



Azim Premji
University

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TEACHING DECIMAL FRACTIONS USING A PAPER KIT

PADMARIYA SHIRALI

A PAPER KIT
APPROACH

**At
Right
Angles**
A Resource for School Mathematics

One often sees mistakes of the following kinds in children's work: $0.2 < 0.199$; $0.4 \times 10 = 0.40$; $0.05 = 0.5$; they read aloud 0.32 as 'point thirty two'. One sees a lack of conceptual understanding even in the case of children who are able to solve problems correctly by applying the rule of aligning the decimal points (addition/subtraction problems) and counting decimal places (multiplication problems). Solving the problems correctly does not necessarily imply that concepts have been fully understood. Curiously, children who are logical may have a larger number of these misconceptions!

Why do these kinds of mistakes happen? One possible reason is that children apply the conceptual understanding they have gained about whole numbers to decimal numbers.

They try to fit new knowledge into existing schema; in this case, the understanding they have built of whole numbers. Here are some examples of this phenomenon.

- In whole numbers a 'longer' number (a number with more digits) is bigger than a 'shorter' number (one with fewer digits); e.g., $1045 > 950$. But this doesn't hold in decimal numbers; e.g., $0.2 > 0.199$.
- In whole numbers, when you multiply a number by 10 you place a zero at the end of the number. Example: $40 \times 10 = 400$. But when we do $.4 \times 10$ we get 4, not .40.
- In whole numbers, a zero placed in *front* of a number has no value and can be dropped. But we cannot drop the zero in 0.05.

Another reason is that they do not have a proper understanding of place value which presupposes multiplicative thinking. That is, they do not understand that the value of a digit gets ten times larger when its position shifts one step towards the left.

Yet another reason could be that they have not internalized the understanding that the decimal and fractional part of a number is less than a whole.

So, how should we introduce decimals? Some teachers use money, others use measurement. But each of these turns out to have some limitations. (We discuss these at the end of this article.)

Let us now list the prerequisite skills that children should have before we teach decimals.

- Basic concepts of fractions:
 - a. part sizes are same and equal
 - b. number of parts that make up a whole
 - c. what numerator and denominator denote
 - d. idea of equivalent fractions.
- Place values: Multiplicative relationship between succeeding places (ten is 10 times a unit, 100 is 10 times a ten, etc.)

GUIDING PRINCIPLES TO FOLLOW IN INTRODUCING DECIMALS AND FRACTIONS

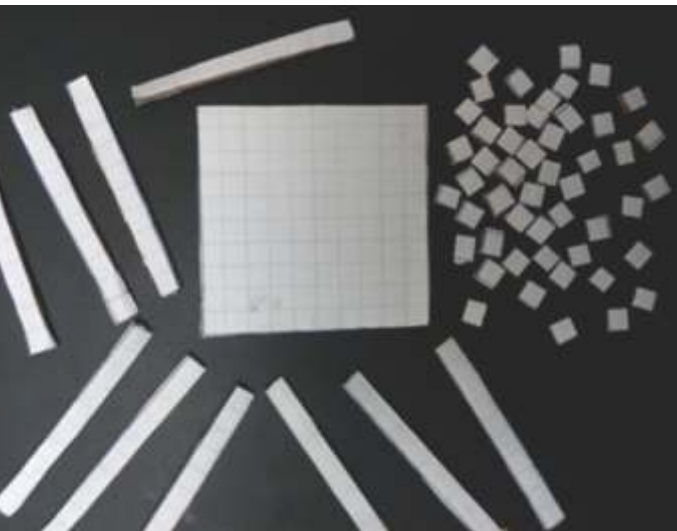
The first step should be through usage of a concrete model, followed by a pictorial representation and lastly the introduction of the abstract symbol. Experience, understanding and verbalization are the sequence to be followed.

Children need to understand that decimal numbers are part of the number system. Fractions and decimal numbers are not internalized by children in the same way as natural numbers (to which they are exposed from childhood). It is necessary for the teacher to contextualize and bring in as many examples as possible of the usage of these numbers in daily life.

