ and $b/2$ of which $b/2$ is smaller.

   Students 3, 4 and 7:
   $l/4$ and $b/2$ of which $l/4$ is smaller.

   Students 5 & 6:
   $l/4$ and $b/4$ of which $b/4$ is smaller.

2. **What is the area of the piece of paper you got?**

   Students 1 and 2: $lb/4$

   Students 3 , 4 and 7: $lb/8$

   Students 5 & 6: $lb/16$

3. **What is the ratio of the area of your piece of paper to the area of the original A4 sheet?**

   Students 1 and 2: 1:4.

   Students 3, 4 and 7: 1:8.

   Students 5 & 6: 1: 16.

4. **Instructions for folding most of the shapes require you to start by folding off a square. What are the dimensions of the largest possible square for your piece of paper in terms of l and b?**

   Students 1 and 2: $\frac{b}{2} \times \frac{b}{2}$.

   Students 3, 4 and 7: $\frac{l}{4}$ and $\frac{l}{4}$

   Students 5 & 6: $\frac{b}{4}$ and $\frac{b}{4}$

5. **Creating this square requires you to fold off a small rectangular extension – what are the dimensions of this rectangle in terms of l and b?**

   Students 1 and 2: $\frac{l-b}{2}$ and $\frac{b}{2}$.

   Students 3 , 4 and 7: $\frac{l}{4}$ and $\frac{b}{2} - \frac{l}{4}$.

   Students 5 & 6: $\frac{b}{4}$ and $\frac{l-b}{4}$.

Answers to the specific questions to be attempted by the students making the triangle.:

1. **Obtain the area of the square that you mark off in terms of l and b in two different ways. Show your calculations.**

   Students 1 and 2 : $\frac{b}{2} \times \frac{b}{2} = \frac{b^2}{4}$

   Student 3 : $\frac{l}{4} \times \frac{l}{4} = \frac{l^2}{16}$

   Students 5 & 6 : $\frac{b}{4} \times \frac{b}{4} = \frac{b^2}{16}$

   The second method is to subtract the area of the rectangular extension from the area of the original piece. Students should be able to show that the answer is the same by both methods.

2. **Mountain folds are made along two creases. Why do these folds result in a triangle (surmounting a rectangle)?**

The mountain folds are along the diagonals of the square which divide the square into four congruent triangles. This allows two of the triangles to be tucked in and the remaining two to be superimposed exactly.

3. **Once the extra paper is tucked in, what kind of triangle do you get?**
   The triangle is isosceles and right-angled.
4. **What are the angles of this triangle? 45°, 45°, 90°**
5. **What are the lengths of the sides of the triangle?**
   (Hint: You will need to use Pythagoras' theorem for this)

   Students 1 and 2: $\frac{b}{2}, \frac{\sqrt{2}}{4}b, \frac{\sqrt{2}}{4}b$

   Student 3: $\frac{l}{4}, \frac{\sqrt{2}}{8}l, \frac{\sqrt{2}}{8}l$

   Students 5 & 6: $\frac{b}{4}, \frac{\sqrt{2}}{8}b, \frac{\sqrt{2}}{8}b$

6. **Find the area of the triangle in two different ways. The formula 'half base times height' gives the area of the triangles as**

   Students 1 and 2: $\frac{1}{2} \times \frac{\sqrt{2}}{4}b \times \frac{\sqrt{2}}{4}b = \frac{b^2}{16}$

   Student 3: $\frac{1}{2} \times \frac{\sqrt{2}}{8}l \times \frac{\sqrt{2}}{8}l = \frac{l^2}{64}$

   Students 5 & 6: $\frac{1}{2} \times \frac{\sqrt{2}}{8}b \times \frac{\sqrt{2}}{8}b = \frac{b^2}{64}$

Each triangle is $\frac{1}{4}$ the area of the square and the answers can be verified either using this or by using 'half base times height' with the longest side as the base and the height being the height of the rectangle tucked in.

