The Conjecturing CLASSROOM

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I first came across the idea of a conjecturing classroom during my teacher training (PGCE) in the UK. We were watching a video clip of 6 and 7 year old students in a mathematics class. The classroom looked ordinary, children sitting in rows, a bit of chatting and minor disruptions, books and pencils and erasers strewn across desks, the teacher at the front without any spectacular resources, holding an unfussy whiteboard pen. Nowhere were the exciting new ideas I had heard during my course thus far, such as creative ways of organizing children and resources, the magic of group work, the use of manipulatives, etc. It looked like a mundane, regular class. Then the teacher began with this: "Who can remember Lucy's conjecture from yesterday?" Several hands sprang up and the teacher wrote a version of what they were saying, on the board, in quotations: "Every number can be written as the sum of consecutive numbers" and above that she wrote "Lucy's conjecture."

I was blown away. The language used by the teacher and these 6 and 7 year olds was nothing like I imagined in a mathematics classroom. They talked about how to 'test the conjecture' and 'strategies' for how to 'prove or disprove' it. As the class progressed, the flow was often interrupted by the teacher adding other students' related conjectures: "It only works for positive numbers" or "Every number can be written as a multiplication of other numbers." Every time she would write above the statement "Eli's conjecture" or "Kian's conjecture." There were frequent shouts such as, "I've disproved it – doesn't work for 6" and then another child might say after a few minutes "It does!" and they would rush to compare their findings. As the lesson came to a close, a few children had noticed that the conjecture seemed to be

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true for all odd numbers, and were even close to a proof of sorts, which one of them tried to explain on the board. And that was it.

While reflecting on the lesson, I realised what had appealed to me was the agency of the children. It was by any standard quite chaotic and even a bit noisy, but it did seem, at least to a bright-eyed new teacher watching from the safe distance of a screen, that the children were all involved, and they seemed to be involved in creating mathematics. They also seemed to be comfortable with trying new things, without being sure of where they were heading exactly. And, I thought, they felt important, their conjectures were being named after them, others in the class were giving those conjectures due consideration. They really looked like a bunch of mini mathematicians, another word that the teacher used often to describe her students or to encourage them – "How would a good mathematician record his/her work?"

I knew then this was the kind of classroom I wanted, and I began in earnest when I had my own class, armed with carefully thought out lesson plans full of investigations and open-ended tasks. Of course, it was a disaster. It would take me a long time to realize how much thought and work that teacher must have had to put in to get the conjecturing classroom of my dreams. She seemed all laidback and easy-going, but in fact she was doing a huge amount; it was just all so subtle.

I believe now that in order to create a conjecturing classroom, I, the teacher must be fully invested in this. If I am, then I will value a thoughtful conjecture as much as the one I know to be true. I must be equally joyous when a conjecture is proven or disproven. I must be willing to let students go off in the direction they have become interested in, even if it is different from what the rest of the class are doing. To have a conjecturing classroom is to value curiosity and allow children to pursue their curious questions to the full. Of course, not every lesson can or even should be like this; there are times when I need to guide children more pointedly, for the sake of the curriculum or to prepare them for a whole new concept. However I want my students to feel completely comfortable and at ease and full of questions when exploring mathematics, and for that there needs to be a lot of opportunity for conjecturing. Also it is necessary to have a set of guidelines in the class. The tasks that promote conjecturing require a fair amount of independent work by students, and having clear guidelines helps them stay on task. The guidelines should also address respecting fellow classmates. Children are all on board and agree with guidelines like "always listen when someone is talking/explaining" and "allow someone to finish an explanation;" if you think someone is wrong, use respectful language like "I think there might be an error in your working" or "I have noticed that what you say doesn't work for such and such, I think it is because such and such." In the heat of the moment, when someone shouts out "you're totally wrong!" with much glee, a quick reminder of the guidelines and children are more than willing to comply. Particularly if they were all part of and agreed with the class guidelines at the start of the school year.

I began teaching the 7th standard in Rishi Valley School last year. Here is a brief account of how the year progressed, with snippets from three lessons, in pursuit of a conjecturing classroom. I was teaching both sections of class 7 - class 7A and class 7B – in parallel, often teaching the same lesson back-to-back. The accounts are an amalgamation of things that happened in both classes, although in fact very often the lessons unfolded in similar ways. Hence I will talk about both 7A and 7B as one class.