ACTIVITY ONE

Purpose:
Preparation of the decimal kit.

Materials required:
Square dot paper or square grid paper (notebooks with Square Grid paper are available in the market and are used generally in primary sections) minimum of 3 sheets.



Help the children to prepare a decimal kit. This action itself helps in gaining understanding of tenth and hundredth and the relationship of these to the whole. It is better to prepare a kit than procure a ready-made one. Teacher should also prepare a kit consisting of several whole grids (to represent units), tenths and hundredths for demonstration purposes.

Paste these sheets on firm card paper. Then ask the children to:

- Outline a 10 x 10 grid which will be considered as the whole, and cut it out.
- Outline one more 10 x 10 grid, divide it into 10 equal parts, and cut out the 10 parts.
- Outline one more 10 x 10 grid, divide it into 100 equal parts, and cut out the 100 parts.

(If teachers find this activity time consuming they can give it as home work. Children will be thrilled to get homework which requires them to draw and cut.)

The process of this kit making should help each child in visualizing that a whole is 10 times as large as a tenth, and a tenth is 10 times as large as a hundredth.

ACTIVITY TWO

Purpose:
To teach tenth and hundredth and discuss their relationship with the whole.

Materials required:
Decimal kit

(This is at the oral level. Symbolic representation is introduced after the students become fully familiar with the language, the words tenth and hundredth.)

Use the kit to pose various questions:

Ask them to pick up one strip and tell them that it is called a tenth. Point out that each strip is one-tenth of the whole. Show pictures of grids with different numbers of tenths shaded, and get them to read the number of tenths shaded. Ask them "How many tenths make a whole?" Let them count and realize that 10 tenths make a whole.

Connection between the name 'tenth' and the process of the whole being made into ten equal parts should be established clearly.

Now ask them to pick up a small square and tell them that it is called a hundredth. Point out that each small square is one-hundredth of the whole. Lead them to realize that 100 hundredths make a whole. Show pictures of grids with different numbers of hundredths shaded, and get them to read the number of hundredths shaded.

Connection between the name 'hundredth' and the process of the whole being made into hundred equal parts should be established clearly.

Now ask how many hundredths make a tenth, how many hundredths make 2 tenth, etc.

Encourage them to use their materials initially while answering the questions.

The words 'tenth' and 'hundredth' are new for children and teachers need to stress the endings of the words so that children do not confuse them with the words 'ten' and 'hundred' which they are already familiar with. Point out that ten is 10 units, and that a tenth is less than a whole, it is one part of a whole which has been made into 10 parts. Similarly, point out that hundred is 100 units, whereas a hundredth is one part of a whole which has been made into 100 equal parts.

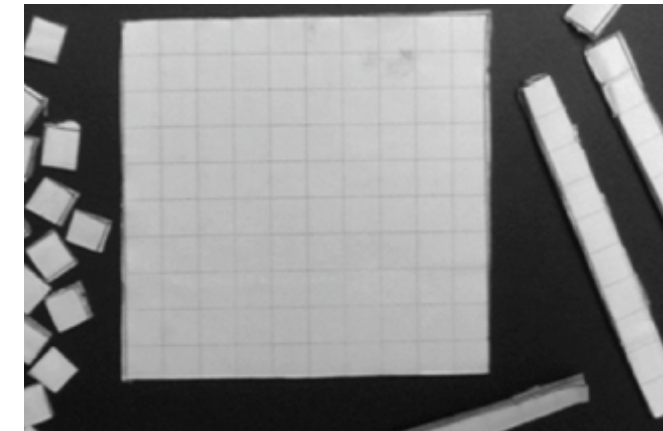
The teacher must emphasize the fact that a tenth and a hundredth are less than a whole. The proper understanding of these relationships paves the way for conceptual understanding of decimals.