Answers to the specific questions to be attempted by the students making the square:

1. The three creases shown all pass through one point. What is this point?
   This point is the centre of the rectangular piece of paper.
2. The procedure for making a square involves marking off two rectangles on either side of a central quadrilateral. What are the dimensions of these two rectangles? Each is $l/4$ and ½ $(b/2 - l/4)$
3. What angle does the valley fold in step 3 make with the vertical and the horizontal? 45° and 135°
4. How do these angles enable the valley fold to create the fourth and fifth steps of Fig. 4?
   The angles on either side of the valley fold are equal since the diagonals are perpendicular to each other. This allows the folding in of a triangle congruent to the external triangle.
5. The valley fold marks off a quadrilateral with two rectangles on either side in steps 5-9 of Fig. 4. What are the sides of this quadrilateral?
   The sides of this quadrilateral are half the smaller side of the original rectangle i.e. $l/8$.
6. What type of quadrilateral is this? Give reasons for your answer.
   A square since its 4 sides are equal (each is $l/8$ and perpendicular (by folding the 45 degree angles are doubled). Also it is made up of 4 pairs of congruent superimposed isosceles right triangles.
7. What is its area?
   Its area is which is the product of the sides Its area can also be obtained by taking ¼ (area of the original rectangle minus the area of the rectangular extensions).

   $$\text{i.e. } \tfrac{1}{4}\left(lb/8 - 2\left(l/4 \times \tfrac{1}{2}\left(b/2 - l/4\right)\right)\right)$$
   $$= lb/32 - lb/32 + l^2/64 = l^2/64$$

Note: Only the most able eighth standard students can attempt this level of algebra which is why only one method of calculating area is asked for.

Answers to the specific questions to be attempted by the students making the parallelogram:

1. When the paper is folded into three, what are the dimensions of the larger rectangle that is obtained at the bottom (in step 3) ?

   $$\frac{l}{4} \text{ and } \frac{1}{2}\left(\frac{l}{4}\right).$$

2. Obtain the area of the rectangle that you get in terms of l and b in two different ways (if possible). Show your calculations. Multiplying the above, we get . Else, we can find ½ (area of original rectangle area of first fold)

   $$= \frac{1}{2}\left(\frac{lb}{8} - \frac{l}{4} \times \left(\frac{b}{2} - \frac{l}{4}\right)\right) = \frac{l^2}{32}.$$

3. What is the ratio of the length to the breadth of this rectangle? 2:1

4. How does folding the two triangles give a figure whose opposite sides are equal and parallel? The rectangle consists of two congruent squares (since the length is twice the breadth). The triangles are folded along the diagonals of these adjacent squares. The diagonals are equal in length and inclined at to the base of the rectangle.

5. What different kinds of polygons emerge during this folding? Sketch or photograph them. A heptagon, a pentagon, two adjacent squares and the parallelogram

Putting the pieces together

Once the individual projects were submitted and graded, the groups of 7 students can come together for more traditional tangram projects. Useful ideas for these may be obtained from [2], [3] and [4] and many more are available on the Internet. The groups work on these with the individual pieces created by them. The outcome does depend on each piece so at this point a shoddily constructed block will affect the whole. Students may choose to redo their individual projects at this stage. Of course, the planned descriptors for the rubric will again focus on process rather than product but the group will gain points for cooperative work and for demonstrating group responsibility and individual responsibility.

References:

[1] Swati Sircar: Sr. Lecturer, Azim Premji University and Chang Wen Wu: Origamist extraordinaire

[2] Hands on Math for Class 6: Jonaki B Ghosh, Haneet Gandhi, Tandeep Kaur

[3] Hands on Math for Class 7: Jonaki B Ghosh, Haneet Gandhi, Tandeep Kaur

[4] Math Masti Booklet 2: math4all (www.math4all.in)

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