Lesson 1: Hiccup Numbers

The very first lesson I had with them was an investigation on what I called 'hiccup numbers.' It turned out that some of them had worked on this investigation, although in a different way, the previous year, but it was still a good introduction to conjecturing. I told the class to choose a three-digit number. I then introduced a new (and made up) verb 'to hiccup'. I illustrated this with an example: 142 'hiccupped' is 142142. I asked them to hiccup their chosen three-digit number and then divide it by 13, then divide the answer by 11, and finally divide that answer by 7. Their little heads were all bent over in serious concentration, furiously applying the long division algorithm that most seemed to be fairly comfortable with (I had a chance to go around and notice and help those who were struggling). Soon enough, one little head looked up with an expression of surprised elation. And a few others bobbed up too. They had remembered my instructions while setting our class guidelines, to never shout out the answer or "I'm done" or "I've got it" but to catch my eye and show me a thumbs-up sign to let me know they were done, without putting off the rest of the class. These little rituals are important in a conjecturing classroom, because the aim is for the entire class to be conjecturing and to do that, everyone in the class must keep to a minimum the usual habits in a classroom that might be demoralizing for others. I went over and they exclaimed in excited whispers that they had got back their original number. "Hmm interesting...try another?" I offered. This task led very quickly to one of the students conjecturing "it always works." I insisted on a well-worded conjecture that had clarity and was not superfluous and after several attempts and with the help of others in the class we had a conjecture: "A three digit number that is hiccupped will give back the original number when divided by 13, 11 and 7 successively." I wrote this on the board and above it I wrote "Ajay's conjecture." I gave a little speech about conjectures and proofs and how the aim of a mathematician, after noticing patterns and formulating a conjecture is to prove or disprove it. These 11-year olds scoffed and laughed a bit nervously at this prosaic and grand language and my referring to them as mathematicians, but I didn't laugh. I was deadly serious. The children took up this language themselves remarkably quickly!

[A little aside, the act of naming a conjecture after the student might seem gimmicky, but I really think this is a wonderful thing to do, to hand ownership of the lesson and the maths to students. After initial excitement over 'their' conjecture at the beginning of the year, students got used to it, and there were delightful instances of students referring to their classmates' conjecture from a previous lesson, overheard talk of 'so and so's conjecture' on the way to lunch, a child excitedly telling me they had disproved their own conjecture. It did seem to me that students felt less proud of having their conjectures named after them, as the novelty wore off, but what remained was a sense of courage for students to create their own ideas.]

Ajay's conjecture was quickly verified by others, one or two children said "it doesn't work for such and such number" then others would dive-in to check that it did indeed work for that number. A few trivial variations of the conjecture were introduced, for example "it doesn't matter in which order you do the divisions." Many students were able to explain why the order didn't matter. Many then also noticed that the divisor was in fact the number 1001 and a few went on to appreciate how this magic trick worked and so proved the conjecture. A new conjecture, "the hiccup number for a 4-digit number is 10001" was proposed.

The mathematical content in this lesson was well within the grasp of students in this class, but my aim for the lesson was to introduce a behaviour – a conjecturing behaviour. The lesson wasn't about adding to their mathematical knowledge. It did, however, prove to be a good exercise to practice long division, which some students were struggling with, and this self-checking task gave them a lot of 'practice with purpose.' I liked this task because although Ajay was the first to articulate his conjecture, nearly everyone in the class had come to the same conclusion or rather suspicion that this would always happen. It was good for their self-esteem and confidence – they could make conjectures.

Throughout the year, I used the magnificent site nrich (www.nrich.maths.org) and the curriculum mapping on it to delve into tasks where children could explore mathematics related to the topic we were doing. Here are a few examples.

1. A few investigations that led to rich discussions, conjecturing and playing around with expressions:
   a. Perimeter Expressions (https://nrich.maths.org/7283);
   b. Number Pyramids (https://nrich.maths.org/2281);
   c. Always a Multiple (https://nrich.maths.org/7208).
2. Some investigations which were great at the end of the year after introducing quadratics:
   a. Multiplication Square (https://nrich.maths.org/2821);
   b. Always Perfect (https://nrich.maths.org/2034).
3. Investigations that proved very useful in the teaching of geometry:
   a. Changing Areas, Changing Volumes (https://nrich.maths.org/7535);
   b. Warmsnug Double Glazing (https://nrich.maths.org/4889);
   c. Painted Cube (https://nrich.maths.org/2322); this is a classic and remains one of my favorites; it encouraged much discussion and conjecturing on area and volume;
   d. Cyclic Quadrilaterals (https://nrich.maths.org/6624);
   e. Tilted Squares (https://nrich.maths.org/2293) (more on this one later).