You can pose other questions which require summation like:

- How many hundredths are there in 4 tenths and 5 hundredth?
- How many tenths are needed to make 2 wholes? How many tenths are needed to make 3 wholes?
- How many hundredths will make 3 tenth? How many hundredths will make 7 tenth?
- By how much more is 8 hundredth bigger than 2 hundredth?
- How much more is 1 tenth than 1 hundredth?
- How much less is 9 hundredth than 1 tenth?
- How many times a hundredth is 1 tenth?

You may find the children stumbling on questions which differentiate between 'how much more' and 'how many times' but these are areas we need to consciously strengthen. One requires additive thinking, whereas the other requires multiplicative thinking.

Even at the end of primary school children are not fully conversant with multiplicative thinking and often resort to additive thinking.

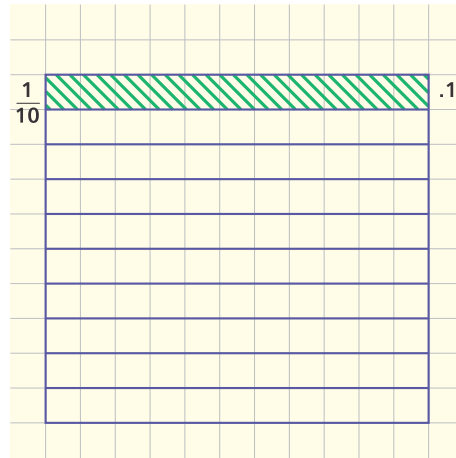


ACTIVITY THREE

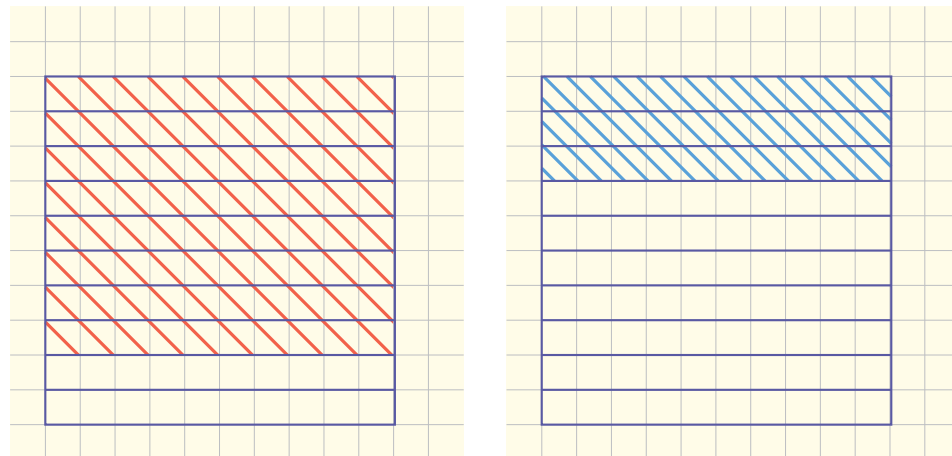
Purpose:
To establish the relationship between fraction and decimal (tenth), to use the symbol and the name.

Materials required:
Square grid paper, colour pencils,
Number line

Ask children to outline a 10 x 10 square grid and divide it into 10 equal parts. Ask them to shade one tenth. Let them record the information as shown in the picture.



Give them practice in shading different tenths and recording the information.

