in the classroom

Stimulating student learning
Open-Ended Questions

A class-room strategy for promoting divergent thinking

Can anything beat the feel-good factor of finding the perfect response to a question in the classroom? This article describes how giving students the opportunity to explore and study a concept through open-ended questions gives them a variety of paths to understanding and bursts the myth that the shortest road to mathematical success is the 'right answer'.

R. Athmaraman

This writer has a mobile phone, which when unlocked, displays the question, "How are you today?" This is one of the simplest examples of an *open-ended question*. We can think of other such examples, such as: "What did you feel after reading this article?", "How interesting was your mathematics class yesterday?" These are in direct contrast with what are known as "*closed-ended*" questions, such as "What is the colour of the silk-sari you purchased yesterday?" "How much increment did you receive in your salary?"

For a maths teacher, open-ended questions offer several advantages, one of which is that they encourage students to speak and express themselves at length, and this is absent in traditional teaching. We illustrate this with a small example of a question for a 7-year old.

Teacher A: What is 7 plus 6?

Learner: 13.

The question is specific and the respondent has simply to state a 'fact'. The question is straightforward, and the answer is simple and predictable. Teacher A, in fact, indirectly controls the response of the learner. This variety of question is closed-ended.

Here is teacher B who uses the same fact in a different way.

Teacher B : Give me two numbers that add up to 13.

The question provides for an assortment of correct and sensible answers. A learner comes out with the answer "9+4" and this ignites an admirable discussion with other learners. They enthusiastically suggest more pairs of numbers with the total 13. The same question will receive a different kind of response when asked to a 12-year old. The student may then give the combination 5.6+7.4, or even $2^3 + \sqrt{25}$. Often in a situation such as this, one can find learners competing with one another to exhibit their perception, comprehension and awareness. There is a lot of 'why' and 'how' from the novice. The query encourages students to dig into their understanding and impressions. The reason is that the question is open-ended. It presents a challenging situation to the student, who thereby has control over the response, quite unlike the situation created by a closed-ended question.

An enterprising teacher will bring into play adequate number of open ended questions to motivate, introduce or clarify concepts. Such questions encourage *Divergent and Reflective Thinking*. When they are employed in the mathematics class, the instructor can expect a range of responses and can thereby make progressive cognitive demands on students. They help learners put their heads together to make sense of mathematics.

The learners recognize the defining characteristics of the underlying concept, discuss various ideas, reason mathematically and ready themselves to conjecture, invent and solve problems. Concepts of mathematics get connected to their areas of application. This is a consequence of the fact that when learners respond to open-ended questions, they look into the background of the underlying concept.

By their intrinsic nature, the open-ended questions are a versatile tool for teachers handling any level of mathematics. The modus operandi of teacher B, who asked for a pair of numbers with a sum of 13, is so strikingly simple that it can be adopted at any stage of instruction for any grade. Here are some examples:

- Find two numbers whose product is 1.5
- The difference of two fractions is 4/5. What could the fractions be?
- Find an algebraic expression which has $(2x - 3)$ as a factor.
- Give the measures of a pair of angles that are supplementary.
- List two vectors whose scalar product is 10.
- The probability of an event is 2/3; what could be the event?
- The sine of an angle is 1/2. Find the angle.
- Provide an instance of a situation where L'Hopital's Rule will be needed.
- Give an example of a non-commutative group.

Observe that the responses to the above questions demand not only the comprehension of concepts but also a command of the processes and skills for applying and manipulating them. Additionally they train the learner's mind to logically justify his or her viewpoint and the solution. This remarkable advantage makes the open-ended questions superior to other varieties.

Teachers are quite familiar with the technique of asking traditional type of closed-ended questions such as "Find the LCM of 12 and 15", "What is the arithmetic mean of 5, 13, 26 and 103", etc. With a little more planning and innovation, one can 'create' quite a variety of open-ended questions just by slightly altering the traditional presentation. Let us list some examples:

Here are some more examples:

Closed-Ended	Open-Ended
Add the first three natural numbers which are not multiples of 3.	The sum of three natural numbers, none of which is a multiple of 3, is 20. What are the numbers?
What is the HCF of 24 and 36?	Can 6 be the HCF of 24 and some number n? Can 7 be the HCF of 24 and some number n?
How many lines of symmetry does a trapezium have?	Give an example for a quadrilateral that has no line of symmetry.
The perimeter of a rectangle is 28 cm and its length is 8 cm. Find its area.	The perimeter of a rectangle is 28 cm. What might be its area?
Round 23.45 to the nearest tenth.	What number when rounded to the nearest tenth will give 23.5? Substantiate your solution.
Which is bigger, $\frac{1}{\sqrt{3}}$ or $\left(\frac{1}{\sqrt{3}}\right)^2$?	Can the square of a number be smaller than the number itself? Justify your answer.
Draw a rectangle and the middle lines of its sides. Then colour 75% of it in red.	Draw a rectangle and colour 75% of the rectangle. Do you get a unique answer? Explain.
Draw a triangle whose sides are 5 cm, 6 cm and 7 cm in length.	Two of the sides of a triangle are 5 cm and 6 cm long. Draw the triangle. Argue how your construction is appropriate.
What is the shape of a manhole cover?	Why are manhole covers circular? List a few objects or tools around you whose shapes directly relate to their uses.
State the line of symmetry of the quadratic function $x^2 + 4$	Find a quadratic function whose line of symmetry is the y axis.
Show that the roots of the quadratic $x^2 - 9$ are equidistant from the origin	Find a quadratic function whose roots are equidistant from the origin.

1. For a quadratic function, what is the connection between the following two properties: "Line of symmetry is the y axis" and "Roots are equidistant from the origin"? What kinds of quadratics have these properties?
2. Let $p(x) = x^3 - x^2 + ax + b$. When $p(x)$ is divided by $x - 2$ the remainder is 12. Can a and b be found using this information? If not, what further information would suffice to find a and b?
3. Let A and B be two points in the complex plane corresponding to the points $\{1 - i, \frac{i}{1-i}\}$. Find a complex number z such that if C is the point corresponding to z, then ΔABC is right angled.

From the examples, one might have noticed the following: *An open-ended problem may yield multiple answers*. Such a problem, requiring divergent thinking, may be solved by many different methods. There will be a great need for investigative and reflective thinking and decision making, to justify the process and the product.

Open-ended questions are not to be confused with 'opening' questions. Opening questions are simply starting points to probe into the background knowledge in the topic to be introduced, the past experiences and the recall of the learner. They are mostly closed-ended, although some starters could be open-ended. However, experience tells us that commencing a class with open-ended questions can spark mathematical communication.

A teacher should use a judicious combination of closed-ended and open-ended questions. Closed-ended questions alone may not provide a real assessment of instruction. *It is necessary for the teacher to wait for the responses of the students when an open-ended question is asked, and not to hurry the student.* Without this allowance of time, the teacher may miss opportunities to spot learning difficulties as well as patterns of valid but divergent thinking in the learners.

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ATHMARAMAN R (M.Phil, M.Ed) started his career in 1962 and has worked as a teacher, headmaster, researcher, academic advisor and math educational consultant. He has served as Secretary of the Association of Math Teachers of India (AMTI) and is currently the editor of Junior Mathematician. He has authored several books and written textbooks for NCERT and Tamil Nadu State Board. He may be contacted at athmaramanr@yahoo.com