The last two investigations (3d and 3e) were particularly important in the experience of turning conjectures into proofs.

Lesson 2: A Silent Lesson

In the middle of the school year, after a fair number of these tasks and many conjectures, I did a 'silent lesson.' A silent lesson is another way of getting students to predict what is coming and then articulate it with words and algebra (no speaking allowed) and although the conjectures from this lesson aren't wordy, students are encouraged to test others' theories or formulae as well as make their own. I will describe what happened in the lesson: I entered the class and instead of doing my usual countdown to get the class quiet, I stood silently, with a calm and composed face, not revealing any emotion, in front of the class, looking down at the children with a beatific smile. "Shall we get our books, *Akka*?" and all other questions were met with a smile and a finger to my lips. The class fell silent as well, any chatty stragglers were reprimanded by other students and all I had to do was to wait patiently. When I had every single eye focused on me, wondering what on earth I was up to, I put my finger to my lips again, held up my piece of chalk, and made a slow and dramatic movement to the board. On it I wrote, slowly and deliberately, pausing to think after each arrow:

I looked around happily at the class, then wrote:

$1 \rightarrow 7$

$2 \rightarrow 12$

$3 \rightarrow ?$

I looked puzzled then, and offered up my chalk to the class. Someone asked a question and I dramatically put my finger to my lips. A hand went up and I gave the child my chalk. He wrote 17 where the question mark was. I walked slowly to the board, and drew a smiley face next to the 17. The class laughed and I immediately put my finger to my lips. I then wrote:

4 → ?

A different child wrote 22. I drew a smiley face and after going through 5, 6 and 7, with nearly all hands up, I wrote:

37 → ?

Until now, children were noticing how the pattern was changing horizontally (add 5 each time), I wanted them to think now about the formula or general term applied vertically. After a few incorrect answers and sad faces next to these, a few children had worked out the formula and were beginning to get the answers when I put down different numbers. I then drew a box on the right hand corner of the board and wrote: "Give hints to your fellow classmates." With some of these hints more children felt confident about the formula and then I wrote:

n → ?

We carried on in this vein, with the formulae getting more and more complex. Towards the end of the lesson, after finding a particularly tricky formula involving squares, I held out the chalk and when a student came to tentatively take it, I went and sat in her place, thus indicating she could make up a formula!

The reason for the silence is (apart from the novelty of it) to focus children's attention on the pattern. Although the entire class was conducted in absolute silence, the children were exhausted at the end, because they were constantly testing their and other children's formulae, constantly adapting their idea with every sad face. The aim of the lesson was, apart from conjecturing, to gain fluency in writing algebraic expressions.

Lesson 3: Tilted Squares

A third lesson I will briefly describe is one we did towards the end of the year, an investigation that leads to the discovery of the Pythagoras theorem. On nrich it is called Tilted Squares (https://nrich.maths.org/2293). It is a very rich investigation, but too much to describe in detail here. The nrich site has a video of it being introduced that is enlightening to watch.

In the course of working on the task, students made up their own notation to describe different types of tilted squares. For example, '3 ↑ 1 →' meant "3 up and 1 across" or a tilted square that can be drawn on dotty paper by going 3 dots up and 1 across from each corner; another students' notation to describe the same square was + + + –. Students made conjectures about ways to find the areas of different sizes or 'tilts' of squares (some looking at the 'straight up' square around the tilted one and subtracting triangles and others looking at the square inside the tilted square and adding triangles). In the course of the lesson they agreed, after some whole-class sharing of strategies, that a table would be an efficient way to keep track of any emerging patterns, and soon the board was full of conjectures! When students finally used their recently honed skills of expanding brackets to come up with the Pythagoras' theorem, there was a real sense of achievement. Many students came to the theorem independently, at different times, and using varied methods, some with the help of fellow classmates or with some guidance from me. When I revealed to them that this conjecture/theorem already had a name and was one of the most famous ones in mathematics, they were having none of it. That one belonged to them, if anything it was, and always would be to them, 7A's Theorem.

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