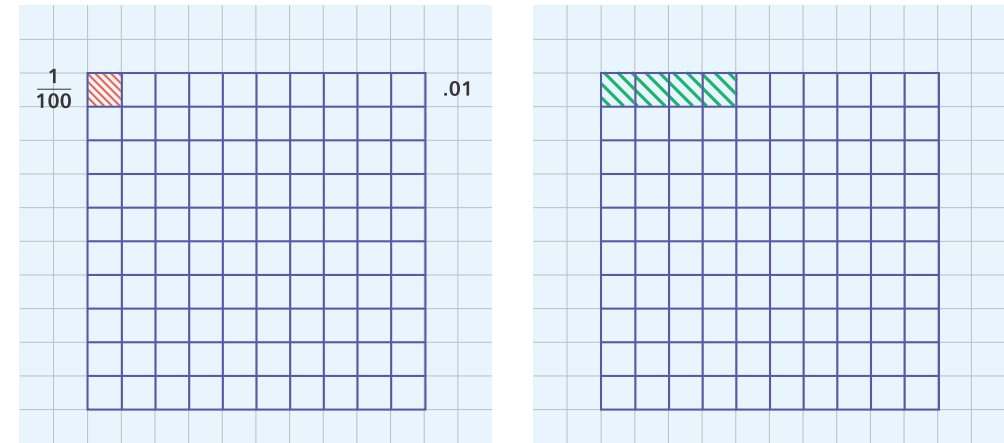


ACTIVITY FOUR

Purpose:
To establish the relationship between fraction and decimal (hundredth), to use the symbol and the name.

Materials required:
Square grid paper, colour pencils

Ask children to outline a 10 x 10 square grid and divide it into 100 equal parts. Ask them to shade one hundredth. Let them record the information as shown in the picture.



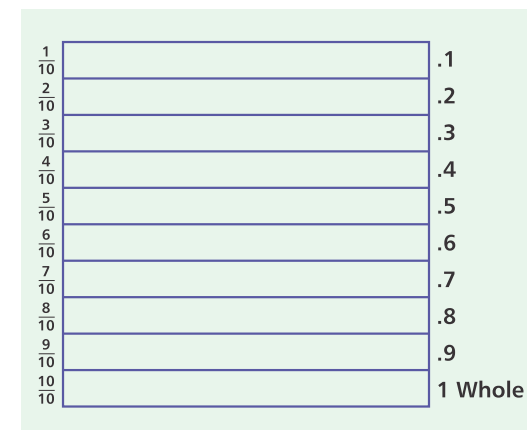
Give them practice in shading different hundredths and recording the information.

ACTIVITY FIVE

Purpose:
To prepare a single grid that consolidates the relationship between fractions and decimals for future reference.

Materials required:
Square grid paper, colour pencils

Ask children to outline a 10 x 10 square grid and divide it into 10 equal parts.

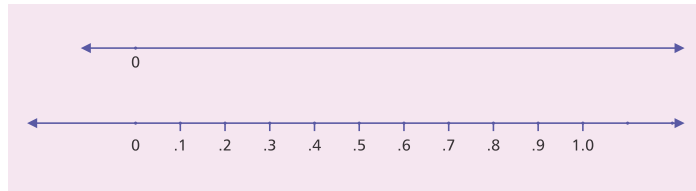


Let them record the information as shown in the picture.

ACTIVITY SIX

Purpose:
To illustrate that decimal numbers are less than a whole and lie between 0 and 1.
To build estimation skills

Materials required:
Two long number lines as shown in the picture, made from paper or cloth, one showing 1 whole made into 10 equal parts and another without any markings; coloured bulletin board pins.



Initially show the number line which has been divided into parts and point out the different decimal numbers.

Ask children to point different tenths like .1, .5, .9, etc., on the number line.

Similarly you can also make and use a hundredth number line.

Ask children to point different hundredths like .25, .36, .78, etc., on the number line.

ESTIMATION GAME

Turn the number line strip over so that the number line with markings cannot be seen by children.

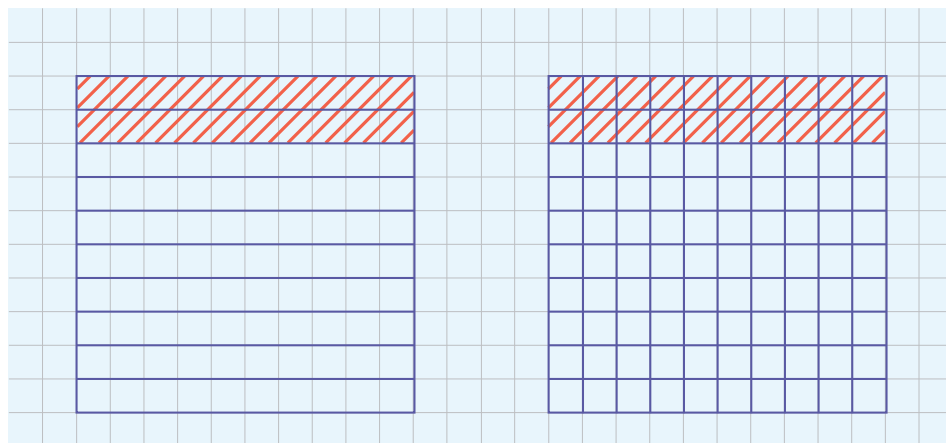
Now pose the question "Where will .2 be?" Let the child try to estimate where it will lie and mark that part with a pin. Pose more questions of that kind and let different children guess the positions of their numbers.

Now turn up the folded part and children can see for themselves how close their estimates were to the actual. Whoever comes closest to their guess is the winner.

ACTIVITY SEVEN

Purpose:
To compare decimal numbers, to establish the equivalence of numbers like .2 and .20.

Materials required:
Square grid paper, colour pencils

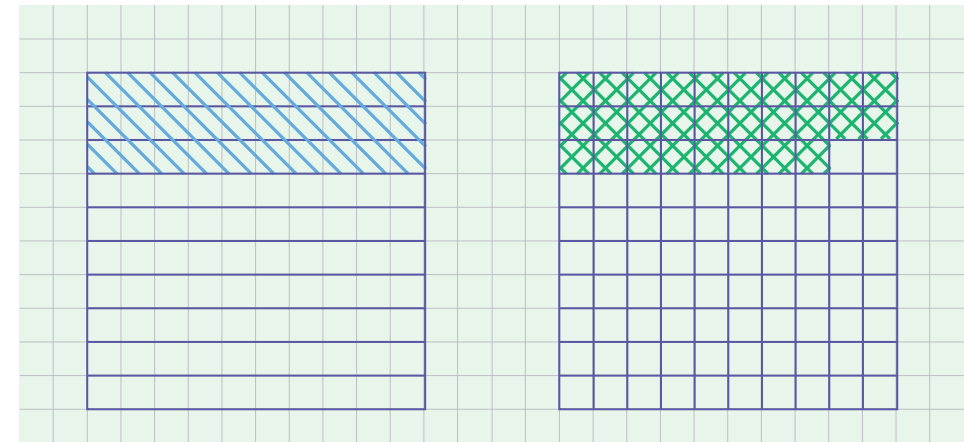


Ask children to shade 2 tenths in the left side grid (of the book) and write the decimal number underneath.

Ask them to shade 2 hundredths in the adjacent grid (right side of the book) and write the decimal number underneath. Let them compare and state the result.

Pose the question "Which is greater, .3 or .28?"

After they respond ask them to shade 3 tenths on one side and 28 hundredths on the other side and verify if their answer was right.



Pose another such question and ask them to justify the answer. Note if they are able to use language correctly. Also check if they are first comparing the tenths and then the hundredths.

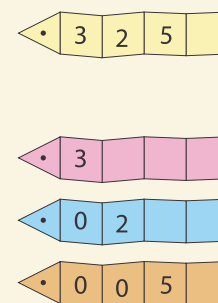
Elicit the rule for comparison of decimal numbers from them. Give many exercises which require them to draw and compare to help in the visualization process.

EXTENSION: Now give questions which have mixed decimal numbers.

ACTIVITY EIGHT

Purpose:
To show mixed numbers Involving whole and decimal parts.

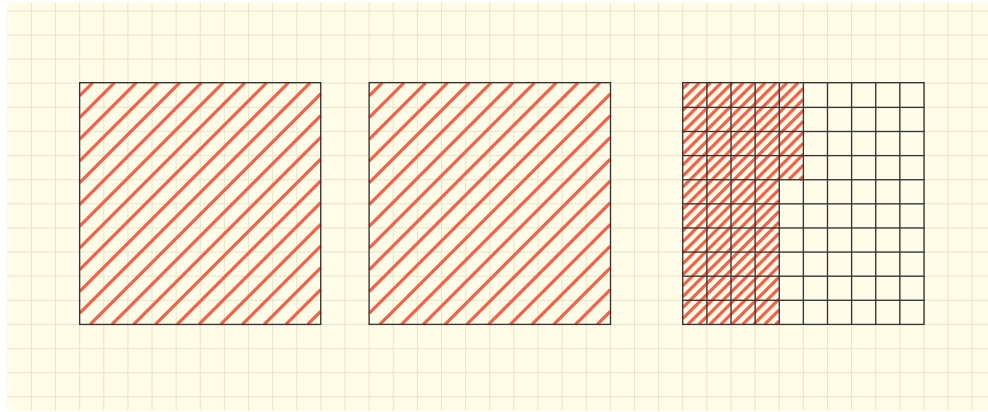
Materials required:
Square grid sheets, Number cards.



Place the corresponding number cards of a decimal number to show the breakup of these numbers.

Demonstrate a mixed decimal number (involving whole and decimal part) using whole grid, tenths and hundredths as shown in the picture. Record it on a place value chart which can also be made on the black board.

Teach the children how to read the number in different ways, for example: 2.44.



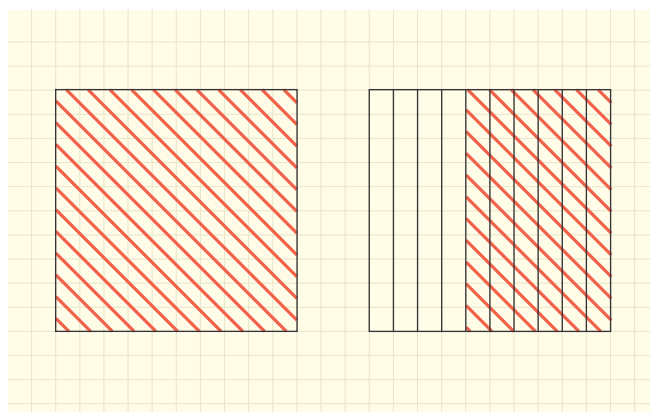
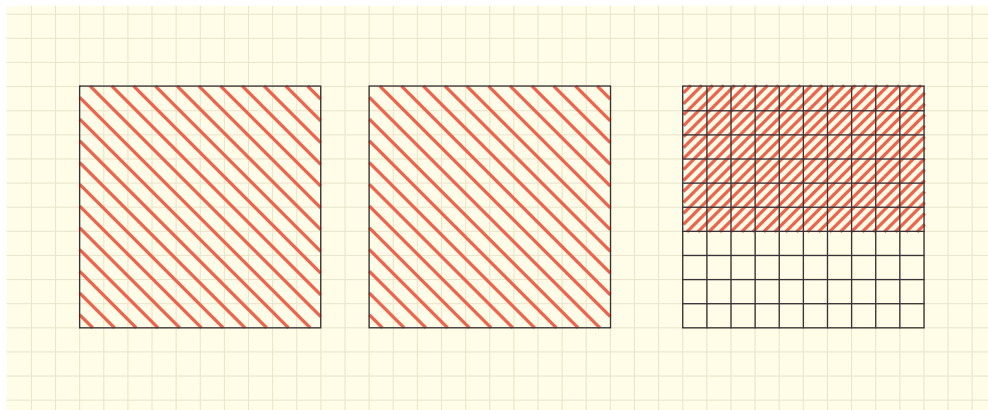
- Two point four four (it should not be read as two point forty four)
- Two units and forty four hundredth
- Two units, four tenths and four hundredths

Now show another combination which has only a whole and hundredths, e.g., 2.06.

Show how we use zero as a place holder and that the zero in the tenth place signifies that there are no tenths in the number. Since we have six hundredths we place the zero in the tenths place.

Use other examples which combine whole and hundredth alone, e.g., 3.08.

Now discuss examples which have whole and tenths alone, e.g., 4.2. At this point raise the question whether we need to place a zero in the hundredth place to indicate the lack of hundredths.



ACTIVITY NINE

Purpose:
To extend decimals to thousandth place and other places.

Materials required:
Grid showing thousandth, Place value chart.

To prepare a grid showing thousandth: Let children make a 100 square grid with slightly larger squares. Ask them to make 10 parts of one of the hundredth squares to demonstrate a thousandth. This needs to be done only once for them to understand how small a thousandth is.

Hunds	Tens	Units	Tenth	Hdth	Thousth
100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
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- Discuss the place value chart shown above. Point out that as one moves from left to right on the place value chart each place is one-tenth ($\frac{1}{10}$) of the previous place. Unit is $\frac{1}{10}$ of ten, ten is $\frac{1}{10}$ of hundred, and Hundred is $\frac{1}{10}$ of thousand.
- Now pose the question “what is $\frac{1}{10}$ of a unit called?”
A tenth.
- Next ask the question “what is $\frac{1}{10}$ of a tenth called?”
A hundredth.

- Now ask “What will be $\frac{1}{10}$ of a hundredth?” A thousandth.
- One can extend this question further by asking, “What is $\frac{1}{10}$ of a thousandth?”

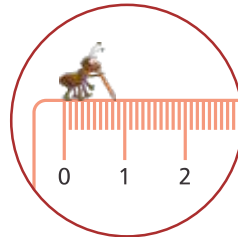
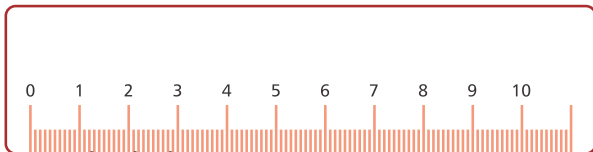
You can lead them to discover that there is no end to this process and one can thus have infinitely smaller decimal numbers.

The place value chart comes of use while teaching multiplication and division of numbers by 10, 100, 1000, etc. Here one can say numbers become 10 times smaller when they move to the right. So when units are divided by ten we need to create a space for them to move and we do so by introducing the decimal point. It is important for the teacher and student to understand that the place value grid is invariant (that is, fixed), and it is the numbers that move to the right when multiplied by 10, and to the left when divided by 10. Then it is easier to understand where it is necessary to put a 0 as a place holder and where it is not.

ACTIVITY TEN

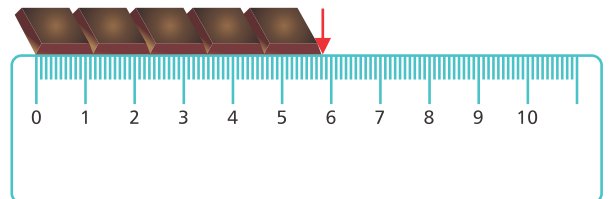
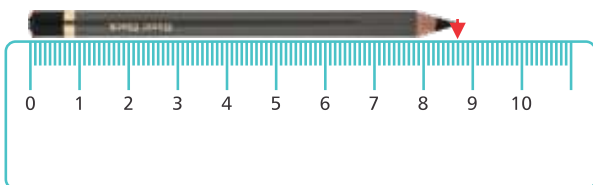
Practical activities:
Do plenty of measurement activities and price related activities to give practice in using decimals.

Materials required:
Scale, small objects, money



Measurement provides a natural context to introduce decimals, through the idea of accuracy (i.e., if a length is between 9cm and 10cm, and one is interested in finding the exact measurement, maybe to make a photo frame, one would have to further divide the gap; and 10 seems a convenient number because of our base 10 number system). However measurement doesn't give a conceptual understanding of place value.

Usage of money has also a similar difficulty because the relationship of 100 paise to 1 rupee cannot be represented concretely and still requires abstract understanding. i.e., a 10 paise coin is one tenth of a rupee and could be written as 0.1, and one paise is one hundredth of a rupee and can be written as 0.01. We can discuss that 0.1 and 0.10 both represent the same amount and the misconception that 0.10 is point ten paise can be addressed. (Here the multiplicative property of place value can be explained in more detail.)



Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre, in Rishi Valley School (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com